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EXTENDING THE NAMING GAME IN SOCIAL NETWORKS TO MULTIPLE HEARERS PER SPEAKER

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ABSTRACT

Social conventions govern numerous behaviors of humans engaging in day-to-day activities from how they greet to languages they speak. The classical Naming Game algorithm has been defined with inherently sequential semantics where agents engage in pairwise interactions and reach global consensus in the absence of any outside coordinating authority. In this paper, we extend the classic Naming Game to multiple hearers per speaker in each conversation even while allowing simultaneous "speaking" and "hearing". We simulate the impact on the number of conversations for convergence by varying the number of hearers and investigate the impact of multiple network types and agent population sizes on the global convergence. The results show that our extended model combining simultaneous conversations and multiple hearers per speaker per conversation makes the words diffuse at a much faster rate and leads to significantly faster consensus formation.

1 INTRODUCTION

1.1 Background

The Naming Game is a multi-agent model where individuals engage in pairwise interactions to reach a global consensus on the name of a single object (Baronchelli et al. 2005). It also helps to investigate the system dynamics in social networks of autonomous agents. The conventional Naming Game is played by a set of N agents who interact pairwise to negotiate conventional forms associated with a set of meanings. These agents, in the absence of any central control, mutually convey information to each other about objects in the environment and evolve their language. An example of such a game is that a population must reach a consensus on the name of an object by interacting locally with each other. For simplicity, the language evolution is restricted in this article to: (1) naming only one object at a time, and (2) not considering homonyms.

The Naming Game model helps individuals self-organize to produce global coordination while interacting locally (Baronchelli 2016). It is well understood that it is possible to bootstrap and successfully accomplish a naming exercise in a completely autonomous fashion with no external coordinator. However, there are a few questions that need to be addressed: How does this happen? Will a consensus ever be reached? How long does it take to reach a consensus? These are some of the important questions in theory and practice but answering them is not easy. It has been observed that new words spread and compete with each other to eventually converge to a single accepted word (Lass 1997) (or to exactly as many words as there are islands in the network graph formed by the communication links among the agents). It has

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been proposed by Steels (2015) that the linguistic communication capability of humans is an emergent phenomenon, using a framework where mutually agreeable words are shared by speakers and listeners and evolve over time (Steels 1995). Others such as Lu et al. (2009) focus on the impact of communities on the outcome of consensus formation in social networks, concluding that stronger community networks hinder global agreement while the Naming Game evolves as clusters of coexisting opinions.

Investigating social dynamics using the Naming Game has been well explored in the literature. Many variants of this game have been studied. In this paper, we use the Naming Game to study the convergence behaviour of multiple agents while interacting locally and attaining consensus. The Naming Game is a minimal model employing local communications. For example, one of the earliest applications of this game has been in the context of robots where one robot acts as a speaker and the other as a hearer (Steels and Loetzsch 2012). The speaker draws the attention of the hearer by naming a characteristic feature of an object. More recently, in systems of human agents, the Naming Game has been used to describe the phenomenon of collaborative tagging and bookmarking on social networks (Cattuto et al. 2007; Golder and Huberman 2006). One of the most popular usages of the Naming Game has been in the context of language formation and the spread of dialects. In a broader sense, the Naming Game can be employed to investigate the large-scale patterns arising from local interactions at individual levels. An interesting example is the pop vs. soda controversy (Thiel and Sleep 2007). The common feature that has been explored and well studied in most of these studies is the inherent sequential one-to-one conversations present in the classical algorithm to reach global consensus. Another study by Baronchelli (2011) investigates how the states of the agents are updated after an interaction where the authors conclude that slightly modifying the rules can dramatically alter the overall dynamics.

1.2 Our Contributions

This classical version of the Naming Game has been studied on a variety of networks including general graphs (Baronchelli et al. 2006), complete graphs (Baronchelli et al. 2006; Baronchelli et al. 2005), small world networks (Dall'Asta et al. 2006; Lin et al. 2006) and scale-free networks (Dall'Asta et al. 2006; Baronchelli et al. 2006). On all these complex networks, the authors assumed that the agents would engage only in pairwise interactions. The major contribution of our simulation study presented here is to extend the model of every conversation in the Naming Game to involve multiple hearers per speaker, and to use simulation to evaluate its dynamics in complex networks, including complete (random), distance-based and small-world networks. We study the global convergence on all these networks while steadily increasing the population size and analyzing the convergence results at each population size.

