# MULTIPLE-MODEL DESCRIPTION AND ALGORITHMS OF SHIP-BUILDING MANUFACTORY SCHEDULING

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## Introduction

We present the new scheduling multiple-model description of complex technical-organizational system (CTOS) [1-2]. We implemented our results (dynamic models and algorithms of scheduling) for *ship-building manufactory which is interpreted as* a networked controlled system that is described through differential equations based on a *dynamic* description of the job execution. The job execution is characterized by (1) execution results (e.g., volume, time, etc.), (2) capacity consumption of the resources, and (3) CTOS flows resulting from the delivery to the customer.

We propose to use a two stage scheduling procedure in line with [3]. A job control model (M1) is first used to assign jobs to suppliers, and then a flow control model (M2) is used to schedule the processing of assigned orders subject to capacity restrictions of the production and transportation resources. The basic interaction of these two models is that after the solving the job control model, the found control variables are used in the constraints of the flow control model. In additional models of resource and channel control, the material supply to resources and its consumption as well as setup times are represented.

## Multiple-model description

### *1 A Dynamic Model of Job Control (model M1)*

We consider the mathematical model of job control. We denote the job state variable $x_{i\mu}^{(o)}$, where $(o)$ – indicates the relation to jobs (orders). The execution dynamics of the job $D_{\mu}^{(i)}$ can be expressed as (1).

$$\frac{dx_{i\mu}^{(o)}}{dt} = \dot{x}_{i\mu}^{(o)} = \sum_{j=1}^{n} \varepsilon_{ij}(t) u_{i\mu j}^{(o)} \tag{1}$$

where $\varepsilon_{ij}(t)$ is an element of the preset matrix time function of time-spatial constraints, $u_{i\mu j}^{(o)}(t)$ is a 0–1 assignment control variable.

Let us introduce equation (2) to assess the total resource availability time:

$$\dot{x}_{j}^{(o)} = \sum_{i=1}^{\bar{n}} \sum_{\substack{\eta=1 \\ \eta \neq i}}^{\bar{n}} \sum_{\mu=1}^{s_i} \sum_{\rho=1}^{p_i} (u_{i\mu j}^{(o)}) \tag{2}$$

Equation (2) represents resource utilization in job execution dynamics. The variable $x_{j}^{(o)}$ characterizes the total employment time of the *j*-supplier. The control actions are constrained as follows:

$$\sum_{i=1}^{\bar{n}} \sum_{\mu=1}^{s_i} u_{i\mu j}^{(o)}(t) \le 1, \forall j; \ \sum_{j=1}^{n} u_{i\mu j}^{(o)}(t) \le 1, \forall i, \forall \mu \tag{3}$$

$$\sum_{j=1}^{n} u_{i\mu j}^{(o)} [ \sum_{\alpha \in \Gamma_{i\mu_1}^{-}} (a_{i\alpha}^{(o)} - x_{i\alpha}^{(o)}) + \prod_{\beta \in \Gamma_{i\mu_2}^{-}} (a_{i\beta}^{(o)} - x_{i\beta}^{(o)}) ] = 0 \tag{4}$$

$$u_{i\mu j}^{(o)}(t) \in \{0,1\}; \tag{5}$$

where $\Gamma_{i\mu_1}^{-}$, $\Gamma_{i\mu_2}^{-}$ are the sets of job numbers which immediately precede the job $D_{\mu}^{(i)}$ subject to accomplishing of all the predecessor jobs or at least one of the jobs correspondingly, and

$a_{i\alpha}^{(o)}, a_{i\beta}^{(o)}$ are the planned lot-sizes. Constraint (3) refers to the allocation problem constraint according to the problem statement (i.e., only a single order can be processed at any time by the manufacturer). Constraint (4) determines the precedence relations, more over this constraint implies the blocking of operation $D_{\mu}^{(i)}$ until the previous operations $D_{\alpha}^{(i)}, D_{\beta}^{(i)}$ have been executed. If $u_{i\mu j}^{(o)}(t) = 1$, all the predecessor jobs of the operation $D_{\mu}^{(i)}$ have been executed. Note that these constraints are identical to those in MP models.

