Modeling complex "Sustainability" for the study of robust absolute stability of nonlinear impulsive control systems

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Abstract: This report examines the complex computer simulations, implemented in rapid application development environment Delphi. The system is designed for the study of robust absolute stability of one- and multidimensional nonlinear impulsive control system (NICS). The complex transfer functions are realized by converting to the p-z-and w-transform transfer function of the linear part of the pulse, obtaining the matrix transfer function to check the criterion for absolute stability of multidimensional NICS with interval coefficients. Implemented a graphical representation of the roots of the characteristic equation obtained in the complex plane, which allows us to estimate the degree of stability of the system. Entering the initial data is realized through a variety of forms, which are entered in the box corresponding to the coefficients of the transfer functions of the original system and other data.

Keywords: nonlinear pulse control system, absolute stability, robust stability, interval polynomial root locus, the criteria for absolute stability.

1 Introduction

At the present time in various sectors of the economy are widely used nonlinear automatic control systems whose dynamics are described by nonlinear differential equations of high order. Especially the rapid development of the theory of the study were NICS. Increasing requirements for dynamic precision autopilot, the complication of the mathematical description of designed control system causes an increase in cost and time of their design, resulting in intense development and introduction of computer-aided design (CAD), which provide for enhancing the quality of design time and reducing its cost. The availability of advanced application software of CAD depends on the efficiency of ACS. However, existing in this system nonlinearities make it difficult to check the robust absolute stability of NICS. Development focused on the use of computer methods of robust absolute stability on which the problem can be created - and object-oriented CAD software, is thus an urgent task.

2 Problem Formulations

Consider the class of NICS, described by the following equation

$$\vec{\sigma}[n] = \vec{f}[n] - \sum_{l=0}^{n} G[n-l]\vec{\Phi}(\vec{\sigma}[l], l), \quad n = 0,1,2,..., \qquad (1)$$

where - $\vec{\sigma}, \vec{f}, \vec{\Phi} - m -$dimensional vectors with errors, external influences and characteristics of the nonlinear element. The input process $\vec{f}[n]$ contains components $f_i[n]$ that are vanishing function of time, that is $\lim_{n\to\infty}\vec{f}[n]=\vec{0}$. The continuous part of the system is characterized by a matrix of impulse response functions $G[n]$ and the corresponding transition matrix processes $H[n]$, as well as the matrix of transfer functions $W(w)$, or the corresponding matrix frequency characteristics $W(j\nu)$, where - $\nu = tg\frac{\omega T_0}{2}$ – frequency. For the class $W(w)$ assumes a stable matrix, that is all its components have poles in the left half w.

The vector-valued function $\vec{\Phi}(\vec{\sigma}[n],n)$ belongs to a class of nonlinearities Φ_{rk}, if its components satisfy

$$0 \le r_{ii} \le \frac{\Phi_i(\sigma_i[n],n)}{\sigma_i[n]} \le k_{ii}, \forall \sigma_i[n]=0, k_{ii}>0; \tag{2}$$

$$\Phi_i(\sigma[n],n)=\Phi_i(\sigma_i[n],n); \Phi_i(0)=0.$$

If, in (2), we have the inequalities

$$0 \le r'_{ii} \le \frac{d\Phi_i(\sigma_i[n],n)}{d\sigma_i[n]} \le k'_{ii}, i=1,2,...,m, \tag{3}$$

that $\vec{\Phi}(\vec{\sigma}[n],n) \in \Phi'_{rk} \subseteq \Phi_{rk}$.

The problem of obtaining criteria for absolute stability of multidimensional NICS described by equation (1) with non-linearities in the class (2,3), further modification of the obtained algebraic criteria and bring them to a form convenient for the study of both absolute and robust absolute stability.

3 Problem Solution

3.1 A mathematical model study of absolute stability of NICS

In [1] it is shown that any system described by equation (1) with the nonlinearities of the class Φ_{0k}, it is absolutely stable with respect to the input, if the inequality

$$\pi(j\nu)=\Omega \bullet \{W(j\nu)+k^{-1}\}>0,\ \forall \nu \in [0,\infty]. \tag{4}$$

where $\pi(j\nu)$ – the hermitian matrix associated with the linear part and the other parameters of the system;

$\Omega \bullet$ - hermitian operator that performs the selection operation of the complex hermitian matrix;

$\bullet$ - matrix interface code.