2 CLASSICAL MODEL

2.1 Classical / Sequential Algorithm

The classical Naming Game operates on a set of N dictionaries, one dictionary per agent, in a sequential set of conversations. These dictionaries evolve over the number of conversations to converge to a single, identical word in every dictionary. The individuals may start with completely dissimilar dictionaries (potentially empty, or with words that are unique per agent). Furthermore, during the game's evolution, the dictionaries may grow bigger or shrink in size as a result of the conversation rules. The game is carried out as a series of steps of conversations. At each step, the dictionaries of the agents may increase in size or shrink to only one single word. The game is carried out until consensus is reached, that is, every dictionary contains the same unique word.

The conversation begins when the speaker picks a random word from its dictionary or invents a new word if its dictionary is empty. The speaker conveys this word to the hearer. The hearer consults its own dictionary to see if the word in the conversation is present in its dictionary. If such a word exists in its dictionary, then both the speaker and hearer decide to abandon their original dictionaries and retain only the word conveyed by the speaker in the conversation. This event is termed as a "success" in the Naming

Game since the hearer's dictionary contains the speaker's word in the conversation and this event helps in the pruning of words in the system, moving it forward towards consensus. However, if the hearer's dictionary does not contain such a word, then the word gets added to the hearer's dictionary. The speaker's dictionary is not changed in this scenario. This event is termed as a "failure". Refer to Figure 1 for illustrative examples of success and failure.

Note that, in the sequential conversation model, a hearer completes the updates to its dictionary before another conversation starts, which implies that the aforementioned success and failure are the only two possibilities. This corresponds to the classical model (Baronchelli 2016). However, an additional possibility arises when two or more conversations may be active at the same time, which raises the possibility that a hearer may be engaged in a conversation when a different speaker may initiate another conversation. Additional variation of the model is therefore needed for asynchronous conversations in which the hearer may have an ongoing conversation when a conversation is initiated (Perumalla 2017).

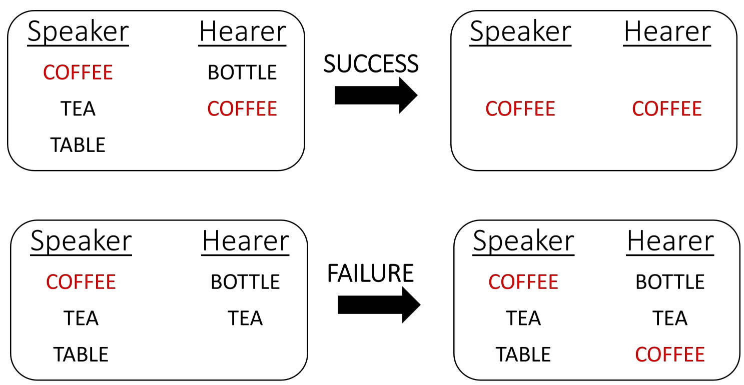

Figure 1: Examples of dynamics in a successful and a failed scenario in the basic Naming Game.

The following steps are executed sequentially in a loop until consensus is reached.

1. A random pair of speaker and hearer is selected from the population.
2. The speaker randomly selects one of the words from its dictionary (or invents a new word if the dictionary is empty).
3. The speaker conveys the word to the hearer.
4. The hearer consults its own dictionary and one of two actions takes place:
   (a) Success: If the hearer's dictionary contains that word, both the hearer and speaker would abandon their original dictionaries and would replace their dictionaries with only the word in the conversation.
   (b) Failure: If the hearer's dictionary does not contain the word in the conversation, the hearer adds the word to its dictionary. The speaker's dictionary is not updated in this scenario.

Remember that one of the simplifying assumptions of our model is the absence of homonymy. This ensures that no two agents would use a different word for the same object. However, since homonymy is an important aspect of human interaction, we can think of "words in a context" to approximate the concept of homonymy. An important point to note here is that this game operates from a global point of view; that is, each conversation depends upon the previous conversations and each successful conversation may alter the individuals' dictionaries considerably. Another important assumption of our work is that the words would be randomly extracted from the speaker's dictionary. Many previous studies have attempted to assign weights to the extraction of these words. For the sake of simplicity, the model we define here does not use weights.

