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MODELLING OF SOCIAL-ECONOMIC SYSTEMS USING OF MULTIDIMENSIONAL STATISTICAL METHODS

Introduction.

The basic idea of simulation modelling is creation of abstract model of the real object or system being investigated, which reflects the basic characteristics of the object. The application of imitation modelling is connected with the fact that frequently it is not possible to provide a definite description of the behaviour of the economic system being investigated. When investigating the dynamic behaviour of the economic system, i.e. by making definite changes of parameters of the system under investigation, researchers frequently observe the existence of incidental factors affecting the character of behaviour of the system. After establishing the character of behaviour of factors (incl. also incidental) describing the characteristics of the system being researched, it is possible to undertake its imitation modelling.

Parametric methods. In most cases parametric methods of modelling are used to model economic problems, i.e., assuming that the laws of distribution of incidental values characterising the economic process are known. Application of parametric methods of research of economic systems is rather well described in special literature; therefore we find it more reasonable to focus on the possibilities of the application of non-parametric methods of research in imitation modelling of economic systems.

Non-parametric methods. Recently, to model the behaviour of economic systems, wide use is made of non-parametric methods, namely, the methods of local regression. These methods do not impose initial restrictions on the functional type of regression and thus allow determining the regressive procedure based on the changing data. In such a way, by means of the non-parametric approach it is possible to avoid specification (establishment of parameters of the system being investigated) of the model, which is typically encountered in the parametric approach. Thus the non-parametric method allows a wide variety of types of non-linear behaviour of the system, which are most frequently observed in the behaviour of real economic systems. Figure 1 shows how, derived from the block charts of distribution of two incidental values – factors X_1 and X_2, it is possible to realise nonparametric method of modelling for creating a bivariate common distribution, considering the dependence between the factors.

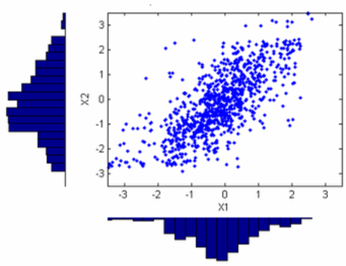

Figure 1: Example illustrating the process of modelling of bivariate incidental value, based on nonparametric evaluation of their distribution – the histograms

Technique of non-parametric modelling

The technique of non-parametric modelling is sufficiently thoroughly described in many books and articles devoted to such a mathematical object as copula. In the common case, distribution of each incidental value may be represented by means of a non-parametric method – a block chart (Figure 2).

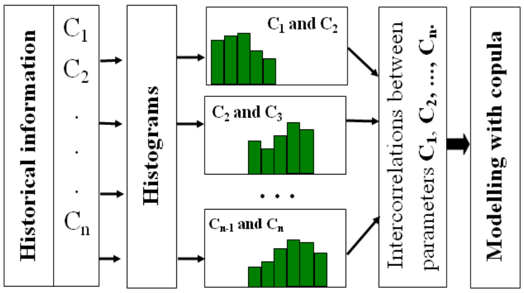

Figure 2: The nonparametric method of modelling with copula

Copulas. In the real world, there is often a non-linear dependence between different variables and correlation cannot be an appropriate measure of co-dependency. Therefore linear Spearmen's correlation coefficient is a limited measure of dependence. It is not surprising that alternative methods (the copula method) for capturing co-dependency have been considered. The concept of copulas comes from Sklar in 1959. In rough terms, a copula is a function with certain special properties.

$$C:[0,1]^n \rightarrow [0,1]. \tag{1}$$

Definition of a two-dimensional copula. A two-dimensional copula is a two-dimensional distribution function C with uniformly distributed marginals U(0,1) on [0,1]. Thus a copula is a function $C:\ [0,1]^2 \rightarrow [0,1]$ satisfying the following three properties (conditions):

1. For every $u, v \in [0,1]$: $C(u,0) = C(0,v) = 0$, $C(u,1) = u$ and $C(1,v) = v$.
2. $C(u,v)$ is increasing in u and v.
3. For every $u_1, u_2, v_1, v_2 \in [0,1]$ with $u_1 \le u_2$ and $v_1 \le v_2$ we have:

$C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1) \ge 0$.

