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# A LATTICE BOLTZMANN ADVECTION DIFFUSION MODEL FOR OCEAN OIL SPILL SURFACE TRANSPORT PREDICTION

Zhanyang Zhang
Michael E. Kress

Doctoral Program in Computer Science
City University of New York
365 5th Avenue
New York, NY 10016, USA

Tobias Schäfer

Doctoral Program in Physics
City University of New York
365 5th Avenue
New York, NY 10016, USA

## ABSTRACT

The focus of our study is to investigate the feasibility and effectiveness of using Lattice Boltzmann Advection Diffusion Equation (LBM-ADE) to model and simulate ocean oil spill transport at the surface level. We present some preliminary results from a prototype model and simulation in limited scale (a sub area of Gulf of Mexico) with assimilation of real ocean current data from the Unified Wave Interface-Coupled Model (UWIN-CM). We validate our model in a benchmark study against GNOME, a tool developed and used by NOAA for ocean oil spill forecast, under two scenarios: (i) a Gaussian hill concentration using a linear ocean current with the analytical solution as a reference; (ii) a Gaussian hill concentration using real ocean current from (UWIN-CM). Our benchmark results in both cases show the LBM-ADE model solutions are very close to the targeted analytical and GNOME solutions with the same initial oil spill and location.

## 1 INTRODUCTION

Ocean oil spills have devastating impacts on marine ecosystems and human society in the surrounding coastal areas. The 2010 Deep Water Horizon spill in the Gulf of Mexico lasted 87 days and was estimated to have released over 3 million barrels of oil. It impacted over 1,600 miles of coastline, killed over 8,000 marine animals/seabirds and caused direct economic loss from fishing and tour industries estimated at tens of billions of dollars (Graham and Reilly 2011). In addition, the impacts to long-term public health and quality of life of millions of people are still unknown.

The transport of oil spilled into the ocean is a complex process that depends in a critical way on the current, wind, temperature and chemical composition of the oil and seawater (Mishra and Kumar 2015). Weathering further compounds the complexity of this process, a phenomenon which involves evaporation, emulsification, dissolution, oxidation and microbial processes. Spilled oil in oceans undergoes these physical, chemical and biological processes and will be transformed into substances with physical and chemical characteristics that differ from the original source material (National Research Council 2003). The National Research Council study indicates the fate of oil spilled into oceans as going through three major phases: (i) after oil introduced into the oceans; (ii) transport the resulting degradation oil away from the source; and (iii) incorporate the residual substances into compartments of the earth's surface system. These compartments involve dissolution in the hydrosphere, deposition in the lithosphere, volatilization into the atmosphere, and ingestion by organisms in the biosphere.

It is a costly and time consuming process to contain and collect spilled oil in the ocean to minimize the damage and stop or delay the oil reaching coastal areas. Two of the commonly used methods in the Gulf oil spill were using floating booms and chemical dispersants. These methods become less effective if they were not effectively deployed in early days right after oil spills. To effectively deploy limited resources, it

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is critical to be able to predict the oil transportation paths in the ocean. Therefore, we focus our study on investigating the feasibility and effectiveness of using the Lattice Boltzmann Advection Diffusion Equation (LBM-ADE) to model and simulate ocean oil spill transport at surface level.

During the first phase of our research, We developed an LBM-ADE model and simulation that is capable of providing numerical solutions as an LBM-ADE solver. To validate the model, we performed a benchmark test using a Gaussian Hill concentration with a simplified velocity field first. However, the ocean surface current constitutes a much more complex velocity field that is temporal-spatial dependent. We tested the LBM-ADE solver against a Finite Differential Method (FDM) ADE solver using a perturbation of the Taylor-Green velocity field. To the best of our knowledge, no such benchmark has been done in the past for an LBM-ADE model using a velocity field as complex as the perturbed Taylor-Green field. Our first phase study shows the LBM-ADE model achieves great results in comparisons vs analytical solution in Gaussian Hill case and vs the FDM-ADE solution in Taylor-Green case (Zhang et al. 2020).

In the next phase of our study, we want to bring the LBM-ADE model one step closer to solving ocean oil spill problems. We developed a prototype model and simulation in limited scale, a sub area of the Gulf of Mexico, with assimilation of real data from the UWIN-CM ocean current model (University of Washington 2018). We conducted two simulation experiments in comparison of the LBM-ADE model vs the GNOME (General NOAA Operational Modeling Environment), a modeling tool which the Office of Response and Restoration's (ORR) Emergency Response Division has been using to predict the possible oil transports on ocean surface (NOAA ORR 2012). GNOME is an ADE based tool with Lagrangian particles, which relies on the accuracy of the ocean surface current velocity to produce quality results. We achieved great results using Gaussian Hill concentrations with two ocean current scenarios: linear ocean current and real ocean current from (UWIN-CM). These results suggest that the LBM-ADE is a promising model that is capable of predicting spilled oil transport on an ocean surface.