2.2 Limitations of Classical Model

The inherently sequential classical model deviates from some realistic considerations, including:

- Asynchronous conversations: A key assumption of the classical model is that the conversations happen sequentially. At each step, a random pair of speaker and hearer engage in conversation and update their dictionaries. However, in reality, the conversations in the system may happen simultaneously. In other words, multiple conversations may be updating dictionaries at the same time across the system. In particular, it is possible for a speaker in one conversation to be a hearer in another conversation.
- Pairwise conversations: The classical algorithm assumes that at each step only one pair of speaker and hearer will engage in a conversation. However, with the larger population sizes in social networks, group conversations are much more common. Some agents may act as "influencers" and impact the dictionaries of more than one hearer after each conversation. The dictionary of the speaker may or may not change.
- Instantaneous Conversations: The classical model assumes that the interactions take place instantaneously. However, typically a non-zero amount of time elapses during a conversation. Moreover, during this time gap, agents may engage in other conversations.

3 MODEL ENHANCEMENTS

Our present work addresses the first two limitations of the classical model as discussed in Section 2.2. We relax the assumptions of one-to-one hearer-speaker mapping as well as the sequentiality of conversations. Parallel asynchronous conversations have already been introduced previously by Perumalla (2017). We extend his model and further relax the assumption of pairwise conversations. Our work introduces the parameter that there can be more than one hearer in each conversation; in other words, a conversation contains one speaker and $H \geq 1$ hearers of the same word chosen by the speaker. Although many peer-to-peer simulation models have been developed in the past (Andelfinger et al. 2014; Cecin et al. 2006), a parallel multiple hearers conversation model has not been previously studied. In this model, conversations happening simultaneously, that is, multiple conversations may be updating dictionaries at the same time.

3.1 Multiple Hearers Per Speaker

Considering the increasingly sophisticated interfaces of social networks, that is, multiple people engaging in very dynamic discussions simultaneously, pairwise-only interactions is a significant limitation on the Naming Game model. Our work extends the Naming Game in social networks to multiple hearers per conversation. We assume that in any conversation, there will be one speaker and multiple hearers. In this extended model, the following steps are repeated in a loop, until consensus is reached.

1. Multiple simultaneous groups are identified, with each group containing one speaker and multiple hearers. The agents in each of these groups may be connected using the definitions of social networks that we will described shortly in Section 3.4.
2. The speaker in each group would randomly pick a word from its dictionary and convey the word to all the hearers of the group.
3. All the hearers who have such a word in their dictionaries would abandon their original dictionaries and would only retain the word in the conversation.
4. All those hearers who do not have such a word in their dictionaries would add the word to their existing dictionaries.
5. The speaker (in any group) would update its dictionary only if each and every one of the hearers in its group has such a word in their dictionaries. In this case, all the hearers and the speaker would abandon their original dictionaries and would only retain the word in the conversation.

All the steps are repeated until convergence is reached. An important point to note here is that in addition to simultaneous asynchronous conversations, in the case of multiple hearers', after each conversation, more than one hearer in each group may update their dictionaries. This speeds up the process of global consensus and word diffusion even more.

In Figure 2, we compare the working of the extended model to the classical Naming Game model. On the left, we see the classical model when agents (speakers) engage in pairwise conversations with other agents. An important component of the classical model is that a speaker cannot be a hearer at the same time. In the extended model, we extend the conversations to more than one hearer. On the right, we see two cases: (a) every speaker interacting with two hearers, and (b) every speaker interacting with three hearers. A snapshot of the system at any given time appears in the form of a 2-partite graph with each node on the left having exactly H edges to the nodes on the right. Each agent on the left will randomly interact with agents on the right, excluding itself. An important thing to note is that a speaker in one conversation can be a hearer in some other conversation at the same time. This is the concurrency that was introduced by Perumalla (2017).