If we let $\{W(j\nu)+k^{-1}\} - \Pi(j\nu)$, then (4), by virtue of the hermitian operator can be written as

$$\Pi(j\nu)+\Pi^T(-j\nu)>0, \forall \nu \in [0,\infty], \tag{5}$$

that is requires the positive definiteness of a hermitian matrix. The total matrix $\Pi(j\nu)+\Pi^T(-j\nu)$ is positive definite if all principal minors Δ_i (i = 1,2,...m) of its determinant are positive.

Absolute stability criteria for multidimensional nonlinear impulse automatic systems can be derived from (5), for which this inequality must be converted to the total matrix of the form.

$$\Pi(j\nu)+\Pi^T(-j\nu)=\begin{Vmatrix} \overline{\Pi}_{11}(j\nu) & \overline{\Pi}_{12}(j\nu) & ... & \overline{\Pi}_{1m}(j\nu) \\ \overline{\Pi}_{21}(j\nu) & \overline{\Pi}_{22}(j\nu) & ... & \overline{\Pi}_{2m}(j\nu) \\ ... & & ... & ... \\ \overline{\Pi}_{m1}(j\nu) & \overline{\Pi}_{m2}(j\nu) & ... & \overline{\Pi}_{mm}(j\nu) \end{Vmatrix} \tag{6}$$

where $\overline{\Pi}_{ii}=(W_{ii}(j\nu)+k_i^{-1}+W_{ii}(-j\nu)+k_i^{-1})$, $\overline{\Pi}_{ij}=(W_{ij}(j\nu)+W_{ji}(-j\nu))$;

$$W_{ij}(w)=\frac{\sum\limits_{l=0}^{n_{ij}} a_i w^{n-i}}{\sum\limits_{l=0}^{n_{ij}} b_i w^{n-i}} \quad . \tag{7}$$

For the criteria of absolute stability of multidimensional NICS necessary to carry out the computation of determinants of matrices of the form (6).

In the case m = 1, that is, for the one-dimensional system, we have

$$\begin{aligned}\Delta_1 &= \Pi_{11}(jv)+\Pi_{11}^T(-jv)=\Pi_{11}(jv)+\Pi_{11}(-jv)=\\ &=2k^{-1}+2\,\mathrm{Re}\,W_{11}(jv)>0,\quad \forall v\in[0,\infty]\end{aligned} \tag{8}$$

which leads to an analogue of the criterion Y. Tsypkin

$$\mathrm{Re}W(j\nu)+k^{-1}>0,\ \forall \nu\in[0,\infty]. \tag{9}$$

In the case m = 2, that is for the two-dimensional system, we have

$$\begin{aligned}\Delta_2 &= 4\,\mathrm{Re}\Pi_{11}(j\nu)\,\mathrm{Re}\Pi_{22}(j\nu)-(\Pi_{12}(j\nu)+\\ &+\Pi_{21}(-j\nu))(\Pi_{21}(j\nu)+\Pi_{12}(-j\nu))\end{aligned}$$

After transformation we obtain

$$\begin{aligned}\Delta_2 &= 4\left[\mathrm{Re}W_{11}(j\nu)+k_1^{-1}\right]\left[\mathrm{Re}W_{22}(j\nu)+k_2^{-1}\right]-\\ &-\left|W_{12}(j\nu)+W_{21}(-j\nu)\right|^2>0.\end{aligned} \tag{10}$$

Given the expression (6-7), the real part of the transfer function can be written to the coefficients of the numerator and denominator in the form

$$\mathrm{Re}W_{ij}(j\nu)=\frac{\sum\limits_{l=0}^{n}\sum\limits_{i,k=0}^{i+k=2l}[\xi_{il}a_ib_k]\nu^{2(n-l)}}{\sum\limits_{l=0}^{n}\sum\limits_{i,k=0}^{i+k=2l}[\xi_{il}b_ib_k]\nu^{2(n-l)}} \quad . \tag{11}$$

This transformation allows us to reduce the criteria for absolute stability of NICS (8-10) to the analytical mind, which checks can be carried out in various ways [2].