Our previous work shows LBM Navier-Stokes Equation (LBM-NSE) model applies mostly to the time immediately after the spill and near the source (Zhang et al. 2020). We are also looking into the possibilities of using LBM-NSE as well as LBM-ADE to model multi species and multi phase flows which can be applied to modeling the ocean oil weathering process, such as mixed water and oil droplets in ocean water columns. At this time, we present our model results of ocean oil spill transport at surface level. We expect to present the weathering component in future work.

## 2 RELATED WORKS

There are many publications regarding using LBM to model and solve ocean flow problems. Notably, Wolf-Gladrow's work (Wolf-Gladrow 2005) used the LBM to solve the linearized Munk Problem (Munk 1950). In another LBM application of ocean models, Nuraiman (2017) used an 1D Shallow Water Equation representation of the Navier Stokes Equations coupled to the 2D Navier Stokes Equation to form an LBM model using the Bhatnagar, Gross and Krook (BGK) kinetic theory (Cercignani 1988). But to our knowledge there have been only a few studies of oil spill tracking using the LBM. One of the most comprehensive studies was done by (Maslo et al. 2014) and showed good agreement between simulated results and satellite observations from an oil spill in the Gulf of Beirut on July 15, 2006. Their LBM model used a two relaxation parameter technique to facilitate numerical stability. In addition, a flux limiter computational technique was used to resolve sharp numerical boundaries, which led to negative densities. Further, an interpolation technique was used to permit a non-square lattice to resolve the flow along the elongated coastline studied. In addition, Ha and Ku (2012) used an LBM model to simulate an advective-diffusion formulation of the spread of an oil slick on the sea surface and confirmed the functionality of their model. Further, Li et al. (2017) solve the 2D convection-diffusion equation using the LBM. Other advection-diffusion equation solutions are presented by Dedits et al. (2015). While not specifically studying oil transport, Li and Huang (2009) used a coupled LBM formulation of the Shallow Water Equation and Contamination Concentration Transport. Excellent agreement was obtained between numerical predictions and analytical solutions in the pure diffusion problem and convection–diffusion problem. Banda and Seaid (2012) also developed an LBM model to solve shallow water equations as the depth-averaged incompressible Navier-Stokes equations with conservation of momentum under the assumption that the vertical scale is much smaller than any typical horizontal scale and the pressure is hydrostatic. Then they apply their shallow water model to simulate pollutant transport in the Strait of Gibraltar.

Our literature review shows most of ocean oil spill and contamination transport models are based on the ADE. In addition to the above cited research work, GNOME, the modeling tool of the Office of Response and Restoration's (ORR) Emergency Response Division has been used to predict the possible route, or trajectory, a pollutant might transport on the surface of water. GNOME is an ADE based tool, which relies on the accuracy of the ocean surface current velocity field to produce quality results. Also, GNOME is based on simulating an ensemble of stochastic particles such that the accuracy depends on the number of particles in the simulation. In contrast, LBM computes the distribution function directly and is expected to be computationally more efficient in situations where GNOME would require an exceedingly large number of particles in order to provide an accurate solution.

ADIOS (Automated Data Inquiry for Oil Spills) was developed by the NOAA to model oil weathering process (NOAA ORR 2019). It is an oil spill response tool that models how different types of oil weather (undergo physical and chemical changes) in the marine environment. As the National Research Council (2003) report has described, oil weathering is a process that spilled oil will be transformed into substances with mixed water and oil droplets which need to be modeled as multi-species fluids.

An important aspect of the LBM is the ability to implement multicomponent fluids including immiscible fluids like oil and water (Succi 2001; Shan and Chen 1993). Simulating oil transport in seawater current presents a number of very significant challenges including identifying the boundary layer between the seawater and oil. This challenge is complicated because the oil seawater mixture changes over time due to the effects of tides and wind as well as exposure to the atmosphere with solar radiation and temperature. Even without these complex physical and environmental effects, identifying the interface between the two dominant fluids, seawater and oil, is a difficult free boundary problem. Assuming that the oil and water are immiscible, at least at the initial point of the release, Leclaire et al. (2012) provided a numerical evaluation of two recoloring operators in calculating the two fluid flows. The technical aspects of the LBM are based on a revised collision operator that uses a BGK single relaxation parameter technique for each fluid separately followed by a step which redistributes the two fluids to define an interface based on the gradient of the density of fluids. This step solves the free boundary problem and prepares the model for the streaming step.

The above mentioned related work lead us to believe that LBM, as a kinetic theory based modeling technique, can support multimodal analysis for ocean oil spills including NSE and ADE based models, as well as multi-fluid models. It makes the LBM a potential uniform platform that can be applied to all three phases of ocean oil spill as described in the National Research Council (2003) report.

## 3 LATTICE BOLTZMANN METHODOLOGY

We use the LBM to model ocean oil pollutants as a set of particles with certain density and mass located on a virtual grid (lattice) that maps over an area of ocean with boundary conditions representing coastal lines or islands. This model makes it possible to track particle spatial positions and microscopic momenta from a continuum to just a handful and similarly discrete in distinct steps. Particle positions are confined to the nodes of the lattice. Variations in momentum that could have been due to a continuum of velocity directions and magnitudes and varying particle mass are reduced (in a simple 2D model) to 9 directions and a single particle mass (Sukop and Thorne 2007). Figure 1 shows the Cartesian lattice and the velocities $e_a$ (where a = 0, 1 ... 8) is a direction index and $e_0 = 0$ denotes particles at rest. This model is known as D2Q9 as it is 2 dimensional and contains 9 velocities. It can be generalized to a 3 dimensional model as D3Q27 if we replace the lattice in D2Q9 with a cube with length, width and height are one lattice unit.