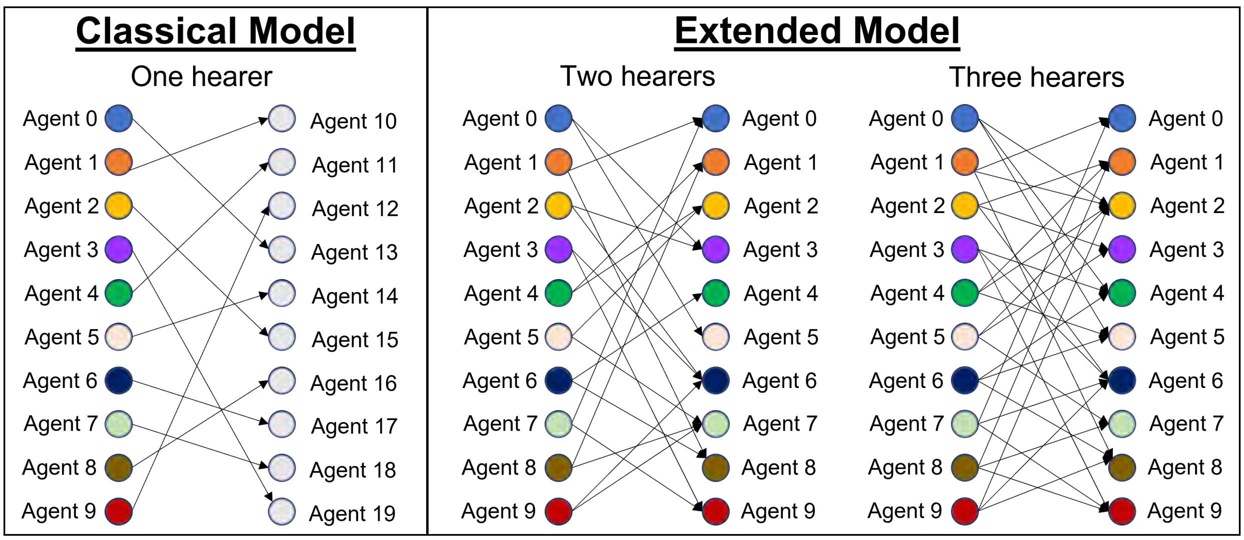

Figure 2: On the left, we see the classical model where agents engage in pairwise interactions. Each agent interacts with one other agent. Note that an agent cannot be a hearer and speaker at the same time. We extend this model to a multiple hearers model where one agent is interacting with two or three (or in general $H \geq 2$ other agents. In the extended model, a speaker in one conversation can be a hearer in another conversation.

3.2 Model Parameters and Measures

For a base case, we have taken several simplifying assumptions. Here, we list the parameters used in the model configuration along with their semantics.

- *Population size* (N): This denotes the number of agents N in this model. As a base case, we consider the system to consist of 1000 agents.
- *Size of initial dictionary*: At the beginning of each game, each agent will be initialized with a single globally unique word in its dictionary.
- *Number of unique words in the system* (N_d): This is the number of unique words N_d in the entire system at a given time in the simulation. The dictionaries of each agent form subsets of these N_d unique words at any time during the game. In the initial configurations for all experiments, we

set each agent's dictionary to contain exactly one unique word, giving $N_d = 1000$ at $t = 0$ for the system of 1000 agents.

- *Number of words in the system* (N_w): This is the sum of dictionary sizes of all agents, which gives the total number of words N_w in the system at time t.
- *Number of contacts*: Each agent (speaker) may reach out to any other agent from the entire population. That is, we assume a complete graph network of potential conversation links.

3.3 Basic Phenomenon

The dynamics of the game are characterized by three temporal regimes: (1) initially, the words are picked from the dictionaries; (2) the words are spread throughout the system; (3) the words undergo final convergence towards global consensus, where all agents eventually possess the same unique word. The main quantities that describe this evolution are N_d and N_w (Baronchelli et al. 2006).

We observe these two quantities N_d and N_W after each conversation while multiple pairs of speakers and hearers engage in conversations and update their dictionaries. This process takes place repeatedly until convergence is reached, as defined in Section 3.1. We investigate the convergence behaviour using the model parameters described in Section 3.2. To identify the number of unique words, we compute the union of the dictionaries of all agents at each step, thereby collapsing duplicate words into one. Consensus is declared to be reached when $N_d = 1$ and $N_w = N$ (number of agents).

3.4 Network Types

Before we look at the results of the model, we define the network types by which agents may be connected. The relation between the agents may not always be random. The individuals who are connected by some relation may be more prone to connect or interact with each other. We explored this aspect by defining our agent-based model on standard network types as discussed below.

- Random: In a random network type, the agents are randomly connected with a given average number of connections per agent (see Newman (2003) for the theory of random networks and Wilensky and Rand (2015) for the use of random networks in agent-based models).
- Distance-based: Any two agents are connected only if the distance between them is less than a given maximum distance (Honarkhah and Caers 2010).
- Small world: In a small world network, any two agents can reach each other through a short sequence of acquaintances. The agents are connected to a given number of closest agents with some agents' connections "rewired" to long-distance agents (Watts and Strogatz 1998).

4 SIMULATION RESULTS

The aim of the Naming Game is that the game would terminate when all the agents possess the same unique word in their dictionaries. This is what we called as "convergence". In this section, we develop the baseline convergence behavior first of the classical model involving only one hearer per speaker. Next, we extend this model to multiple hearers, increasing the number of hearers and comparing the results. We further investigate the impact of network types on the rate of convergence and, finally, the impact of population size on a multiple hearers model. All the models discussed here are developed using AnyLogic simulation software.

