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# OPTIMIZING ARTERIAL TRAFFIC SIGNAL SETTINGS: SHOTGUN VERSION FOR SIMULTANEOUS PERTURBATION STOCHASTIC APPROXIMATION APPROACH

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## ABSTRACT

The recent advancement in hardware computation speed has allowed stochastic microscopic traffic simulators to be embedded in signal optimization systems. In this study, stochastic perturbation simulation approximations (SPSA), an efficient difference-typed gradient-based searching, has been applied in the signal solver of a signal optimization system due to (i) its lower required total number of replications and (ii) the capability to conduct a variance reduction technique (VRT). The case study has shown that the objective value, in terms of road users' delay, indeed improves over iterations. Since the gradient-based method may be trapped in a local optimum, this study has further applied the shotgun mechanism that provides better solutions in the subject stage to proceed to the next stage. By offering the shotgun process, the quality of the solution can be further improved.

## 1 INTRODUCTION

### 1.1 Background

Notwithstanding the deterministic macroscopic traffic flow models' computational efficiency in optimizing traffic signal settings for road networks, their ability to model real-world phenomena is diminished as the system being modeled is extremely intricate (Park et al. 2001; Stevanovic et al. 2007). Such a phenomenon is caused by the rougher abstraction level of the macroscopic system, that is, modeling the bulk behavior of road users as platoons (Robertson 1969), at the link level (Liu and Chang 2011) or at the road segment level (Lo et al. 2001). For instance, when modeling the spill-out of left-turners from their designated turning bays, which block the through-going traffic (Fig. 1) it is difficult to find an off-the-shelf deterministic macroscopic model that can be readily applied (Liu and Chang 2011). Conversely, stochastic microscopic simulators that model individual decisions can flexibly model complicated phenomena (Stevanovic et al. 2013) as long as the network topology and road user (agent) are properly customized and calibrated. The same notions are also valid for other complex transportation systems such as (i) "actuated signals," generating variable green duration (McTrans 2008) depending on the stochastic nature of traffic arrival and (ii) complicated interactions between multimodal road users (buses, passenger cars, pedestrians, bicycles, etc.) within a single system (Stevanovic et al. 2008).

One of the most popular frameworks for signal optimization systems for network traffic consists of two modules: (a) *signal optimizer* and (b) traffic model (Wong 1996) as the simulator (Fig. 2a). Conventionally, deterministic macroscopic models are applied, such as one of the earliest models – the platoon dispersion model (PDM) proposed by Roberson (1969), the cell transmission model (Lo et al. 2001; Binning et al.

2011), or the congested version for PDM (Binning et al. 2010) (Fig. 2b). The corresponding *signal optimizers* are hill-climb, gradient-based (Wong 1995; Wong 1996) genetic algorithms (GA) (McTrans 2008) or simulated annealing (SA) (Binning et al. 2010) to iteratively find optimal solutions, with each iteration searching for improvement and evaluating the performance index (P.I., or the objective function), after executing one or more replications. Despite their computational efficiency, these macroscopic simulations have limitations in describing complicated phenomena or elaborating on simulation details. Failing to model details in the simulator would generate solutions for traffic signals that deviate from the optimum.

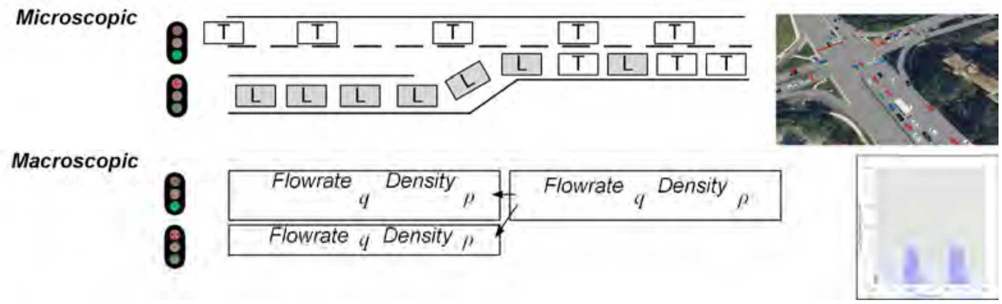


Figure 1: Left-turn spillback.

Recently, some prominent studies (e.g., Stevanovic et al. 2007; McTrans 2008; Stevanovic et al. 2008; Stevanovic et al. 2009; Stevanovic et al. 2013) have advanced in applying microscopic stochastic simulations to handle complicated systems for network signal optimization. These can be categorized as the *VISGAOST* framework (Fig. 2c). By embedding a microscopic simulator in the lower level, many studies are capable of optimizing signal settings. For example, (i) the *Direct CORSIM Optimization* (McTrans 2008) and the work by Stevanovic et al. (2007) can optimize parameters for “actuated control,” which has varying green durations in each *cycle*; (ii) the work by Stevanovic et al. (2008) can optimize parameters for transit signal priorities that offer buses or trams based on the real-time position of the public transit; and (iii) *VISGAOST* can evaluate intersection safety in the lower level and optimize the safety-aimed signal settings. In spite of their novelty, their total computation time (wall-clock time) is still far from the expectations of the traffic practitioners, as the users expect the optimization to be accomplished within a few hours, while *VISGAOST* still requires days of wall-clock time, e.g., 20 days reported in the work of Stevanovic et al. (2009).