The next step is to incorporate the single-species distribution function f, which has only nine discrete 'bins' instead of being a continuous function. The distribution function can conveniently be thought of as a histogram representing a frequency of occurrence. For example the shaded area in Figure 1 shows a likely oil pollutant propagation pattern after one time step.

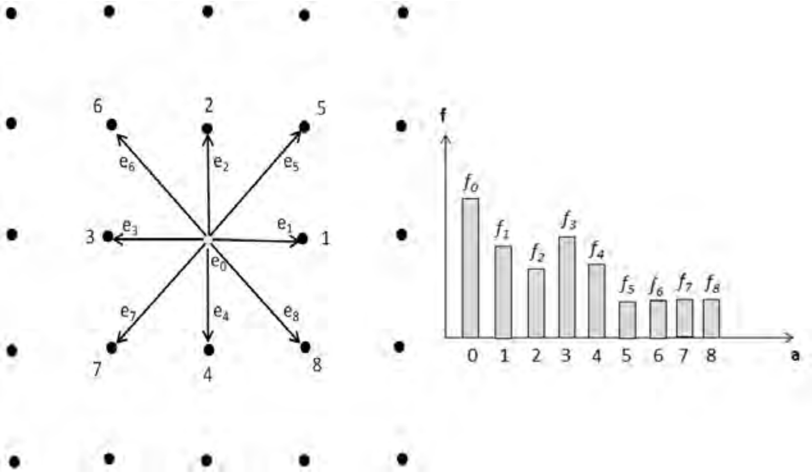

Figure 1: The particle distribution function represents the percentage of particles in the velocity bins.

Accordingly, the macroscopic fluid density is defined as:

$$\rho = \sum_{a=0}^{8} f_a \tag{1}$$

The macroscopic velocity $u$ is an average of the microscopic velocities $e_a$ weighted by the directional densities $f_a$ as defined as:

$$u = \frac{1}{\rho} \sum_{a=0}^{8} f_a e_a \tag{2}$$

This simple equation allows us to pass from the discrete microscopic velocities that comprise the LBM back to a continuum of macroscopic velocities representing the fluid's motion.

When incorporating external forces, such as wind, gravity and others, that interact with the ocean water, Equation (3) can be modify as:

$$u = \frac{1}{\rho} \sum_{a=0}^{8} f_a e_a + \frac{F \Delta t}{2\rho} \tag{3}$$

where the first term is the velocity due to mass density redistribution with conservation of momentum and the second term is due to external forces (Kruger et al. 2017).

Equation (3) is a generalization of the LBM that is applicable to both NSE and ADE models. In the LBM NSE model, Equation 3 provides a mathematical base for developing a velocity projection schema that integrates the ocean surface velocity into an LBM model as an external input, and then uses it to update local equilibrium distribution functions $f^{eq}$. On the other hand, in the LBM ADE model, we ignore the first term in (3) and only consider the second term as an advective velocity resulting from external forces since ADE only conserves mass, not momentum.

The next steps are streaming and collision of the particles via the distribution function. The simplest approach to approximate the collision can be defined as:

$$f_a(x + e_a \Delta t, t + \Delta t) = f_a(x,t) - \frac{f_a(x,t) - f_a^{eq}(x,t)}{\tau} \tag{4}$$

where $\tau$ is a relaxation time used in the BGK operator. Although they can be combined into a single statement as above, collision and streaming steps must be separated if solid boundaries are present because the bounce back boundary condition is a separate collision. Collision of the fluid particles is considered as a relaxation towards a local equilibrium. The parameter $\tau$ is a relaxation time to reach equilibrium. A D2Q9 equilibrium distribution function $f^{eq}$ is defined as:

$$f_a^{eq}(x,t) = w_a \rho(x,t)[1 + 3\frac{e_a u}{c^2} + \frac{2(e_a u)^2}{9c^4} + \frac{3u^2}{2c^2}] \tag{5}$$

where the weights $w_a = (\frac{4}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{36}, \frac{1}{36}, \frac{1}{36}, \frac{1}{36})$ and c is the velocity on the lattice, one lattice unit per time step $(lu/\Delta t)$ in the simplest implementation. Note that if the macroscopic velocity u = 0, the equilibrium $f_a^{eq}$ are simply the weights times the fluid density.