***Corollary 1.*** *The analysis of constraints (4) shows that control* $\mathbf{u}(t)$ *is switching on only when the necessary predecessor operations have been executed.* $\sum_{j=1}^{n} u_{i\mu_j}^{(o)} \sum_{\alpha \in \Gamma_{i\mu 1}^{-}} (a_{i\alpha}^{(o)} - x_{i\alpha}^{(o)}(t)) = 0$ *guarantees the total processing of all the predecessor operations, and* $\sum_{j=1}^{n} u_{i\mu_j}^{(o)} \prod_{\beta \in \Gamma_{i\mu 2}^{-}} (a_{i\beta}^{(o)} - x_{i\beta}^{(o)}) = 0$ *of at least one of the predecessor operations.*

According to equation (5), controls contain the values of the *Boolean variables*. In order to assess the results of job execution, we define the following initial and end conditions at the moments $t = T_0$, $t = T_f$:

$$x_{i\mu}^{(o)}(T_0) = 0; \; x_{i\mu}^{(o)}(T_f) = a_{i\mu}^{(o)}; \; . \qquad (6)$$

Conditions (6) reflect the desired end state. The right parts of equations are predetermined at the planning stage subject to the lot-sizes of each job.

According to the problem statement, let us introduce the following performance indicators (7)–(9):

$$J_1^{(o)} = \frac{1}{2} \sum_{i=1}^{\bar{n}} \sum_{\mu=1}^{s_i} [(a_{i\mu}^{(o)} - x_{i\mu}^{(o)}(T_f))^2], \qquad (7)$$

$$J_2^{(o)} = \sum_{i=1}^{\bar{n}} \sum_{\mu=1}^{s_i} \sum_{j=1}^{n} \int_{T_0}^{T_f} \alpha_{i\mu}^{(o)}(\tau) u_{i\mu j}^{(o)}(\tau) d\tau , \qquad (8)$$

$$J_3^{(o)} = \frac{1}{2} \sum_{j=1}^{n} (T - x_j^{(o)}(T_f))^2 \qquad (9)$$

.

The performance indicator (7) characterizes the accuracy of the end conditions' accomplishment, i.e. the service level of *ship-building manufactory*. The goal function (8) refers to the estimation of an job's execution time with regard to the planned supply terms and reflects the delivery reliability, i.e., the accomplishing the delivery to the fixed due dates. The functions $\alpha_{i\mu}^{(o)}(\tau)$ is assumed to be known characterizes the fulfilment of time conditions for different jobs and time points of the penalties increase due to breaking supply terms respectively. The indicator (10) estimates the equal resource utilization in the *ship-building manufactory*.

***2 A Dynamic Model of Flow Control (model M2)***

We consider the mathematical model of flow control in the form of equation (10):

$$\dot{x}_{i\mu j}^{(f)} = u_{i\mu j}^{(f)}, \; \dot{x}_{ij\eta\rho}^{(f)} = u_{ij\eta\rho}^{(f)} \qquad (10)$$

We denote the flow state variable $\dot{x}_{i\mu j}^{(f)}$, where $(f)$ indicates the relation of the variable *x* to flows.

The control actions are constrained by maximal capacities and intensities as follows:

$$\sum_{i=1}^{\bar{n}}\sum_{\mu=1}^{s_i} u_{i\mu j}^{(f)}(t) \le \tilde{\tilde{R}}_{1j}^{(f)}, \sum_{\rho=1}^{p_i} u_{ij\eta\rho}^{(f)}(t) \le \tilde{\tilde{R}}_{1j\eta}^{(f)}, \tag{11}$$

$$0 \le u_{i\mu j}^{(f)}(t) \le c_{i\mu j}^{(f)} \cdot u_{i\mu j}^{(o)}, 0 \le u_{ij\eta\rho}^{(f)}(t) \le c_{i\eta\rho j}^{(f)} \cdot u_{i\eta\rho j}^{(o)}. \tag{12}$$

where $\tilde{\tilde{R}}_{1j}^{(f)}$ is the total potential intensity of the resource $C^{(j)}$, $\tilde{\tilde{R}}_{1j\eta}^{(f)}$ is the maximal potential channel intensity to deliver products to the customer $\overline{B}^{(\eta)}$ of results of *ship-building manufactory*, $c_{i\mu j}^{(f)}$ is the maximal potential capacity of the resource $C^{(j)}$ for the job $D_{\mu}^{(i)}$, and $c_{i\eta\rho j}^{(f)}$ is the total potential capacity of the channel delivering the product flow $P_{<s_i,\rho>}^{(j,\eta)}$ of the job $D_{\mu}^{(i)}$ to the customer $\overline{B}^{(\eta)}$ of results of *ship-building manufactory*.