By substituting in (9) the expressions (11) we can reduce the absolute stability criterion for a multidimensional system to the following equation, which can be represented in the following general form

$$P_1(x)=A_1(x)+\sum_{j=2}^{n}h_jB_j(x)=0, \tag{12}$$

where - $A_1(x), B_j(x)$ polynomials, $h_j=c_jk_j, x=\nu^2$.

Moreover, the polynomials $A(x), B(x)$ can be written as

$$A(x)=\sum_{l=0}^{n}\sum_{i,k=0}^{i+k=2l}(\xi_{il}b_ib_k)\nu^{2(n-l)};\ B(x)=\sum_{l=0}^{n}\sum_{i,k=0}^{i+k=2l}(\xi_{il}a_ib_k)\nu^{2(n-l)} \quad . \tag{13}$$

Absolute stability criteria for systems of higher dimensions are considered in [5].

If the polynomial expressions (9.12) there are no positive real roots, then the criterion for absolute stability and the system under study is absolutely stable.

3.2 Mathematical model studies of robust absolute stability of NICS

To investigate the robust stability of nonlinear impulsive control system is necessary to obtain the transfer function of interval coefficients, which are then used in the appropriate criteria for absolute stability. Get the interval values of the coefficients of the numerator and denominator of the transfer function, we can use theorem V.L. Kharitonov [3].

Consider the real interval polynomial of the form

$$P(s)=\sum_{i=0}^{n} a_i s^i, a_i \in [\underline{a_i}, \overline{a_i}], \underline{a_i} \le \overline{a_i} \tag{14}$$

Typically, the study of interval polynomials taken to refer to the weak and strong Kharitonov's theorem.

Weak Kharitonov's theorem. A necessary and sufficient condition for robust stability of polynomials (14) is a Hurwitz polynomial for all corners are $a_i = \underline{a_i}$ either $a_i = \overline{a_i}$ $\forall i$. Total (14) can form 2^{n+1} a corner polynomials.

Strong Kharitonov's theorem. A necessary and sufficient condition for robust stability of a family of polynomials (14) is Hurwitz following four polynomials

$$P_1(s)=\underline{a_0}+\overline{a_1}s+\overline{a_2}s^2+\underline{a_3}s^3+\underline{a_4}s^4+\dots$$
$$P_2(s)=\underline{a_0}+\underline{a_1}s+\overline{a_2}s^2+\overline{a_3}s^3+\underline{a_4}s^4+\dots$$
$$P_3(s)=\overline{a_0}+\overline{a_1}s+\underline{a_2}s^2+\underline{a_3}s^3+\overline{a_4}s^4+\dots$$
$$P_4(s)=\overline{a_0}+\underline{a_1}s+\underline{a_2}s^2+\overline{a_3}s^3+\overline{a_4}s^4+\dots$$

Thus, in general, the study of robust stability of one-dimensional NICS can be reduced to checking the following analytical expression [4]

$$P_i(x)=A_i(x)+hB_j(x),\ i=1,\dots,4,\ \mathrm{j}=\mathrm{i}+1,\dots,4 \tag{15}$$

where h – the variable parameter, which varies from 0 to ∞.

$$A_i(x)=(a_0+\Delta a_0)+(a_1+\Delta a_1)x+\dots+(a_n+\Delta a_n)x^n,$$
$$B_j(x)=(b_0+\Delta b_0)+(b_1+\Delta b_1)x+\dots+(b_n+\Delta b_n)x^n$$

where a, b_i – the nominal values of the coefficients, variations of which have limitations,

$$-\alpha_i \le \Delta a_i \le \alpha_i,\ -\beta_i \le \Delta b_i \le \beta_i.$$

Analytical expressions for the study of robust stability of multivariate NICS written similarly, taking into account the dimension of the system.
Because the study of robust absolute stability, in this case is carried out using *w*-conversion, you must first implement *z*-transform of the system, and then obtain the corresponding coefficients in the *w*-form.