One of the most fundamental reasons for *VISGAOST* to demand a large number of replications is the lack of variance reduction techniques (VRT) that can control the variance of the stochastic simulations; hence, *VISGAOST* requires each evaluation of the objective function (fitness value) to execute multiple (e.g., five in Stevanovic et al. 2007) replications to generate a confidence level. As such, how to genuinely apply VRT (Law and Kelton 2000) in the *signal optimizer* should be addressed in this study.

### 1.2 Aim of the Study

To address the aforementioned issues, this study aims to construct a traffic signal optimization system that can embed stochastic microscopic simulation, which not only simulates fine detail but also reduces total wall-clock time. Therefore, the proposed system should fulfill the requirements (i) capable of modeling complicated interactions in road networks; (ii) only two required replications for each iteration; and (iii) capable of conducting VRT, which is vital in reducing the required samples for stochastic simulations.

### 1.3 Proposed Structure

The method of stochastic perturbation simulation approximations (SPSA) has been selected as the *signal optimizer* to support the stochastic microscopic traffic simulation of the lower level in this study, in order to increase the efficiency as follows:

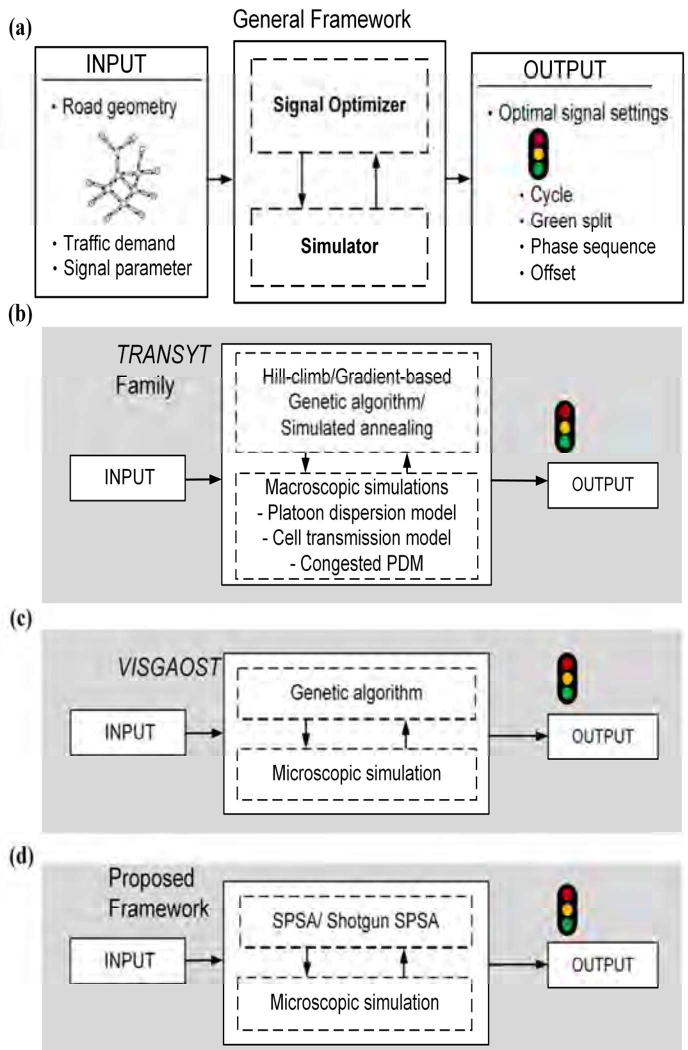


Figure 2: Framework of signal optimization system. (a) General framework; (b) TRANSYT family; (c) *VISGAOST*; and (d) the proposed method.

- Fewer evaluations per iteration: It is conceivable that GA needs to evaluate the objective function (fitness value) of each chromosome in the non-elite population. On the other hand, general gradient-based search methods do not have such a computation burden; and
- Fewer replications per evaluation: Recognizing that the simulator is stochastic, GA needs to execute multiple (e.g., five in Stevanovic et al. 2007) replications to create a confidence level in order to generate a relatively certain fitness value. On the other hand, a naïve gradient-based search without an analytical gradient form needs to evaluate the finite difference values of each variable to approximate the gradient. Consequently, the number of replications needed in one iteration is (asymptotically) proportional to the dimension of the search space. Such requirement contributes to the computation burden given the large number of decision variables in a road network. In contrast, SPSA requires only two replications per evaluation. Such an advantage is pronounced when the network size is even larger.

Notably, solely applying SPSA is not sufficient to obtain a desirable solution for the signal optimization problem, since the objective function (usually total delay, total travel time, or system throughput) is not a convex function of the signal variables, in general. Albeit the convergence of SPSA to an optimum can be guaranteed, these solutions are local under the settings of non-convex functions. In this regard, this study has further introduced the shotgun procedure (Binning et al. 2010) for SPSA.

The novelty of the proposed framework lies in embedding SPSA in the *signal optimizer* while applying microscopic simulation to optimize network signal plans. Compared to the state-of-the-art models, VRT can be naturally conducted when applying finite differences. Such a technique is not possible under *VISGAOST* framework, as fitness evaluations are conducted separately for each chromosome.

In addition, this study synthesizes SPSA with the shotgun procedure, which branches the searching points and improves the solutions iteration-by-iteration prior to selecting one or some of them to proceed whilst discarding the rest of them considered not showing potential in attaining a global optimum.