4.1 Classical Model

Here we look at the change in the number of unique words in the system the, N_d, and the total number of words in the system, N_w, over the total number of conversations using base parameters (as defined in Section 3.2). Figure 3 shows the results in the case of a random network with only one hearer. Note that the base case (with only one hearer) would be similar to the classical Naming Game. We see an initial rise followed by a decrease in the number of words only during successful conversations. We expect to see fewer successful conversations at first. However, once the words start to diffuse and the dictionaries start to reduce in size, the convergence happens at a much faster rate. Once the consensus is reached, N_w would equal N since no new words are introduced into the system. The results are consistent with those in previous studies (Baronchelli 2016; Perumalla 2017).

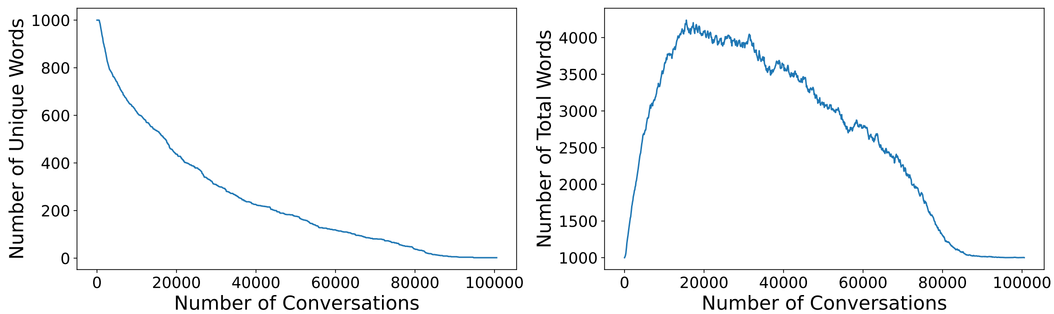

Figure 3: Here we present the plots using the base parameters, with a population size of 1000 where each agent has a unique word in its dictionary in the beginning. This is the same as the classical Naming Game. The plot on the left displays N_d on the vertical axis over the number of conversations, and the plot on the right is N_w on the vertical axis over the number of conversations in a random network using the base parameters.

4.1.1 Extending the Classical Model to Multiple Hearers

Using our new model, we steadily increase the number of hearers per speaker in a conversation and observe the convergence results under each case with a random network. With a population size of $N = 1000$, each agent starts with a unique word in their dictionaries, but in each conversation, there are more hearers than one. Figure 4 shows how varying the number of hearers impacts the number of conversations to convergence. As we would expect, with an increase in the number of hearers, a greater number of agents update their dictionaries after each successful conversation. The number of conversations for convergence reduces drastically from about 100,000 conversations in the classical set up (one hearer) to about 11,000 conversations in our extended model (ten hearers).

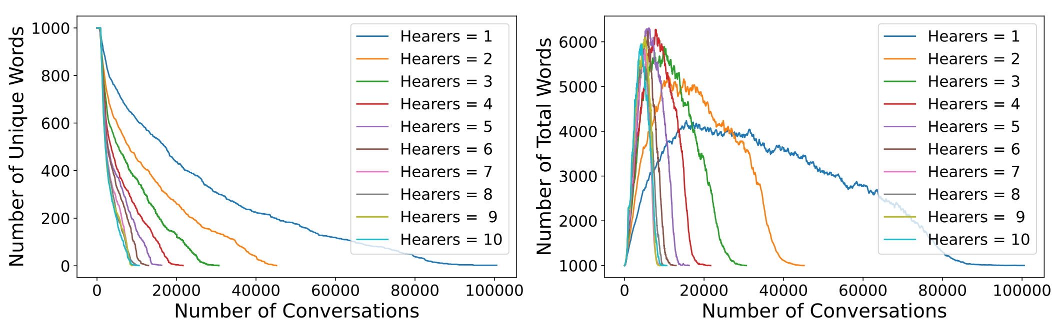

Figure 4: These plots present the convergence when we vary the number of hearers from the classical setting in a game with 1000 agents assuming a random network. We see the impact on the rate of convergence of unique words on the left and total words on the right. As expected, with the increase in the number of hearers, the words diffuse at a faster rate.