Condition 1 provides the restriction for the support of the variables and the marginal uniform distribution. Conditions 2 and 3 correspond to the existence of a nonnegative "density" function. The most useful results of copula theory are Sklar's theorem.

Sklar's theorem - copula's first definition. Let F be a joint multivariate distribution with marginals F_1 and F_2. Then, for any x_1, x_2 there exists a copula C such that

$$F(x_1,x_2) = C(F_1(x_1), F_2(x_2)). \tag{2}$$

Furthermore, if marginals F_1 and F_2 are continuous, the copula C is unique. Conversely, if F_1 and F_2 are marginal distributions and C is a copula, then the function F defined by $C(F_1(x_1), F_2(x_2))$ is a joint distribution function with marginals F_1 and F_2. If we have a random vector $X = (X_1, X_2)$ the copula of their joint distribution function may be extracted from equation (2):

$$C(u_1,u_2) = F(F_1^{-1}(u_1), F_2^{-1}(u_2)), \tag{3}$$

where the F_1^{-1}, F_2^{-1} are the quantile functions of the margins. In most financial cases we can effectively use Archimedean copulas. The Archimedean copulas provide analytical tractability and a large spectrum of different dependence measure. As an example, we shall illustrate one of the most popular Archimedean copulas – the Clayton copula (Figure 3).

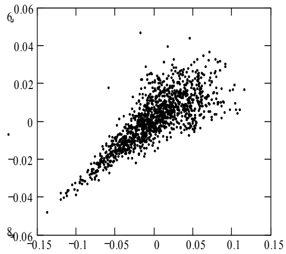

Figure 3: Clayton copula

The Clayton copula. Clayton copula is $C(u_1,u_2)=\left(u_1^{-\theta}+u_2^{-\theta}-1\right)^{-1/\theta}$. Generator for Clayton copula $\varphi(t)=\frac{t^{-\theta}-1}{\theta}$, $\theta\in(0,\infty)$. Kendall's tau can be defined by equation $\tau(\theta)=\frac{\theta}{2+\theta}$.

Example of using copula method for modeling in insurance for agriculture

The example shows the possibilities:
- to establish the insurance coverage for cereal sowings insurance process;
- to evaluate insurance tariffs and the insurance premium;
- to evaluate the dependence structure between the price and yield risks.

Let us consider the modelling scheme of the agricultural insurance fund, which later will allow us to model the process of developing the model and to establish the minimum amount of the insurance fund U (without a state subsidy). The minimum fund amount U guarantees that with certainty γ agricultural losses will be compensated. For modelling the insurance fund, we will use the simplest individual risk modelling scheme. Let us assume that the whole farm insurance fund is satisfactory, given the following conditions:
- the number of registered farms in the fund is constant;
- risks of individual farms are independent;
- payment of premiums is effected at the beginning of the period.

The loss distribution function is equal for all farms.

Let us designate that:
n – number of agreements in the fund;
j – ordinal number of the farm;
p – probability of setting in of the insurance event;
Y_j – possible losses of the farm j. Value Y_j has probability distribution function F(x);
X_j – satisfied loss of the farm j. $Xj = Ind_j*Y_j$;
Ind_j - binary index of the insurance event of the farm j.

By using variable Ind, we can calculate total number N of farms incurring losses:

$$N=\sum_{j=1}^{n} Ind_j \qquad (4)$$

Total amount of losses is:

$$Z=X_1+X_2+\ldots+X_n \qquad (5)$$

or by using indices of setting in of the events:

$$Z=Ind_1\cdot Y_1+Ind_2\cdot Y_2+\ldots+Ind_nY_n=\sum_{j=1}^{n} Ind_j\cdot Y_j \qquad (6)$$

Figure 4 shows that total loss Z is formed in n farms during one time period.