## 2 LITERATURE REVIEW

In review of the literature, the work by Robbins and Monro (1951) is one of the *earliest simulation optimization* methods that aim to estimate *zeros* in noisy environments. It was later reformulated as extrema finding procedure by estimating zero(s) of the gradients. Kiefer-Wolfowitz (1951) proposed the first-order *simulation optimization* method by proceeding to the direction of the gradient, which can be approximated by naïve finite difference, assuming the exact gradient is not computable (cf. Bhatnagar et al. 2012 for a detailed review). Suppose *P* variables need to be optimized, Kiefer-Wolfowitz (K-W) requires (*P*+1) replications to evaluate the gradient, resulting in inefficiency when the dimension (*P*) grows. To reduce the number of replications needed per iteration while preserving the convergence rate (in terms of the number of iterations), Spall (1992) proposed SPSA, requiring only two replications to evaluate the gradient. Such a mechanism randomly selects a direction of perturbation and observes the collective effect of the change in the objective values when perturbated.

SPSA can be extended to the second order to improve the convergence rate, but one must be cautious in treating the inverse of the Hessian matrix. To compute the inverse of Hessian, one can average the eigenvalues of the Hessian matrix and place them on the diagonal only (Zu and Spall 2002) or apply Woodbury Identity to embed the inverse computation into the iteration update (Bhatnagar et al. 2012).

Despite the development in the *simulation optimization* community, the SPSA has not yet been applied to solving network traffic signal optimization, whose various deterministic methods employ macroscopic simulations, including hill-climb (Robertson 1969), first-order search (Lan and Chang 2016; Wong 1996), (second-order) Newton's method (Ceylan et al. 2010), or heuristic approaches with macrosimulations (e.g., Binning et al. 2010; McTrans 2008; Lo et al. 2001) or microsimulations (Park et al. 2001; Stevanovic et al. 2007; McTrans 2008; Stevanovic et al. 2008; Stevanovic et al. 2009; Stevanovic et al. 2009).

## 3 PROBLEM STATEMENT

### 3.1 Decision Variables

Three primary signal variables are defined in the transportation research community (Fig. 3):

- *Common cycle length* $C$: the time it takes for the green phase of a movement to reappear.
- *Green split* $\phi_{i,j}$: the green duration of movement $j$ of intersection $i$.
- *Offset* $\theta_i$: The time of the start of the major movement of intersection $i$, measured from the primary clock.

Notably, this study focuses solely on optimizing *offsets* since, among three kinds of variables, determining the coordination of intersections is the most challenging task. Specifically, *common cycle length* and *green splits* can be determined by a spreadsheet (e.g., Webster 1958), whereas determining *offsets* requires advanced methods such as mathematical programming (e.g., Gartner et al. 1991). Hence, this study focuses on the most challenging part of determining the *offsets*, $\boldsymbol{\theta} \stackrel{\text{def}}{=} [\theta_1, \ldots \theta_i \ldots]$, although *cycle length* and *splits* can also be variables embedded in the system.

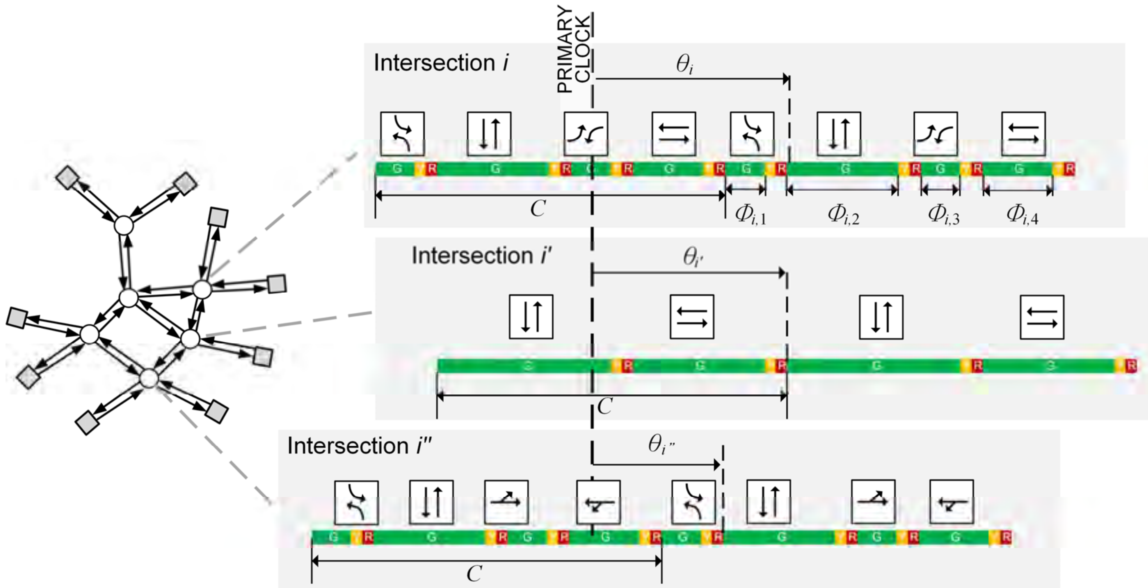


Figure 3: Definition of key variables in signal optimization.