4.2 Varying the Network Types

Next, we investigate the impact of network types on convergence. Our base case assumes that the agents are connected via a complete graph setup. That is, an agent can reach out to and engage in a conversation with any other agent, irrespective of the distance between them, number of connections, etc. However, since the agents may not always be connected randomly, the type of network may also have an impact on the number of conversations to convergence. That is, when individuals are closely connected, we would expect to see convergence at a faster rate. However, if the individuals are connected only to a small number of other individuals, it may take a large number of conversations converge. In order to explore this further, we study the impact of network types on the number of conversations to convergence in this section.

4.2.1 Distance-Based Interaction Network

Recall that, in a distance-based network, any two agents are connected only if the distance between them is smaller than a defined value. Here we consider a case where the agents would only be connected when the distance between them is at most 80 units in the simulation. Consider the results in Figure 5 for the base case (only one hearer) in the case of a distance-based network. Consistence with our expectations, the convergence is observed to happen at a relatively high number of conversations. The words diffuse rather rapidly for the first 100,000 conversations. But, after that phase, it takes a large number of conversations for the model to fully converge. As the game starts, the distance does not constitute a major constraint because there are many connected agents available to engage in successful conversations that would speed up the diffusion of words. As the simulation progresses and the dictionaries of agents start to converge, the maximum distance-based connectivity starts to become a constraint, preventing rapid progress. The dictionaries of the connected agents may already have been converged, and it may require a lot of conversations to happen before the full diffusion of the words takes place across the width of the network.

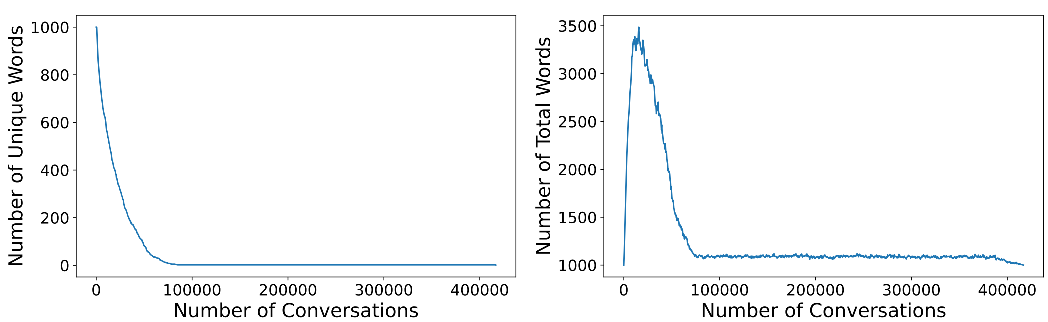

Figure 5: Using the base assumptions of 1000 agents and one hearer per conversation, here we investigate the convergence in a distance-based network where agents are connected only if the distance between them is smaller than 80 units. As we see here, the diffusion of words happens rapidly in the beginning, but, after a point, it takes a very large number of conversations to converge. The distance constraint between the agents is causing the delay in model convergence.

Next, we further explore the impact of varying the number of hearers in a distance-based network. In Figure 6, we steadily increase the number of hearers from one to ten. We observe the faster convergence as we increase the number of hearers. The results here are different than for the random network case. The total number of conversations required here for one hearer case is 400,000 which is much higher than classical set-up (100,000 conversations). However, in the case of ten hearers per speaker the results are similar to the base case (about 13,000 conversations in distance-based vs about 11,000 conversations in random network).

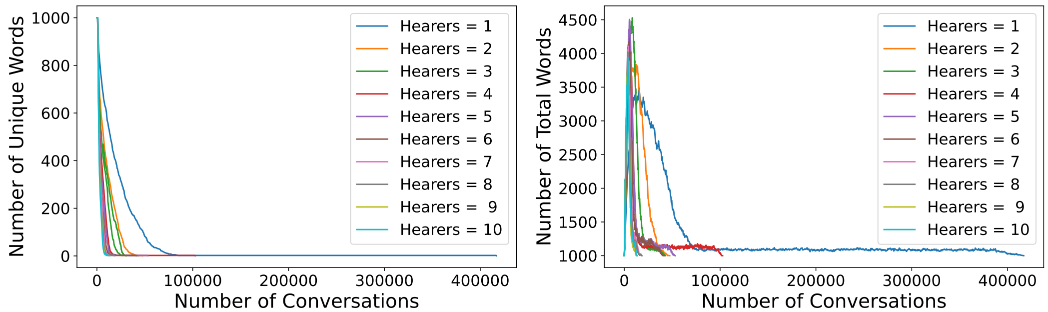

Figure 6: We vary the number of hearers in case of a distance-based network and observe that as we gradually increase the number of hearers from from one to ten, the rate of convergence also increases.