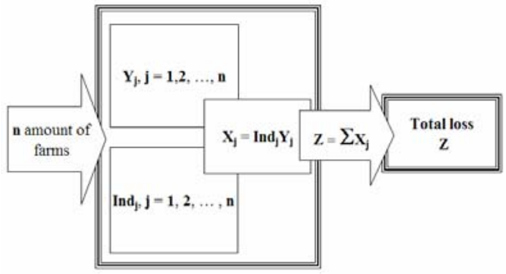

Figure 4: Illustration of process of total loss formation

We are to compensate losses to farms with a certainty γ and are to ensure the required operation of the fund with cash funds L. It means that the amount of the fund after compensations must be positive with a certainty $P(U - Z \geq 0) = \gamma$. The degree of risk of the insurance fund can be established by the variation coefficient:

$$K_{\mathrm{var}}(Z) = \frac{\sigma(Z)}{E(Z)} = \frac{\sqrt{D(Z)}}{E(Z)} \tag{7}$$

where $\sigma(Z)$ - standard deviation from the amount Z (standard error);
E(Z) - mathematical expectation of value Z, which in practice is measured with average value of Z;
D(Z) - variation of value Z.

Yield risks $X_1, X_2, \ldots, X_n$ can be modelled by a family of Beta distributions, whereas price shocks can be modelled by log-normal distributions:

$$f(x) = \frac{x^{n1-1}(1-x)^{n2-1}}{\int_0^1 u^{n1-1}(1-u)^{n2-1}\,du} \tag{8}$$

with parameters n_1 and n_2 in programme MathCad:

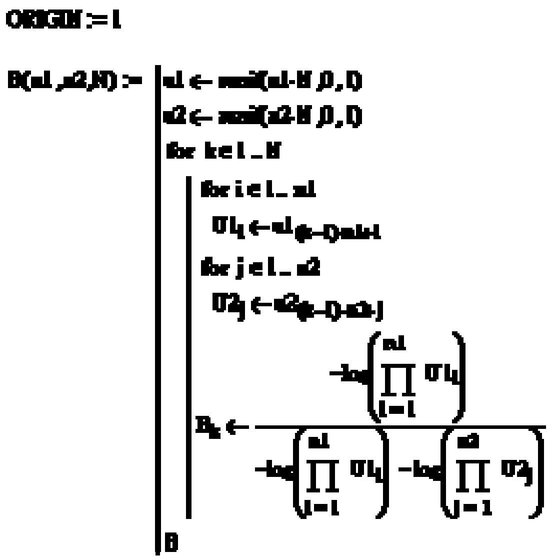

Figure 5: Illustration of random number generation in programme MathCad

The amount of the insurance coverage in cereal sowings insurance depends on the average amount of crop received by years, in which no relevant losses took place. The results of modelling without price risk show that very often variation coefficient of insurance fund K_{var} fluctuates within the range from 20% to 40%, which testifies to the fact that insurance fund, is often not so stable and additional financing is required from the state.

Conclusion

The application of the Monte-Carlo statistical method using copula is sufficiently simple method and frequently allows avoiding from complicated theoretical calculations as well as allows obtaining sufficiently accurate practical results to take appropriate decisions on insurance parameters. In most cases statistical distributions of the parameters describing applied problems aren't normally distributed. Therefore multidimensional copula using allows investigating social and economic problems. Results of modelling using copula, show that estimation of parameters of functioning of social and economic systems are more exact. The example described in the paper shows that the expenses on insurance is possible to reduce to 5-7 %. Therefore statistical modelling with copula method are important for carrying out of the best risk-management and reduction of losses of manufacturers of agricultural production from various risks of manufacture of a crop.

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