### 3.2 The Objective Function

The road users' total delay $J$ is selected as the objective function. The delay is determined by the decision variables (*offsets* $\boldsymbol{\theta}$): $J = J(\boldsymbol{\theta})$. The objective function is measured as the expected value of the delay of all realizations (with random seed $\xi$), or $J(\boldsymbol{\theta}) = \mathbb{E}_{\xi}\,\{\, Y(\boldsymbol{\theta};\xi)\,\}$, where $Y(\boldsymbol{\theta};\xi)$ is the delay of signal setting $\boldsymbol{\theta}$, realized by a particular replication with random seed $\xi$. This delay is obtained from the simulation by summing up the delay of each vehicle $d_l$, where individuals are indexed $l$, or $Y(\boldsymbol{\theta};\xi) = \sum_{l(\xi)} d_l(\boldsymbol{\theta};\xi)$.The delay can be computed, from the simulation, by the difference between free-flow travel time and realized travel time due to obstruction.

### 3.3 The Problem

This research investigates the optimal coordination of signals which minimizes the total (expected) delay of vehicles within the road network:

$$\min_{\substack{\boldsymbol{\theta} \\ 0 \le \theta_i \le \bar{C}}} \; J(\boldsymbol{\theta})$$

where $\bar{C}$ is the *cycle length*. In this study, $\bar{C}$ is a parameter, but it can be generalized to be a variable.

## 4 SOLUTION APPROACH

SPSA (Spall 1992) generates perturbations in each iteration and constructs a sequence of values whose expected values (with respect to random drawing) are gradient. Then, following one of the directions of the perturbation vector, the control variables will converge to optimal solutions asymptotically in distribution. The perturbation element $p$ can be determined randomly by a symmetric Bernoulli distribution (Chau and Fu 2015):

$$\Delta_p = \begin{cases} +1 & w.p.\, 0.5 \\ -1 & w.p.\, 0.5 \end{cases}$$

The gradients are estimated as the difference of the performance of two simulation runs $Y(\boldsymbol{\theta} \pm c\boldsymbol{\Delta}; \xi)$:

$$\widehat{\nabla J}(\boldsymbol{\theta}; \xi) = \left(\frac{Y(\boldsymbol{\theta}+c\boldsymbol{\Delta}; \xi) - Y(\boldsymbol{\theta}-c\boldsymbol{\Delta}; \xi)}{2c}\right) \boldsymbol{\Delta}^{-1},$$

where, $c$ is a scalar, $\boldsymbol{\Delta} \stackrel{\text{def}}{=} [\Delta_{p=1} \dots \Delta_p \dots \Delta_{p=P}]^T$, $\boldsymbol{\Delta}^{-1} \stackrel{\text{def}}{=} [(1/\Delta_{p=1}) \dots (1/\Delta_p) \dots (1/\Delta_{p=P})]^T$. Searching is made by proceeding to the next iteration ($n$+1) by the following equation.

$$\boldsymbol{\theta}^{n+1} = \boldsymbol{\theta}^n - a_n \widehat{\nabla J}(\boldsymbol{\theta}; \xi)$$

where $a_n$ is the step size. The sequence $\{\boldsymbol{\theta}^n\}$ converges to the globally asymptotic stable solution of the ordinary differential equation (ODE) $\dot{\boldsymbol{\theta}} = \nabla J(\boldsymbol{\theta})$ with probability 1 (Bhatnagar et al. 2012).

To obtain convergence in distribution, this study follows the convergence criteria in Bhatnagar et al. (2012), who apply the step size $a_n$ and magnitude of difference $c_n$: $\sum^{\infty} a_n = \infty$, $\sum^{\infty} (a_n/c_n)^2 < \infty$. Notably, unlike the Kiefer-Wolfowitz finite difference (FD) schemes, features of SPSA are (i) a perturbation occuring at denominators instead of numerators; (ii) the indirect gradient estimation $\widehat{\nabla J}(\boldsymbol{\theta}; \xi)$ having the same weight on each basis ($\pm \boldsymbol{e}_p$); and (iii) the possible reduction to two evaluations per iteration. Compared to FD, which requires $(P+1)$ or $(2P)$ estimations per iteration, SPSA requires only 2, independent of the dimensions of the optimization problem.

The SPSA is selected as the methodology of the *signal optimizer* shown in Figure 2(d). Bhatnagar et al. (2012) and Zhu and Spall (2002) summarized the asymptotic normality of the solution: $n^{\beta/2} \cdot (\boldsymbol{\theta}^n - \boldsymbol{\theta}'^*) \sim \mathcal{N}(\mu', \Sigma)$ as $n \to \infty$, where $\boldsymbol{\theta}'^*$ is the true (local) optimal; $\beta$ depends upon gain sequences $\{a_n\}$ and $c$; and $\mathcal{N}$ (.,.) is a Gaussian distribution with mean $\mu'$ and covariance matrix $\Sigma$, depending on Hessian at $\boldsymbol{\theta}^*$. Implementing optimization via microsimulation offers the following advantages:

- *Efficiency*: It has efficient computation due to only two (2) replications needed per iteration.
- *Reduced variance*: VRT can be realized by controlling the random seed, i.e., the same seed applies to both perturbed points, ( $\boldsymbol{\theta} + c\boldsymbol{\Delta}$) and ( $\boldsymbol{\theta} - c\boldsymbol{\Delta}$).
- *Reduced overfitting*: SPSA can reduce the prediction error in evaluating the objective function by altering random seeds in each iteration (yet keeping the same seed within an iteration).

