

# ANALYTICAL METHOD OF CALCULATION OF CONVEYOR LINES IN MARINE TRANSPORT SYSTEMS

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**Setting the problem.** At modern manufacturing enterprises with a flow-based method of production organization, the technological routes for manufacturing products contain  $10^2$  -  $10^3$  technological operations in the inter-operational reserves of which there are thousands of subjects of labor in unfinished production (work-in-process) [3]. Among the variety of models used for designing enterprise management systems with a streamlined method of production organization, PDE-models, which are used to describe the production systems with a flow type of production organization, have a distinct advantage over other models in those cases when the production functions in a non-stationary mode. The production management concepts are implemented for production systems using PDE-models by regulating the input flow of raw materials  $\Phi(t)$  and materials and the processing power of the subjects of labour  $[\chi]_{1\psi}(t,S)$  at each technological operation. It is assumed that the production system is ideal, that is, there is no loss of subjects of labour as a result of the production process (there are no defective products). The system of equations determining the behavior of the parameters of the production line in a one-stage description has the following form [4]:

$$\frac{\partial[\chi]_0(t,S)}{\partial t} - \frac{\partial[\chi]_1(t,S)}{\partial S} = [\chi]_{1\psi}(t,S), \quad (1)$$

under the initial condition

$$[\chi]_0(t_0,S) = \Psi(S), \quad (3)$$

under the boundary condition

$$[\chi]_0(t,S_0) = \Phi(t), \quad (4)$$

where  $[\chi]_0(t,S)$  is the density of the distribution of subjects of labour in unfinished production by technological positions;  $[\chi]_1(t,S)$  is the rate of processing of subjects of labor along technological positions at the time point  $t$ .

The position of the subject of labour in the technological route is characterized by a coordinate  $S \in [0; S_d]$ . When using the coordinates of the value space, the constant  $S_d$  corresponds to the cost of production. In a number of works, as a coordinate determining the position of the object, a dimensionless quantity  $\xi \in [0; 1]$ , the value of which gives an idea about the degree of processing of the subject of labour;  $\xi = 0$  corresponds to the initial stage of processing the subjects of labor;  $\xi = 1$  corresponds to the final stage of subjects of labor processing (the subject of labor turns into a finished product). The initial condition (3) determines the number of subjects of labor at a time point  $t_0$  at each technological operation. The solution to the system of equations (1)–(4) makes it possible to calculate the parameters of the production line without branching. At the same time, most enterprises have technological processes, where the movement of labour subjects is carried out both sequentially and in parallel. The flow lines are a part of the technological routes that can be connected to each other and diverge. The question of describing branched flow lines, even in the simplest form, remains relevant.

**The analysis of the recent research and identification of the previously unsolved parts of the general problem of designing branched conveyor lines.** The solution to the system of equations (1) - (4) for a constant speed of movement of subjects of labour along the production line or along its individual part is presented in, [4]. However, a detailed analysis of the received solution is not given. At the same time, the calculation of the state parameters for conveyor lines is an important production task that requires the use of new methods of solution.

Despite quite a large number of publications devoted to the design of control systems for production lines using PDE-models, the analysis of the conveyor line model has not received sufficient attention. The purpose of this article is to construct an analytical solution to the system of equations (1) - (4) for the case of movement of subjects of labour along the connecting technological routes with a conveyor way of organizing production for each of them. An important and specific task, along with the analysis of the obtained solution, is to demonstrate the dependence of the type of solution on the type of initial and boundary conditions that determine conveyor line functioning. The task of determining the state of the parameters of the conveyor line for an arbitrary moment of time, as well as the task of calculating the duration of the production cycle for manufacturing articles deserve attention as a result of solving the system of equations (1) - (4).

**Main material presentation.** Let us consider the process of manufacturing products, which contains two technological routes of a conveyor type, united into one. We choose the lower conveyor line as the main conveyor line, and the top line as the auxiliary conveyor line. The auxiliary conveyor line is used in a number of cases, one of which allows increasing the productivity of output for the main conveyor line. As a rule, the main line contains a critical route of product manufacturing.