3.3 Displaying the region of stability of the system

To show the stability region under study is used NICS conclusion of the roots of (15) using a modified method of root locus, which allows to investigate both one-and multidimensional systems, and systems with interval parameters.

3.4 Description of software package

Operating window software package contains a toolbar in a single row of buttons is shown in Figure 2.

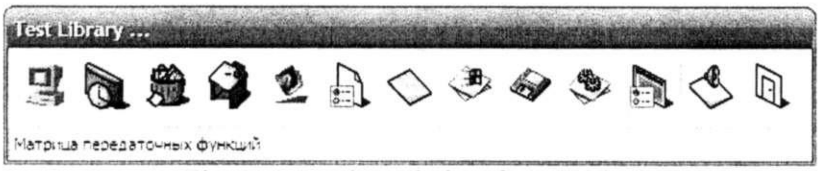

Fig. 2. Operating window for complex

When you hover the mouse over any of the icons, this button becomes convex, and the bottom line, you will be prompted with a description of the planned actions. Data is entered in the respective sub-fields, consistently disclosed by clicking on the desired field or button (Figure 3).

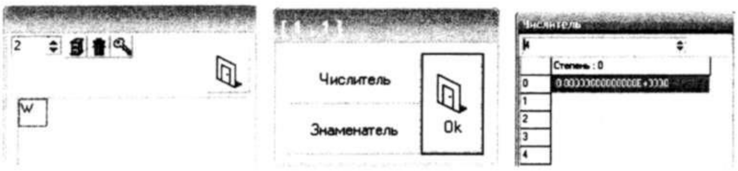

Fig. 3. Type forms for data entry

Here we describe the use of some buttons on the toolbar. The button **Polynomial** provides taking the derivative of the polynomial introduced, calculation of its roots, the computation of several polynomials Sturm. The **roots** of the button allows the conversion of the roots of the polynomial coefficients. The **transfer function** button allows for multiplication and addition of one transfer function of the other, to get the characteristic equation of the system, replace the variable $p \to z$, $z \to w$, $w \to jv$. The button **Interval transfer function** and so the transfer function provides for the construction of root locus with interval coefficients obtain the nominal values of the coefficients, the expansion of the transfer function in the real and imaginary parts of the multiplication by another transfer function with interval coefficients. The button **matrix of transfer functions** allows us to calculate the determinant, to obtain the characteristic equation, to construct the root hodograph dimensional and multidimensional systems.

As an example, Fig. 4, which displays the display area of the roots of a two- NICS with interval coefficients.

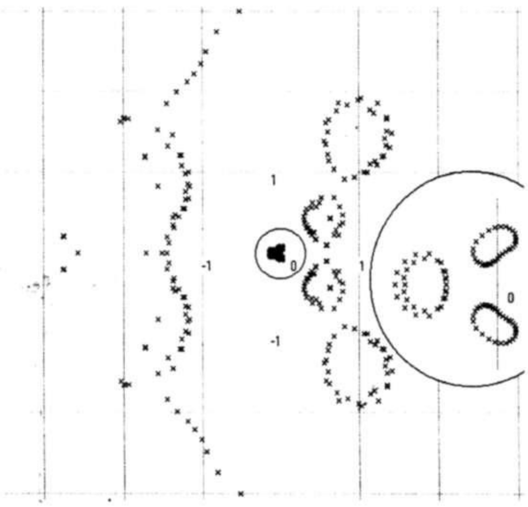

Fig. 4. The region of the roots of the two- NPCS

From Figure 4 shows that there are no roots lie on the positive real axis and therefore investigated NICS robustly absolutely stable.

4. Conclusion

The developed software package, implemented in a language Object Pascal, can run on Windows XP and above. System Requirements 4 MB of HD space and 1-2 MB of RAM. Application software system allows to study both the one-dimensional and multidimensional NICS, to synthesize a corrective device that provides the specified quality indicators, to analyze the absolute and robust stability of NICS.

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