To implement the LBM model as a simulation program, Bao and Meskas (2011) presented an algorithm outline that can be summarized as follows:

1. Initialize $\rho$, $\mu$, $f_a$ and $f_a^{eq}$;
2. Streaming step: move $f_a \rightarrow f_a^*$ in the direction of $e_a$, where $f_a^*$ holds intermediate values of density distribution after the streaming step.
3. Compute macroscopic $\rho$ and $\mu$ from $f_a^*$ using above equations (1) and (2);
4. Compute $f_a^{eq}$ using equation (3);
5. Collision step: calculate the updated distribution function using equation (4): $f_a = f_a^* - \frac{f_a^* - f_a^{eq}}{\tau}$;
6. Repeat steps 2 to 5.

During the streaming and collision step, the boundary nodes require some special treatments for the distribution functions in order to satisfy the imposed macroscopic boundary conditions. The LBM as described has been shown to be second order accurate in time and space to the 2D incompressible Navier Stokes Equations by Kruger et al. (2017) and separately by Wolf-Gladrow (2005).

LBM is a kinetic theory based modeling technique that can provide numerical solutions for a range of flow problems whose underline physics are governed by NSE and/or ADE.

When the NSE and the ADE are applied in near incompressible fluids, they can be expressed as equations (6) and (7).

$$\frac{\partial u}{\partial t} + u\nabla u = -\frac{\nabla p}{\rho} + \nu \nabla^2 u + F \tag{6}$$

where: $u$ is fluid velocity; $P$ is fluid pressure; $\rho$ is fluid density; $\nu$ is fluid kinematic viscosity; and $F$ is an external force.

$$\frac{\partial C}{\partial t} + u\nabla C = D\nabla^2 C + q \tag{7}$$

where: $C$ is mass concentration; $D$ is diffusion coefficient (assume isotropic diffusion); $u$ is fluid velocity as an advection force; and $q$ is a source term.

The LBM-NSE model is defined by the set of equations (1) to (5). While the LBM-NSE model conserves both mass and momentum, the LBM-ADE model only conserves mass (referred as concentration $C$). An LBM-ADE model is defined by the set of equations (8) to (10) as below:

$$C = \sum_{a=0}^{8} g_a \tag{8}$$

$$g_a(x + e_a \Delta t, t + \Delta t) = g_a(x,t) - \frac{g_a(x,t) - g_a^{eq}(x,t)}{\tau_g} \tag{9}$$

$$g_a^{eq}(x,t) = w_a C(x,t)[1 + 3\frac{e_a u}{c^2} + \frac{2(e_a u)^2}{9c^4} + \frac{3u^2}{2c^2}] \tag{10}$$

where the $g_a$ are the directional densities of concentration; $g_a^{eq}$ is the equilibrium density function; $\tau_g$ is the relaxation time and $u$ is a velocity vector due to advection forces, while $e_a$ and $w_a$ are the same as previously defined in (2) and (5).

Since the equations for the NSE (4) and equation for the ADE (9) are the same, the algorithm outlined by Bao and Meskas (2011) is also applicable to the ADE.

## 4 GNOME AND ITS STOCHASTIC MODEL

GNOME (NOAA ORR 2012) is an ADE based model using Lagrangian particles, referred as Lagrangian elements (LEs), to represent oil pollutants. All the LEs trajectories collectively represent the path of oil transport on the ocean surface. In a simplified ocean oil spill scenario without considering weathering process, each LE is moved independently by two forces, namely advection and diffusion. Furthermore GNOME assumes the advection and diffusion processes are independent of each other. GNOME models the advection process using a forward Euler scheme (a first-order Runge-Kutta method). Assume at time step $t$, a LE is at the point $p(x,y,t)$. It calculates the LE position after one time step $t+\Delta t$ at the point $p(x+\Delta x, y+\Delta y, t+\Delta t)$ using the velocity field for each element.

Diffusion is modeled as stochastic processes where a set of LEs engage a 2-D random walk with a displacement probability such that the mean value remains zero, but the variance grows linearly with time. It has been shown that a long series of random steps will converge to a Gaussian distribution with variance growing linearly with time (Csanady 1973). For a given input of diffusion coefficient, GNOME uses a particle random walk simulation, a stochastic process, to approximate the diffusion process.

## 5 LBM-ADE MODEL AND GNOME BENCHMARKS

In a previous study, we designed and developed LBM-NSE and LBM-ADE models and simulations. We benchmarked our LBM-NSE solution for Poiseuille flows against the analytical solutions. For our LBM-ADE model, we benchmarked our LBM-ADE solutions with Gaussian Hill concentration and a linear velocity field against analytical solutions. Since the ocean surface current is a much more complex velocity field that is temporal-spatial dependent, We tested the LBM-ADE solver against a Finite Differential Method ADE (FDM-ADE) solver using a perturbation of the Taylor-Green velocity field. All our benchmark tests achieved great results in comparisons with the analytical solutions and the FDM-ADE solutions (Zhang et al. 2020). Here, we compare our LBM model results to the results obtained by GNOME in a realistic ocean oil spill example.

Encouraged by our previous results, we benchmark our LBM-ADE solution in comparison to GNOME solution in a simplified ocean oil spill (without considering oil weathering) at a location close to the BP oil spill in 2010 at the Gulf of Mexico. This benchmark study was done with two different ocean surface current profiles as advection velocities, one is a linear velocity filed; and the other is temporal-spacial dependent velocity field that comes from a real ocean current data model (UWIN-CM).