### 4.1 Proposed Algorithm 1

The customized SPSA, based on the general algorithm (Spall 1991), has to adapt to the nature of signal settings, where (i) the *offsets* have a periodic nature, i.e., $\theta_i = \theta_i$ mod ($C$) and (ii) the stochastic microscopic traffic simulator can be controlled by a random seed $\xi$. The flowchart of Algorithm 1 (Fig. 4) has major steps following the framework of SPSA: (i) initialization; (ii) generating random perturbation; (iii) evaluating the signal performance from simulations; (iv) proceeding to the new solution; (v) updating the solution; (vi) checking the termination criterion; and (vii) returning the solution. The customized version of the SPSA algorithm, tailored for the signal coordination problem, has the following unique features:

- *Cycle length constraint*: One needs to define $\boldsymbol{\Pi}(\cdot)$: $\theta_p \leftarrow$ round ( $\theta_p$ mod($C$)), i.e., taking modulo operation over *cycle length* $C$, since *offset* variables are periodic. For practical reasons, round to the closest integer since the resolution of the *offsets* signal controller is 1 (sec). Notably, different from other applications, we do not modify the value of the *offset* $\theta_p$ after rounding $\boldsymbol{\Pi}(\cdot)$. Instead, we only round the values to be used in evaluations without replacing $\theta_p$ .
- *Overshoot prevention*: Since the periodic nature of the *offsets*, $Y(\boldsymbol{\theta};\cdot) = Y(\boldsymbol{\theta} + C \cdot \mathbf{1}, \cdot)$, it is illogical to make too large a step toward the next iteration; otherwise, it would be equivalent to proceeding

in the wrong direction. To be specific, $\theta_p^{(n+1)} = \theta_p^{(n)} + A$, where $C/2 \leq A \leq C$, the performance is the same at $\theta'^{n+1}_p = \theta_p^{(n)} - (C - A)$, which is moving in a direction other than A with magnitude $(C - A) > 0$. To prevent backfiring, one can set a threshold ω to prevent overshoot.

- *Threshold of performance difference*: To build a general signal coordination problem, the model must be able to handle road networks of different size and geometry. The parameter $a_0$ reacts to the results of the first estimate that reflect the size of the problem. For example, on a four-lane 6-intersection arterial with a demand volume of around 1,000 veh/hr per direction and 3,600 seconds simulation time, the magnitude of Y difference, $|Y^+ - Y^-|$, is below 25,000 (veh-sec) in 88.5 % of the time. It is expected that this *locking* value would be approximately doubled by the network size. This information is used to adjust $a_0$ to react to the network size and reduce the aforementioned overshooting. $Z$ is a conservative guard if the first iteration produces an extremely low $|Y^+ - Y^-|$ such that $a_0$ is overly set (and with a higher chance of overshooting in the following iterations).
- *Resolution of Signal Controller*: In practice, the traffic signal controller commanding signal lights has a resolution of 1 second for *offsets*. Simulators such as *VISSIM* reflect this fact and allow the *offsets* to be set as integers only. As a result, the parameter of the last iteration can be as fine as $c_N = 0.5$, but it is meaningless to be finer ($(+c_N)$- $(-c_N)$ = 1.0). Hence, $\{c_n\}$ is determined by $c_0$, $c_N$, and $N$, that is, $\{c_n\} = c_0 / n^b$, $b = \ln(N)/\ln(c_0/c_N)$.
- *Common Random Number*: The two evaluations ($Y^+$ and $Y^-$) in the same iteration must be assigned with the same seeds. Assigning a common random number (for instance, in *VISSIM*) allows to evaluate the two sets of *offsets* ($\boldsymbol{\theta}_n \pm c\boldsymbol{\Delta}$) under the same simulation conditions (same realization of *random variates*).