4.2.2 Small World Interaction Network

In this set of experiments, we model a small world network as follows. An agent is connected directly only to 100 other agents. In case of a small world network, most agents are not connected directly, but have many common connections. Here we assume that 5% of the agents will have long-distance connections. Refer to Figure 7 for the results in the case of a single hearer. Similar to the distance-based case and in line with our expectations, the diffusion of words happens drastically in the beginning, but it takes a really large number of conversations to converge fully, in comparison to a random network. That is, after a point the limited number of connections becomes a constraint, and it takes a while before any diffusion of words takes place. Because most of the agents interact only with the nearest neighbors, the model requires a large number of conversations to happen for convergence.

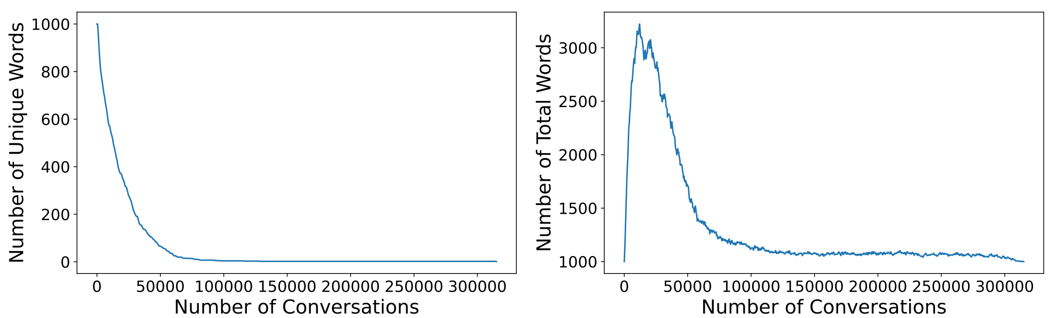

Figure 7: Here we present the results in a small work network, with a population size of 1000 and parallel interactions with one hearer. We assumed that an agent interacts only with the nearest 100 agents and 5% of the agents will have long-distance connections. Since most of the agents will interact with the nearest agents, initially the diffusion of words happens drastically, but after a point it takes a large number of conversations to fully converge.

We further explore the impact of varying the number of hearers on the number of conversations required for convergence (Figure 8). The results again look similar to the distance-based network case; however, the total number of conversations required to reach global convergence is lower than distance-based. The number of conversations required for consensus is reduced about one-tenth from over 300,000 in the case of one hearer to only about 30,000 in the case of ten hearers per agent.

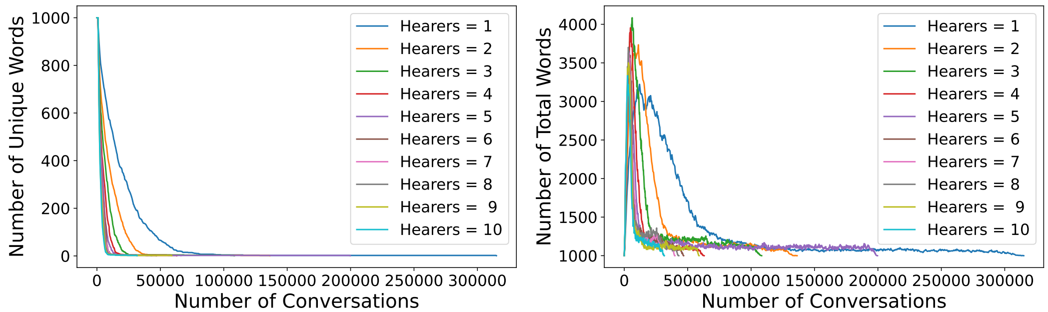

Figure 8: Here we see the impact of varying the number of hearers on the number of unique words (left) and on the number of total words (right). As we would expect, the model would converge faster as we increase the number of hearers.

4.3 Varying the Population Size

Next, we examine the change in number of conversations required for convergence when we vary the size of population (number of agents) from 1,000 to 5,000, 10,000 and finally 20,000 under different number of hearers. Although, in this section we vary the population size, but for simplicity, we have assumed that each agent is connected to (can interact only with) 1000 other agents.

First, we check the results while steadily increasing the number of agents. We consider two cases here: number of hearers = 1 (classical model) and number of hearers = 2. As we observe in Figure 9, we see a steady increase in the number of conversations for convergence when the population size is increased from 1,000 to 20,000 in both cases. However, with two hearers, the convergence is considerably faster.