The end conditions are similar to those in (6) and subject to the units of processing time. The goal functional of the flow control model are defined in the form of equations (13) and (14):

$$J_1^{(f)} = \frac{1}{2}\sum_{i=1}^{\bar{n}}\sum_{\mu=1}^{s_i}\sum_{j=1}^{n}[(a_{i\mu j}^{(f)} - x_{i\mu}^{(f)}(T_f))^2 + \sum_{\substack{\eta=1\\ \eta\ne i}}^{\bar{n}}\sum_{\rho=1}^{p_i}(a_{ij\eta\rho}^{(f)} - x_{ij\eta\rho}^{(f)}(T_f))^2], \tag{13}$$

$$J_2^{(f)} = \frac{1}{2}\sum_{i=1}^{\bar{n}}\sum_{\mu=1}^{s_i}\sum_{j=1}^{n}\int_{T_0}^{T_f} \beta_{i\mu}^{(f)}(\tau) u_{i\mu j}^{(f)}(\tau) d\tau. \tag{14}$$

The economic meaning of these performance indicators correspond to equations (7) and (8). With the help of the weighting performance indicators, a general performance vector can be denoted as (15):

$$\mathbf{J}(\mathbf{x}(t), \mathbf{u}(t) = \left\| J_1^{(o)}, J_2^{(o)}, J_3^{(0)}, J_1^{(f)}, J_2^{(f)} \right\|^{\mathrm{T}}. \tag{15}$$

The partial indicators may be weighted depended on the planning goals and SC strategies. Original methods [1,2] have been used to transform the vector $\mathbf{J}$ to a scalar form $J_G$.

The job shop scheduling problem can be formulated as the following problem of OPC: this is necessary to find an allowable control $\mathbf{u}(t)$, $t \in (T_0, T_f]$ that ensures for the model (1)–(2), and (10) meeting the vector constraint functions $\mathbf{q}^{(1)}(\mathbf{x},\mathbf{u}) = 0$, $\mathbf{q}^{(2)}(\mathbf{x},\mathbf{u}) \le 0$ (3)–(5) and (10–11), and guides the dynamic system (i.e., job shop schedule) $\dot{\mathbf{x}} = \boldsymbol{\varphi}(t,\mathbf{x},\mathbf{u})$ from the initial state to the specified final state. If there are several allowable controls (schedules), then the best one (optimal) should be selected in order to maximize (minimize) $J_G$. In terms of optimal program control (OPC), the program control of job execution is at the same time the job shop schedule. We will refer to this problem of OPC as *PS*.

The formulated model is a linear non-stationary finite-dimensional controlled differential system with the convex area of admissible control. Note that the **boundary problem** *PS* is a standard OPC problem; see [4-6]. In fact, this model is linear in the state and control variables, and the objective is linear. The transfer of non-linearity to the constraint ensures convexity and allows to use interval constraints.

### Algorithm of ship-building manufactory

Necessary optimality conditions can be derived from maximum principle [4, 5]. Consider control system (16).

$$\dot{\mathbf{x}}(t) = f(t, \mathbf{x}(t), \mathbf{u}(t)),\ t_0 \le t \le t_f,\ \mathbf{x}(t_0) = \mathbf{x}_0,\ \mathbf{u}(t) \in U,\ \ J = F(\mathbf{x}(t_f)) \to \mathbf{min} \tag{16}$$

Let us introduce a scalar Hamiltonian function $H$ and conjunctive vector system $\boldsymbol{\psi} \in R^n$ in Eq. (17).