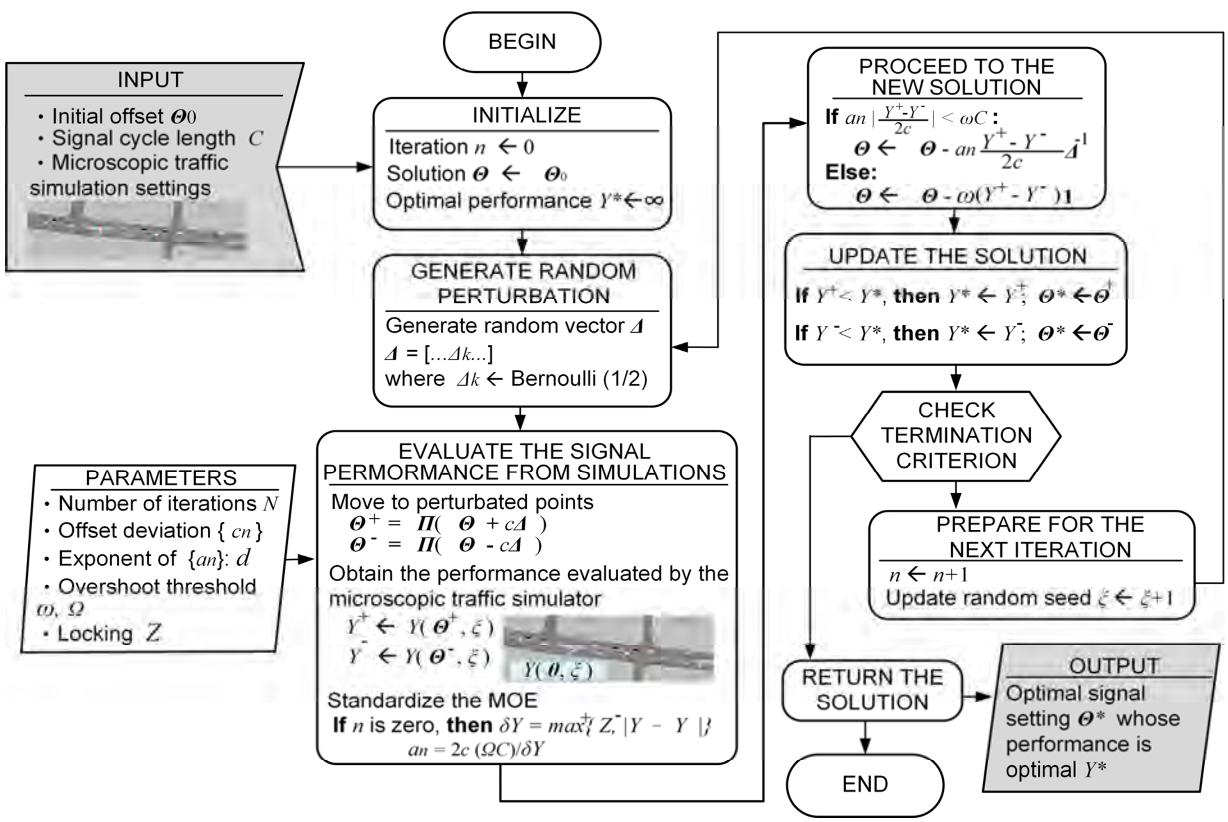


Figure 4: Flow chart of the proposed Algorithm 1.

### 4.2 Proposed Algorithm 2 – Shotgun SPSA

In general, the gradient-based search will be trapped in local optima as most simulation systems are not convex with respect to the decision variables. Shotgun methods, which make multiple shots in the procedure, are an effective method for preventing trapping (Binning et al. 2010). To circumvent trapping, a shotgun version of SPSA has been developed. The proposed Algorithm 2 implements multiple independent procedures of Algorithm 1 with different randomly generated initial points. Then, the search is continued only for the one with the best performance after implementing a few iterations ($M$) among these shots. The proposed Algorithm 2, shotgun SPSA, whose inputs are signal *cycle length C*, the core is a microscopic traffic simulator $Y$(.;.), and outputs are optimal *offsets* $\boldsymbol{\theta}^*$ with performances $Y^*$, is detailed below.

1. *FOR* shot $\sigma$=1 to $Q$:
   Randomly generate initial *offsets* ${}^{\sigma}\boldsymbol{\theta}^0$
   Conduct $M$ iterations:
   ( ${}^{\sigma}\boldsymbol{\theta}^*$ , ${}^{\sigma}Y^*$ )← Algorithm 1 ( ${}^{\sigma}\boldsymbol{\theta}^0$ , $Y$(.;.),$C$; $c_M$=$c_0{'}/M^{lnN/\ln(c_0/c_{N'})}$ , $N$=$M$)
   *END FOR*
2. $\boldsymbol{\theta}'^0 \leftarrow \text{argmin}_{\sigma}\, {}^{\sigma}Y^*$
3. ($\boldsymbol{\theta}^*$ , $Y^*$ ) ← Algorithm 1($\boldsymbol{\theta}'^0$ , $Y$(.;.), $C$ ;$N$=$N' - M$, $c_0$=$c_0{'}/(M+1)^{lnN/\ln(c_0/c_{N'})}$)

In the for-loop, evaluations are made in shots. Afterwards, the best among the shots is selected to proceed with further improvement based on SPSA with the remaining ($N' - M$) iterations.

## 5 CASE STUDY

### 5.1 Experiment Settings

To demonstrate that the proposed procedure can actually solve the signal coordination problem, this study investigates a real-world road network with six signalized intersections. The network has 38 signalized approaches, a total of 10,431 vehicles in 3,600 seconds of simulation time, and the exact turning volume counts, signal parameters, and network topology are reported in Chen et al. (2019). Five *offsets* that can coordinate traffic signals in the system are the control variables in the problem. *VISSIM* (PTV 2018), a microscopic traffic simulation tool, is applied.

Since the *MULTIBAND* (Gartner et al. 1991) model can produce optimal signal setting plans for specific background settings like arterials dominated by two-way through-traffic, this study has selected *MULTIBAND* as the benchmark model to examine the quality of the solutions generated by the proposed algorithms. The experimental setup of *MULTIBAND* is reported by Chen et al. (2019). It should be, however, stressed that the proposed model aims to target broader applications that do not necessarily have a benchmark model to compare.