The subjects of labour with the  $(m-1)$ -technological operation of the main line and the  $M$ -technological operation of the complementary auxiliary line get onto the  $m$ - technological operation of the main line. The system of equations describing the interaction of the joint lines has the form for the auxiliary production line:

$$\frac{\partial [\chi_1]_0(t, S_1)}{\partial t} \frac{\partial [\chi_1]_1(t, S_1)}{\partial S_1} \quad (5)$$

$$[\chi_1]_1(t, S_1) = a_1 [\chi_2]_0(t, S_1),$$

under initial

$$[\chi_1]_0(t_{01}, S_1) = \Psi_1(S_1),$$

or boundary conditions

$$[\chi_1]_0(t, S_{01}) = \Phi_1(t),$$

and for the leading production line:

$$\frac{\partial [\chi_2]_0(t, S_2)}{\partial t} \frac{\partial [\chi_2]_1(t, S_2)}{\partial S_2} (S_2 - S_{02}) [\chi_1]_{1d_1} \quad (6) \quad [\chi_2]_1(t, S_2) = a_2 [\chi_2]_0(t, S_2), \quad (7)$$

under initial

$$[\chi_2]_0(t_{02}, S_2) = \Psi_2(S_2),$$

or boundary conditions

$$[\chi_2]_0(t, S_{02}) = \Phi_2(t),$$

where  $S_k$  ( $0 \leq S_k \leq S_{dk}$ ) are the coordinates of technological positions for the auxiliary ( $k=1$ ) and the main ( $k=2$ ) lines;  $[\chi_k]_0(t, S_k)$ ,  $[\chi_k]_1(t, S_k)$ – the value of inter-operational reserves and the rate of processing of subjects of labour at a technological position that is characterized by the value  $S_k$  for the auxiliary ( $k=1$ ) and the main ( $k=2$ ) lines.

The technological position with the coordinate  $S_k = S_{dk}$  corresponds to the degree of readiness of the subject of labor, that is, the state to which the subject of labor must correspond at the exit of the conveyor line in accordance with production and technological documentation.  $t_{0k}$ ,  $S_{0k}$ – the moment of time and the coordinate of the technological position for which the initial or boundary conditions are determined, respectively. Parameters  $[\chi_k]_0(t, S_k)$  и  $[\chi_k]_1(t, S_k)$  are related to each other by a coefficient  $a_k$ , that determines the conveyor line speed. For conveyors with the constant speed of movement  $a_k = \text{const}$ . The right-hand side of the equation  $(6\delta(S_2 - S_{02})[\chi_1]_1(t, S_{d1}))$  is determined by the receipt of the subjects of labour for the technological position of the main line with the coordinate  $S_2 = S_{02}$  from the auxiliary line to the main line with the rate  $[\chi_1]_1(t, S_{d1})$ , where  $\delta(x)$  is the delta function

$$\int_{-\infty}^{\infty} \delta(S_2 - S_{02}) dS_2 = 1, \quad S_{02} \in [0; S_{d2}].$$

Introducing dimensionless variables, we write the balance equation (6) for the main line in the dimensionless form

$$\frac{\partial [\theta_2]_0(\tau, \xi_2)}{\partial \tau} + \frac{\partial [\theta_2]_0(\tau, \xi_2)}{\partial \xi_2} (\xi_2 - \xi_{02})_1 [\theta_1]_0 \quad (8)$$

$$[\theta_2]_0(\tau_{02}, \xi_2) = \psi_2(\xi_2), \quad (9)$$

$$[\theta_2]_0(\tau, \xi_{02}) = \vartheta_2(\tau). \quad (10)$$

The function  $[\theta_1]_0(\tau, 1)$  is unknown and is determined by solving the balance equation (5) for the auxiliary line that can be written down in a dimensionless form:

$$\frac{\partial [\theta_1]_0(\tau, \xi_1)}{\partial \tau} + \frac{\partial [\theta_1]_0(\tau, \xi_1)}{\partial \xi_1}, \quad g_{01} = g_1 \frac{S_{d2}}{S_{d1}}, \quad (11)$$

$$[\theta_1]_0(\tau_{01}, \xi_1) = \psi_1(\xi_1), \quad (12)$$

$$[\theta_1]_0(\tau, \xi_{01}) = \vartheta_1(\tau). \quad (13)$$

By finding the characteristics that corresponds to a system of equations (11)–(13), making the transformations and after the substitution that gives the identical equalities to the initial and boundary conditions, we receive a closed system of equations. The state of the flow parameters of the leading conveyor line is determined by the state of the parameters of the auxiliary conveyor line. The system of characteristics corresponds to a system of equations (8)–(10):