GNOME is a modeling tool that is widely used by NOAA and EPA teams in response and restoration of ocean oil spills. To do a benchmark study with GNOME, even in a simplified oil spill scenario, it is essential to establish a baseline for future work which uses LBM to model and simulate the oil weathering process in the ocean.

### 5.1 Benchmark Design and Configuration

LBM and GNOME are both ADE based modeling tools that share the same governing physics laws that can be defined as a partial differential equation (7) where we set $q = 0$ except in the initial condition for a one-time oil spill release.

While the LBM model takes a kinetic approach and the GNOME model takes a stochastic approach, they both provide numerical solutions that approximate and converge to the analytical solution of equation (7). But there are differences between them which present several challenges in the benchmark study. To conduct a creditable comparison between LBM and GNOME, we need to carefully design and configure the benchmark experiment taking into consideration the following issues: (i) LBM works in Eulerian specification of flow field, while GNOME works in Lagrangian specification of flow field; (ii) LBM uses units of measurement of space and time in lattice unit, while GNOME uses units of measurement in meter and second; (iii) LBM uses a square lattice to represent a ocean surface area of 1° by 1° in longitude and latitude, while GNOME uses a non-square area of 97.904 km by 111.194 km due to the curvature of the earth's surface in longitude and latitude; and (iv) we need to specify the oil spill volumes and diffusion coefficients that are equivalent in both units of measurement in LBM and GNOME.

We design two benchmark experiments: The first one is to compare LBM-ADE, GNOME and the analytical solutions to a Gaussian hill initial concentration with a linear velocity. The second one is to compare LBM-ADE and GNOME solutions to the same Gaussian hill initial concentration with an ocean current velocity from the UWIN-CM ocean data model.

In the two simulation methods used for the benchmark, the LBM-ADE simulates the oil spill based on solution of the ADE partial differential equation with specified relaxation parameter, $\tau_g$ and associated diffusion coefficient in lattice units, $D_l$. GNOME simulates the oil spill based on a stochastic particle analysis with a combination of particle trajectory tracing and stochastic Brownian motion with specified diffusion coefficient, $D_p$.

### 5.2 LBM-ADE and GNOME Benchmark with a Linear Velocity

We introduced three performance indicators to quantitatively comparing the benchmark results: (i) the maximum value of concentration over time; (ii) the locations of the maximum value of concentration over time; and (iii) Root Mean Square (RMS) to measure the the difference between the concentration of the two models.

The analytical ADE and LBM-ADE solve the ADE equation (7) with an initial condition of a Gaussian hill. While GNOME starts with an initial condition of point release with 10,000 LEs representing the volume of spilled oil.

The analytical solution of ADE with Gaussian hill is given in equation (11)

$$C(X,t) = \frac{\sigma_0^2}{(\sigma_0^2+\sigma_D^2)} C_0 e^{\frac{(-(X-X_0-Ut)^2)}{2(\sigma_0^2+\sigma_D^2)}} \tag{11}$$

For a detailed explanation of the parameters in this equation we refer to (Kruger et al. 2017).

Table 1 shows the numerical quantities in each models side by side where we assume the values of $\tau_g$ are chosen in relation to the diffusion coefficient. All the unit conversions are based on scaling laws (Kruger et al. 2017).

Table 1: A list of numerical quantities in each model.

| Analytical ADE | LBM-ADE | GNOME |
|---|---|---|
| Grid size (200,200) | lattice size (200,200) | (1°, 1°) longitude and latitude |
| Initial oil spill as a Gaussian hill | Initial oil spill as a Gaussian hill | Point source: 10,000 LEs |
| $C_0 = 2.5e^{-6}$, $\sigma_0 = 5.0$ | $C_0 = 2.5e^{-6}$, $\sigma_0 = 5.0$ | initial oil volume: 67,520 m$^3$ |
| Relaxation time: NA | Relaxation time: $\tau_g = 1.0$ | Relaxation time: NA |
| Diffusion: $D_l = .1129$ | $D_l = \frac{c^2}{9}(\tau_g - \frac{\Delta t}{2})$ | $D_p = 101.61 m^2/s$ |
| Lattice velocity, $c = \frac{\Delta X}{\Delta t}$ | Lattice velocity, $c = \frac{\Delta X}{\Delta t}$ | $\Delta t = 15$ minutes |
| Velocity field, u=(0.2758,0.2428) | u=(0.2758,0.2428) | u=(0.15,0.15) in m/s |

In this benchmark study, we introduce a one-time oil spill as a Gaussian hill in the center of the grid at time step $t = 0$ as an initial oil spill in both the analytical ADE and the LBM-ADE. The volume of a Gaussian hill is a function of $C_0$ and $\sigma_0$ which are calculated to be the volume of oil in GNOME. GNOME uses a point source of 10,000 LEs to represent the oil volume. Then we let the models run 296 time steps, with $\Delta t = 15$ minutes, a total of 74 hours.