The parameters used are: number of replications $N$ =40; *offset* deviation of the first iteration $c_0$ =2.5; *offset* deviation of the last iteration $c_N$=0.5; exponent of $\{a_n\}$, i.e. $d$=0.7; overshoot thresholds $\omega$ =0.1 and $\Omega$=0.1; and locking $Z$ = 25,000. For Algorithm 2, the shotgun scatters into five shots and makes a decision at iteration 10. Specific parameters for Algorithm 2 are: number of shots $Q$ = $P$ ($2 \leq Q \leq P$); iterations to end the shots $M$ = 10 ($M$ < $N'$); number of replications $N'$ = 25; $c_0'$ = 2.5; and $c_{N'}$=0.5.

The main purposes of this case study are (a) to investigate the *optimality*, whether the proposed algorithms can achieve the solutions that are considered optimal, based on the results from the benchmark model, and (b) how the shotgun mechanism can improve the quality of the solutions, which can be quantified as (i) the average performances; (ii) the variance of performances; and (iii) the percentage of sample solutions that are higher than one standard deviation (1$\sigma$) or two standard deviations (2$\sigma$) above the mean results from the benchmark model.

## 5.2 Results – Algorithm 1

Among five initial *offsets* (generated randomly) being tested, an optimal vector of *offsets* is $[0,96,112,32,159,120]^T$, corresponding to optimal performance 405.1 kilo-veh-sec, whose evolution over iterations is plotted in Figure 5. The performances have improved in the first few iterations, but are not as significant in the following iterations. The overall performances of all other random initial points are reported in Table 1. To evaluate their precision, 25 different seeds other than those used during optimization are applied. Table 1 reports the validation by running the output signal variables by using random seeds unseen by the procedure. No statistical significance in the mean of validated output between Algorithm 1 and the benchmark is observed. However, a higher standard deviation can be observed from the output of Algorithm 1 compared to the benchmark model.

Table 1: Performance and solution time

| Items | Benchmark | Algorithm 1 | Algorithm 2 |
|---|---|---|---|
| Model Output (kilo-veh-sec) [a] | | | |
| Mean (Stnd. Dev.) | N/A (N/A) | 466.6 (40.0) | 451.0 (29.4) |
| Validation (kilo-veh-sec) [b] | | | |
| Mean (Stnd. Dev.) | $\mu$ =462.6 (36.1) | 493.8 $^{NS}$ (48.1) $^{***}$ | 486.0 $^{NS}$ (42.5) $^{***}$ |
| Solution Quality | | | |
| Percent of the samples above ($\mu$+1$\sigma$) | 13.3% | 48.0% | 28.0% |
| Percent of the samples above ($\mu$+2$\sigma$) | 6.7% | 20.0% | 12.0% |
| Solution Time [c] | | | |
| Mean<br>(Stnd. Dev.) | 0.24 s<br>(N/A) | 2h 44 m 58 s<br>(6m 38s) | 4 h 23 m 48 s<br>(3 m 7s) |

Notes: NS= not significant at 5% level, compared to the benchmark. ***=Significant at 1 % level, compared to the benchmark.
[a] From 5 outputs, starting from randomized initial points, generated by the algorithm.
[b] Total 25 replications. Each output generates 5 replications (5x5) from seeds other than seeds used during the optimization.
[c] Hardware: Intel Core i5-2400 CPU 3.1 GHz with 8G RAM

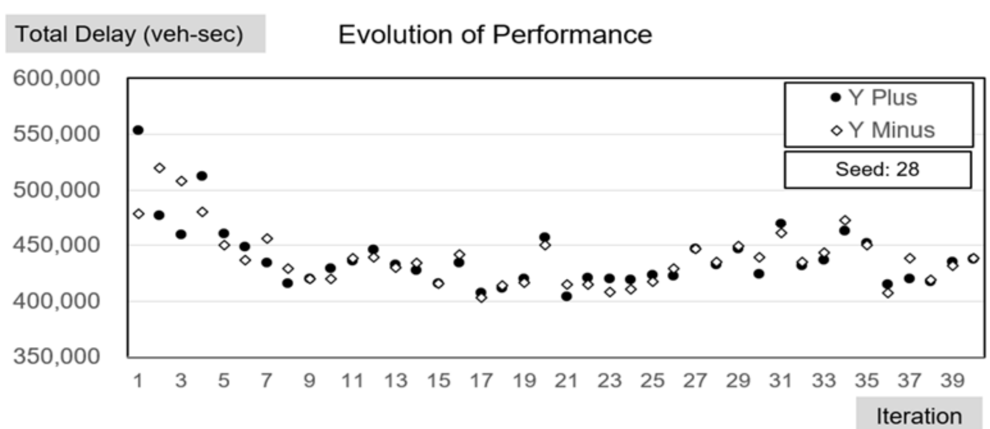


Figure 5: Performance evolution of Algorithm 1.

### 5.3 Results – Algorithm 2

Among five randomized initial starting points, an evolution of the performance function is plotted in Figure 6. The shotgun scatters into five shots and makes a decision at iteration 10: preserving shot 5 with the current best solution and continuing the search until iteration 25. There are no statistically significant differences to the results generated from the benchmark model, yet the F-test (on variances) shows that the output of Algorithm 2 has higher standard deviations than the benchmark model.