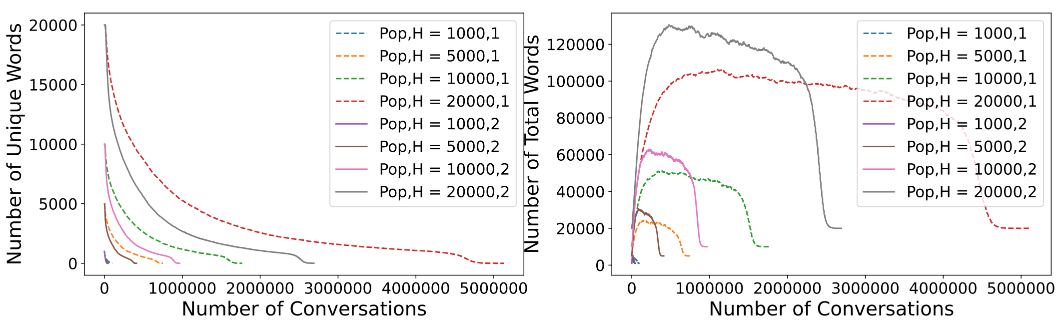

Figure 9: Variation of number of conversations for convergence with the agent population size in case of random network. Each term labeled Pop,H in the plots represents the population size and the number of hearers. We see here that there is a steady increase in the number of conversations required for convergence as the size of the population is increased.

4.4 Varying the Number of Hearers and Population Sizes

Finally, we investigate the impact on the convergence when we vary the number of hearers under different population sizes. We consider the population sizes of 1,000, 5,000, 10,000 and 20,000 agents and examine the results while steadily increasing the number of hearers from one to ten. Figure 10 documents the variation of number of conversations to the number of hearers for different population sizes. We plot the logarithm of the number of conversations since the data is right skewed and plots are easier to interpret in log-scale. We see an exponential decrease in the number of conversations required for convergence with the increase in the number of hearers across all population sizes. This is expected since dictionaries converge faster with the increase in number of hearers. We do observe slight deviations in some configurations, which may be caused by variations among simulation runs. Further analytical treatment is identified as future work.

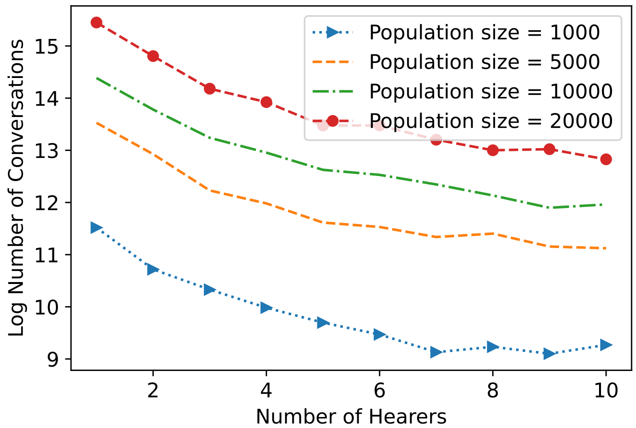

Figure 10: We plot the log of the number of conversations against the number of hearers across different population sizes. We observe that as the number of hearers increases, there is a clear downward trend in the number of conversations for convergence across all the population sizes.

5 SUMMARY AND FUTURE WORK

The Naming Game is a fundamental model in social networks, explaining several basic phenomena such as consensus dynamics and language formation. In this study, we revisited the classical sequential algorithm for the Naming Game to address some of its limitations. In contrast to the minimal Naming Game, we extended the model to include multiple hearers per conversation. We further extended the notion of multiple hearers to parallel conversations where each speaker can be a hearer in another conversation at the same time. This more closely models reality in which agents do not behave sequentially, nor do they always restrict interactions to pairs. We capture the concurrency in multiple-hearer conversations as an essential element of social behaviour models. This extended model enables more realistic networks to be built.

Compared to the traditional pairwise model, multiple hearers model accelerate towards convergence at a much faster rate. When agents engage in group conversations, multiple agents update their dictionaries in each conversation step, making the speed of convergence increase rapidly. We also explored the case of different network types to model inter-agent connectivity beyond random or clique-based interactions. We further extended our analysis to multiple population sizes. Our multiple hearers model on different data sizes serves to guide the introduction or enhancement of multiple hearers in a concurrent environment in social behavioral models. We performed preliminary analysis of our model on different data sizes. As a part of future work, we propose conducting detailed quantitative analysis of the extended model that we introduced in this paper.

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