$$\frac{d\xi_2}{d\tau} = g_2, \quad (14)$$

$$g_2 \frac{d[\theta_2]_0(\tau, \xi_2)}{d\xi_2} = \delta(\xi_2 - \xi_{02}) g_1 [\theta_1]_0(\tau, 1), \quad (15)$$

$$[\theta_2]_0(\tau_{02}, \xi_2) = \psi_2(\xi_2).$$

After the transformations we obtain the solution  $[\theta_2]_0(\tau, \xi_2)$  (22) for the function:

$$[\theta_1]_0(\tau, 0) = \vartheta_1(\tau) = 1; \quad [\theta_1]_0(0, \xi_1) = \psi_1(\xi_1) = 0; \quad (16)$$

$$[\theta_2]_0(0, \xi_2) = \psi_2(\xi_2) = \frac{1}{3} (2 + \sin(2\pi\xi_2)) \quad (17)$$

with the source of receiving subjects of labour on the main line at the point  $\xi_{02} = 0$ .

Conditions (16) for the auxiliary conveyor line indicate that at the initial time point  $\tau = 0$  the auxiliary conveyor line was empty. At point of time  $\tau = 0$  the first technological position receives the subjects of labor with intensity  $g_{01}\vartheta_1(\tau)$ .

**Conclusions and further prospects for the development and improvement of PDE-models of production systems.** The received results of the research are fundamental for the development of control systems for production of a conveyor type, consisting of the main and a number of auxiliary lines. The dependence of the state of the inter-operational reserves of the main conveyor line on the state of the inter-operational reserves of auxiliary conveyor line is shown. The influence of initial and boundary conditions of the auxiliary line on the state parameters of the main conveyor line is considered. Except for the case considered in the present article, the boundary condition (4) can be claimed in the case when the model of the production line takes into account the offloading of the subjects of labor in the form of a semi-finished product from the technological position characterized by coordinate  $S_0$  or the presence of the factors which determine accounting the amount of defective products in the model. The presence of initial (3) and boundary (4) conditions leads to the interrupted character of the solution of the system of equations, which is characteristic for the behavior of the flow parameters of modern production conveyor lines.

The model of the production system of a conveyor type, in which auxiliary lines are used as the subjects of management for ensuring the required output of finished products by the enterprise, is of interest. The main line operates in a continuous mode, while auxiliary conveyor lines in the on / off mode, which allows to regulate the output of finished products.

It should be pointed out that the results of the work are of scientific and practical interest for designing the speed control systems for conveyor belt systems, which found widespread use in sea freight (Fig. 1, 2). An important result of this work is also the method of calculating the duration of the production cycle, based on the use of characteristic equation. This method makes it possible to construct the dependence of the length of production from the distribution of labour subjects along the conveyor line at the moment of time, which determines the arrival of the first subject of labour for processing at the first technological operation. Thus, the duration of the production cycle is not a constant value; it is determined by the load for operating production.

A further perspective for the development of the issue discussed in this paper is the construction of a conveyor line control system and the extension of the offered method for calculating conveyor lines for the system "main line - N-auxiliary lines".



**Fig. 1. Belt conveyors in Jiangsu Taicang Environmental Protection Power Plant**



**Fig. 2. Belt conveyors in Jiaying Port, Zhejiang**

### References

1. **Шахмейстер Л.Г.** Динамика грузопотоков и регулирование скорости ленточных конвейеров. / Л.Г.Шахмейстер, В.Г.Дмитриев, А.К. Лобачёва. М.: Недра, 1972. 173с.
2. **Пигнастый О. М.** Анализ моделей переходных управляемых производственных процессов / О. М. Пигнастый // Научные ведомости Белгородского государственного университета. Белгород: БГУ. 2015. № 35/1. С.133-144. <https://doi.org/10.5281/zenodo.2595561>.
3. **Pihnastyi O.M.** Statistical validity and derivation of balance equations for the two-level model of a production line // O.M.Pihnastyi // Eastern-European Journal of Enterprise

Technologies. Kharkiv: PC "TECHNOLOGY CENTER". - 2016. – vol 5. –№ 4 (83). - P. 17 – 22. <https://doi.org/10.15587/1729-4061.2016.81308>.

4. **Pihnastyi O.M.** Model of conveyer with the regulable speed /O.M. Pihnastyi, V.D. Khodusov // Bulletin of the South Ural State University. Ser.Mathematical Modelling, Programming & Computer Software (Bulletin SUSUMMCS), 2017, vol.10, no.4, P.64-77 <https://doi.org/10.14529/mmp170407>.