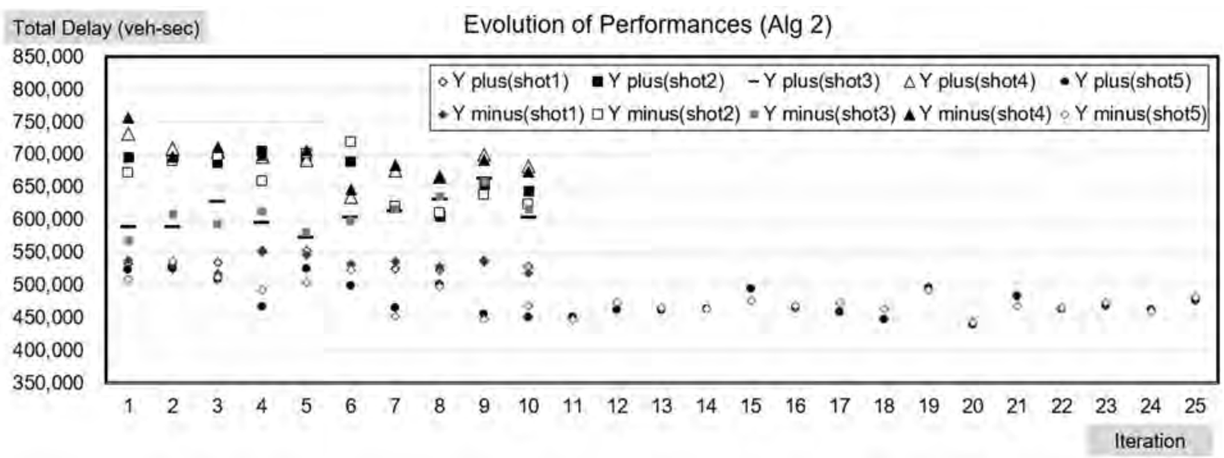


Figure 6: Performance evolution of Algorithm 2.

### 5.4 Discussion

Some observations can be found in the numerical results:

- The proposed models can achieve the optimality of the benchmark model. Compared to the benchmark model, both proposed algorithms generate competitive solutions. Two algorithms show no significant differences, compared to the benchmark model, in the validation runs (25 runs for each algorithm). However, both models show higher standard deviations than the benchmark.
- In terms of the wall-clock time, the proposed algorithms are *not* competitive with the benchmark model. The proposed algorithms need hours to complete the optimization, while the benchmark model consumes only 0.24 s for the same task. However, the proposed algorithms aim to target general *simulation optimization* problems, which are, in general, not solvable by the benchmark model.
- The solutions for Algorithm 1 do not improve after iteration 26, in terms of the value of the objective function. This might suggest that one may develop mechanisms that apply an adaptive step size when the solution stagnates or impose termination criteria to reduce the number of replications.
- Although there is no statistical significance in the differences of the performances (in both mean and standard deviation) between Algorithms 2 and 1, Algorithm 2 has a better quality of solution showing that Algorithm 2 is less likely to converge at local optima. This can be assessed by computing the percentage of sample solutions that deviate from the mean ($\mu$) validation results of the benchmark model by one ($\mu + \sigma$) and two standard deviations ($\mu + 2\sigma$). Numerical results show that 12 % of the samples from Algorithm 2 are higher than two standard deviations above the mean ($\mu$+2$\sigma$) generated from *MULTIBAND*, compared to 20 % of the sampled results from Algorithm 1. The same trend can be observed from the percent of samples that are above one standard deviation higher than the mean ($\mu$+1$\sigma$) of the benchmark. The chance of trapping in local solutions is highly reduced with the shotgun mechanism.

- By applying the shotgun procedure to explore the solution space, Algorithm 2 enhances the solution quality, but at the expense of increasing the number of replications to 124 (compared to 79 in Algorithm 1), resulting in a solution time of 4 h 23 m 48 s (see Table 1).

## 6 CONCLUSION

Coordinating arterial traffic signals via (microscopic) *simulation optimization* captures the detailed phenomena that cannot be easily described by macroscopic simulations. Such a framework is made possible by the current advancement of hardware. However, controlling the total number of replications remains a vital issue, since the users will be expected to accomplish the optimization procedure within a few hours. A framework embedding a microscopic traffic simulator while applying SPSA as the *signal optimizer* has been proposed and has shown its ability to reach optimality in a case study. The main contributions of this study are

- developing a novel framework, embedding the microscopic simulator and SPSA as the *signal optimizer* for network signal optimization that (i) enables more detailed simulations and (ii) reduces the total number of replications compared to metaheuristic algorithms;
- offering VRT in computing approximate gradients and tailored procedures and parameters customized to the problem nature of network signal optimization;
- demonstrating that the methodology of SPSA can indeed be applied to real-world cases of network signal optimization; and
- applying the shotgun mechanism to prevent trapping in local extrema.

Notably, the numerical case in this paper solely shows the solution quality on the optimality, investigating whether the output is comparable with the solution from the benchmark model. The next phase of the study shall (i) study the effect of parameters on the solutions; (ii) investigate warm-starts to further reduce the required number of replications; (iii) conduct ablation such as comparison to the methods without VRT; (iv) provide comprehensive numerical evidence, apart from the theoretical analysis, regarding the required number of iterations to achieve comparable results compared to those generated by K-W and *VISGAOST*. Since the primary reason to adopt microsimulation is to take advantage of its ability to model complicated scenarios, the algorithm shall also be tested with other complicated systems, such as multi-modal traffic environments (pedestrians, buses, bicycle riders, scooters, mopeds, etc.), and shall be tested with various network sizes as major future works.

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