$$H(t,\mathbf{x}(t),\mathbf{u}(t),\psi(t)) = \psi^{\mathrm{T}}(t)\mathbf{f}(t,\mathbf{x}(t),\mathbf{u}(t)),$$

$$\dot{\psi}(t) = -\frac{\partial H}{\partial \mathbf{x}}(t,\mathbf{x}(t),\mathbf{u}(t),\psi(t)), \tag{17}$$

$$\psi(t_f) = -\left.\frac{\partial F(\mathbf{x}(t))}{\partial \mathbf{x}}\right|_{t=t_f}, \tag{18}$$

Conjunctive vector system plays the role of dual models in linear programming. Coefficients of the conjunctive systems can be interpreted as Lagrange multipliers. Under assumptions that $\mathbf{u}(t)$ is optimal control and $\mathbf{x}(t)$ and $\psi(t)$ are the trajectory and conjunctive system satisfying (17) and (18), the function $H(t,\mathbf{x}(t),\mathbf{u}(t),\psi(t))$ reaches its maximum for $\mathbf{x}(t)$ at the point $\mathbf{u}(t)$. Then Eq. (19) holds:

$$\mathbf{u} = \mathbf{u}(t,\mathbf{x}(t),\psi(t)) \tag{19}$$

Subsequently, Eq. (19) is brought into correspondence with (17) and (18). In the result, a two-point boundary problem for a system of ordinary differential equations in regard to $\mathbf{x}(t)$ and $\psi(t)$ is formed. The optimal solution is now bounded by this differential system. Note that Eq. (17)-(19) in general case provide only necessary conditions for optimal solution existence whereas for linear control systems these maximum principles provide both optimality and necessary conditions.

The basic peculiarity of the boundary problem considered is that the initial conditions for the conjunctive variables $\psi(t_0)$ are not given. At the same time, an optimal program control should be calculated subject to the boundary conditions (we omit their presentation and refer to paper [2,7]. To obtain the conjunctive system vector, the Krylov–Chernousko method of successive approximations for an optimal program control problem with a free right end which is based on the joint use of a modified successive approximation method [7] has been used.

**Step 1** An initial solution $\bar{\mathbf{u}}(t),\ t \in (t_0, t_f]$ (a feasible control, in other words, a feasible schedule) is selected and $r = 0$.

**Step 2** As a result of the dynamic model run, $\mathbf{x}^{(r)}(t)$ is received. Besides, if $t = t_f$ then the record value $J_G = J_G^{(r)}$ can be calculated. Then, the transversality conditions (18) are evaluated.

**Step 3** The conjugate system (17) is integrated subject to $\mathbf{u}(t) = \bar{\mathbf{u}}(t)$ and over the interval from $t = t_f$ to $t = t_0$. For the time $t = t_0$, the first approximation $\psi_i^{(r)}(t_0)$ is obtained as a result. Here, the iteration number $r = 0$ is completed.

**Step 4** From the time point $t = t_0$ onwards, the control $\mathbf{u}^{(r+1)}(t)$ is determined ($r = 0,1,2,\ldots$ denotes the number of the iteration). In parallel with the maximization of the Hamiltonian, the main system of equations and the conjugate one are integrated. The maximization involves the solution of several mathematical programming problems at each time point.

The advantage of method of successive approximations is that it allows to implement needle control variations subject to the whole area of feasible control actions subject to the given constraint system, i.e., the area of feasible schedules [8]. Another method of successive approximations advantage is that the search for an optimal control in each iteration is performed in the class of boundary (e.g., pointwise or relay) functions which correspond to the discrete nature of decision making in scheduling. Note that the method of successive approximations in its initial form has not guaranteed the convergence.

## Conclusions

Problems of *ship-building manufactory scheduling* may be challenged by high complexity, combination of continuous and discrete processes, integrated production and transportation operations as well as dynamics and resulting requirements for adaptability. A possibility to address these issues opens the embedding of OPC into *ship-building manufactory* scheduling and using its advantages in combination with advantages of mathematical programming (MP). Under the assumption that the introduction of the dynamic aspect of job arrival can have a significant impact on the solution procedure, this study presented a new original model for *ship-building manufactory* scheduling as OPC of job execution dynamics blended with combinatorial optimization and based on a natural dynamic decomposition of the scheduling problem and its solution with maximum principle in combination with MP.

The proposed substitution lets use fundamental scientific results of the OPC theory in *ship-building manufactory* scheduling.


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