To present the benchmark results, we start with Figure 2 that shows a 3-D visual comparison LBM and analytical ADE solutions for the Gaussian hill at the initial and last steps. Then followed by Figure 3 that shows the locations of maximum concentration over time and the values of the maximum concentration as the Gaussian hill diffuses over time. It is worthy to note that due to stochastic fluctuations in GNOME the LEs scatter around the analytical solution. The results shows the LBM-ADE solution has excellent agreement with solutions in both Analytical ADE and GNOME.

Since in GNOME the initial oil spill is modeled as a point source release, all the LEs are located at the center point of the grid instead of a Gaussian hill distribution as in LBM-ADE and analytical ADE. It is interesting to point out that, regardless this initial oil distribution difference, the GNOME maximum concentration diffusion converges to LBM-ADE and Analytical ADE after a transient period of approximately 40 time steps. At the same time, there is no transient effect in Gaussian hill distance traveled. This makes sense since the transient effect only impacts the diffusion process and has no impact on the advection process.

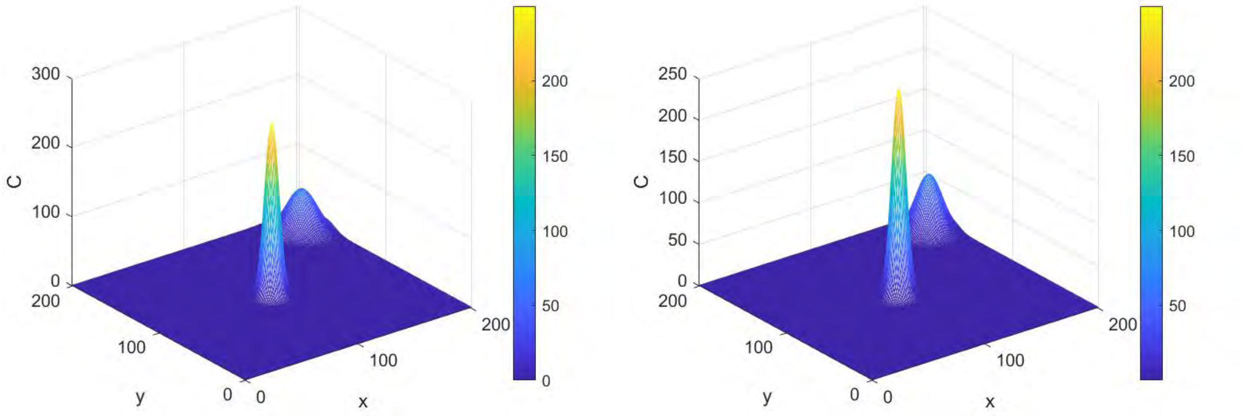

Figure 2: Gaussian hill concentration at initial and final time; (left) LBM-ADE and (right) Analytic ADE.

### 5.3 LBM-ADE and GNOME Comparison with Ocean Current Data

In this study, we compare the LBM-ADE solution with GNOME in the same configuration as in subsection 5.2 except replacing the linear velocity field with a velocity field that comes from a real ocean data model (UWIN-CM). Since the velocity field from UWIN-CM model is temporal-spatial dependent, there is no analytical solution in this case. UWIN-CM is a fully coupled atmosphere–wave–ocean system data model. We used a subset of data from the UWIN-CM ocean model to cover a 1 degree square area of the Gulf of Mexico centered at -88.4 longitude and 28.8 latitude over three days from Feb. 07, 2016 at 16:00:00 til Feb. 10, 2016 at 18:00:00. Figure 4 shows a map of the area in the Gulf of Mexico being modelled with the ocean current profile and simulated oil slick.

We used bi-linear interpolation spatially to generate a velocity field for the LBM computational domain at each time step while $\Delta t = 15$ minutes. The ocean surface velocity field is assimilated in the LBM-ADE model as an advection velocity at each time step. In GNOME the ocean current data is used in the same

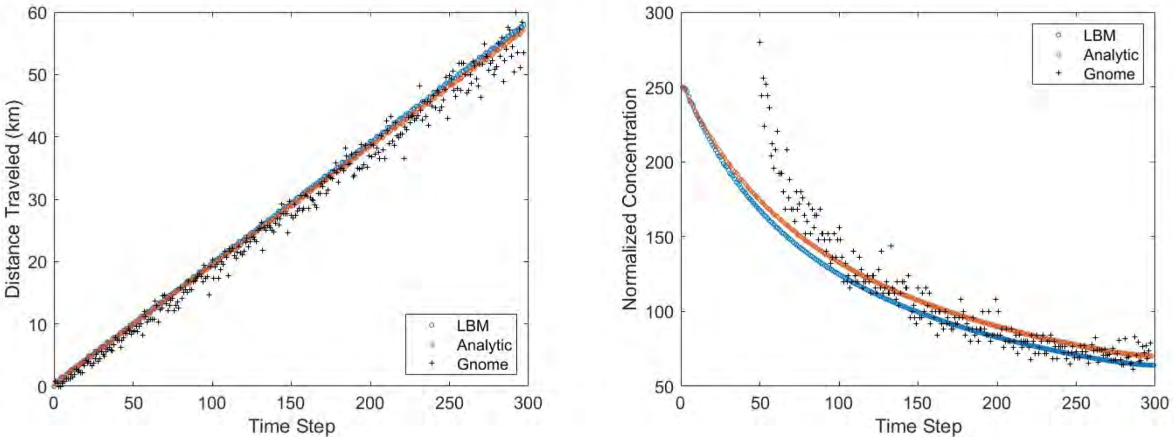

Figure 3: LBM-ADE, Analytical ADE and GNOME comparison, (left) Gaussian hill distance traveled in km; (right) normalized maximum concentration.

way except the velocity data and time steps are stored in a netCDF file, then loaded into the model during model configuration.

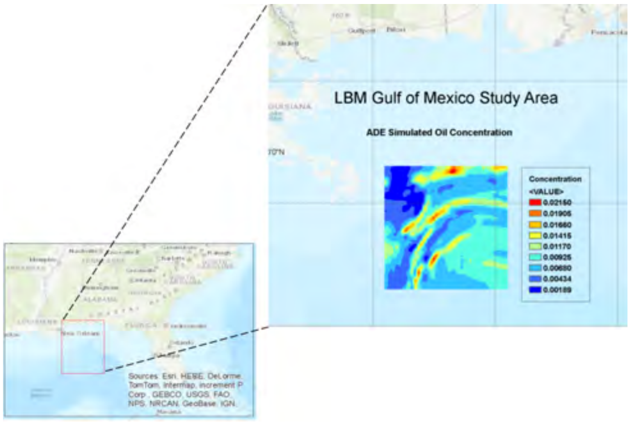

Figure 4: Study area in the Gulf of Mexico from (28.8,-88.4) to (29.8,-87.4) and the ocean profile for a simulated oil slick, normalized surface density from 1.89x10$^{-3}$ (blue) to 2.15x10$^{-3}$ (red).

We show the comparison results of the ADE process in both LBM and GNOME side by side in snap shots at time step 40 and 280 in Figure 5. These times were chosen to omit the GNOME transient effect and the loss of concentration at the north eastern boundary. These figures present a good visual comparison between LBM-ADE and GNOME for a given spilled oil volume and how the oil transport pattern under the diffusion and advection processes with real ocean current data.

Next we quantitatively measure the difference between LBM-ADE and GNOME using RMS performance indicate in selected time steps in Table 2. It shows the RMS value decreases as time step increase. This comparison study shows LBM-ADE and GNOME reaches a very good agreement in this oil spill scenario without weathering process as shown in Figure 6.

Regarding the timing comparison between LBM and GNOME, we found that, in our parameter regime, the computational times of the LBM solver and the GNOME solver are comparable. Both were implemented in Python and run on a laptop computer with 8GB of ram, and an Intel Core Duo T7500,

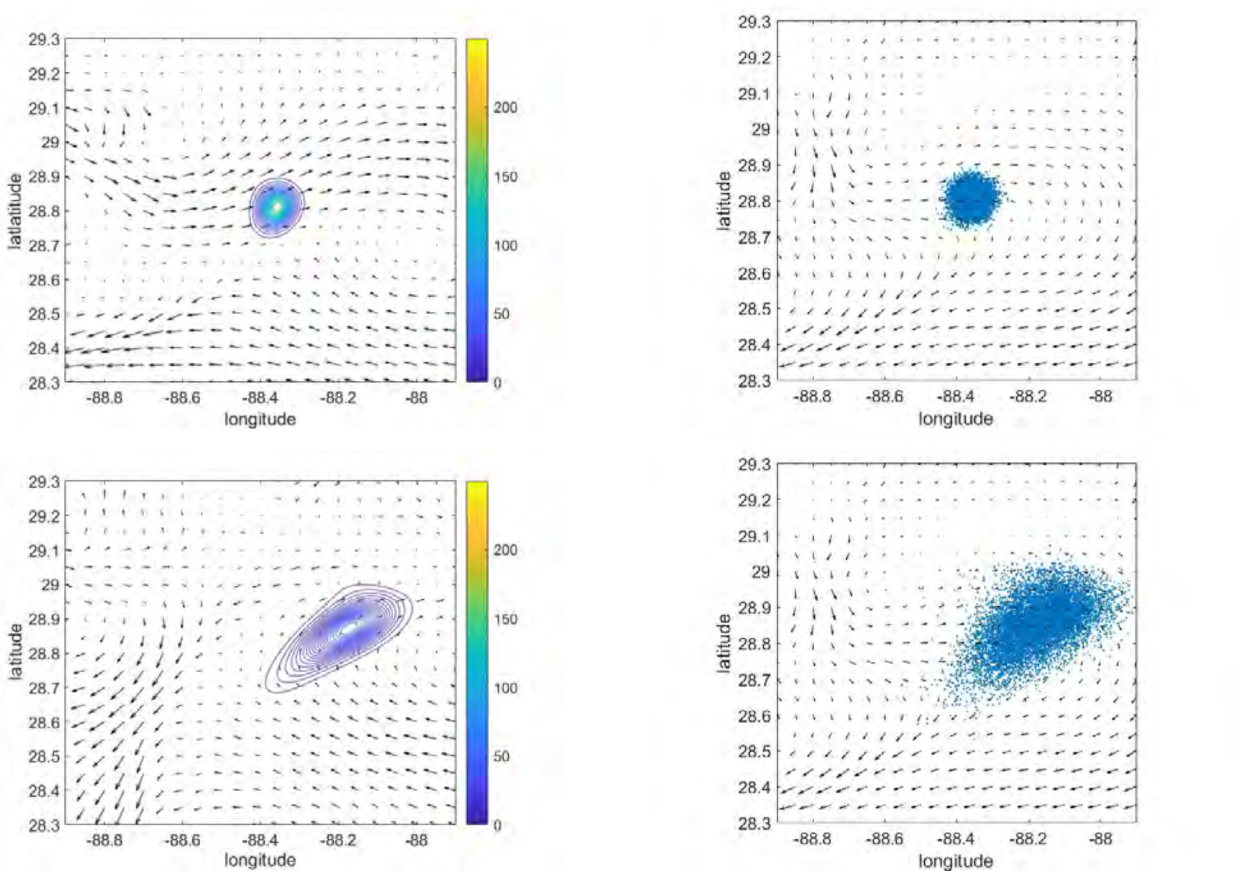

Figure 5: Concentration snap shots at time step 40 and 280, (left) LBM-ADE; (right) GNOME.

Table 2: Comparison of LBM-ADE and GNOME in RMS measures.

| Time Step | RMS(GNOME-LBM) |
|---|---|
| 279 | 2.6878 |
| 280 | 2.6514 |
| 281 | 2.6464 |

2.20GHz processor. As typical runtimes we found for LBM Python = 17.78s on a (200x200) grid and for GNOME PyGnome=19.89s for a simulation with 40,000 particles.

## 6 CONCLUSION

Our study shows that the LBM-ADE can be used to model oil transport on the ocean surface. We compare the LBM-ADE with GNOME in predicting oil spill transport on a surface level in a realistic ocean current scenario. While the LBM-ADE is a kinetic base model and GNOME is a stochastic based model, the numerical solutions are in very close agreement. To verify our approach, we compare the solutions of a LBM-ADE, an analytical ADE, and GNOME with a simplified Gaussian release oil spill using a linear ocean velocity field. Then we compare LBM-ADE and GNOME using a realistic ocean current data from a location in the Gulf of Mexico near the 2010 Deepwater Horizon oil spill. The results show excellent agreement.

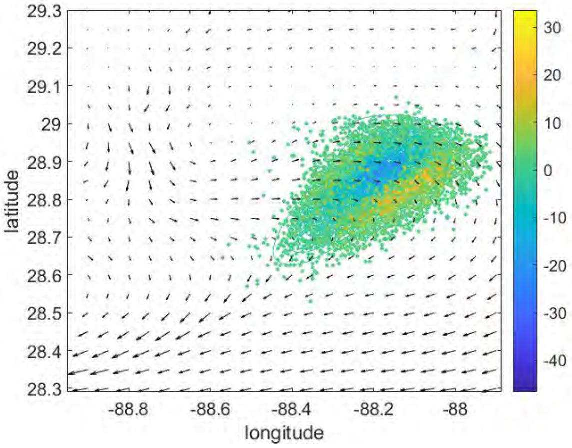

Figure 6: Contour plot of difference in concentration between LBM solution and GNOME solution at time step 280.

This study sets us on firm ground to further our research to include the oil weathering component in the LBM to make it a more realistic ocean oil spill modeling tool.

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## AUTHOR BIOGRAPHIES

**ZHANYANG ZHANG** is an associate professor in the Ph. D. Program in Computer Science at The Graduate Center of the City University of New York and in the Computer Science Department at the College of Staten Island. He earned his Ph.D. in Computer Science from City University of New York. His research interests include wireless communication networks, cyber security, network system modeling/simulation, information theory and data science. His email address is zhanyang.zhang@csi.cuny.edu. His website is http://www.cs.csi.cuny.edu/∼zhangz/.

**TOBIAS SCHÄFER** is a professor in the Ph. D. Program in Physics at The Graduate Center of the City University of New York and in the Department of Mathematics at the College of Staten Island. He earned his Ph.D. in Theoretical Physics at the University of Düsseldorf, Germany. His research interests include fluid dynamics, turbulence, and nonlinear optics. His email address is tobias@math.csi.cuny.edu. His website is http://www.math.csi.cuny.edu/∼tobias.

**MICHAEL KRESS** is an Emeritus Professor in the Ph. D. Program in Computer Science at The Graduate Center of the City University of New York and in the Computer Science Department at the College of Staten Island. He earned his Ph.D. in Magneto-hydrodynamics from The Courant Institute of Mathematical Sciences, of New York University. His research interests include high performance computing and numerical modeling and simulation of interdisciplinary systems including, environmental science applications regarding hurricanes, pollutant transport and transportation modeling as well as psychology and behavioral science using social network analysis. He founded the City University of New York High Performance Computing Center. His email address is Michael.Kress@csi.cuny.edu.