Revisiting M/D/1/N FIFO queue with renovation[1]

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In this short note we revisit the *M/D/1/N* FIFO queue with the general renovation mechanism, which was introduced in [1–3]. This mechanis works as follows. Define *N+1* numbers, say $q_i \geq 0$, $0 \leq i \leq N$, satisfying $\sum_{i=0}^{N} q_i = 1$. Upon a service completion the served customer removes i, $0 \leq i \leq N$ additional customers from the queue with probability q_i and then leaves the system. If upon the service completion the queue is empty the served customer leaves the system having no effect on it.

Roughly speaking a renovation implies that in a queueing system each customer, having received service, may remove some additional work from the system (i.e. may renovate it). This makes renovation look like a variant of an active queue management (AQM) scheme. But not exactly the same since renovation manages the queue after service completions, whereas in most of the AQM schemes decisions are made upon arrivals.

To our best knowledge the comparison of renovation with known active queue mechanisms (like, for example, Random Early Detection) has not been made before in the literature. The first step in this direction is to refine renovation procedure because in its default variant (from [1–3]) it is not particularly suitable for the queue management. For example, after the renovation the system may become empty and this is meaningless from the practical point of view (since it increases the system idle time). In the next section we discuss the required refinements and analytic expressions for the following performance characteristics: stationary loss rate, moments of the number in the system. Moments of consecutive losses, waiting/sojourn time (as introduced in [4]) are out of scope.

Mathematical model

We consider the system consisting of one queue of finite capacity N, served by single server. Customers arrive at the system according to the Poisson flow of rate λ. If a customer sees the system ull it is lost, otherwise it

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occupies one place in the queue if the server is buty and the server if it is idle. Service times are constant, equal to $d > 0$. Upon service completion one customer from the head of the queue enters server.

For the purpose of the active queue management, a variant of the general renovation mechanism (described above) is implemented in the system. Firstly notice that after the renovation, the queue may become empty and thus the server will be idle until the next arrival. From the practical point of view it is more appealing to leave at least one customer in the system after the renovation. Secondly, it may happen that upon the service completion it is required to remove more customers, than are actually waiting in the queue. For such a conflict we will consider separately two resolution options

Option 1. If upon service completion there are $1 \le i \le N$ customers waiting in the queue, then

- with probability q_0 nothing happens;
- with probability q_j, $0 < j < i$, exactly j customers from the queue leave the system and those customers are chosen successively starting from the head of the queue;
- with probability $Q_i=q_i+q_{i+1}+\ldots+q_N$, exactly $(i\text{-}1)$ customers from the queue leave the system. Again those customers are chosen successively starting from the head of the queue.

Option 2. If upon service completion there are $1 \le i \le N$ customers waiting in the queue, then

- with probability $q_0 + Q_i$ nothing happens;
- with probability q_j, $0 < j < i$, exactly j customers from the queue leave the system and those customers are chosen successively starting from the head of the queue.

For both options the served customer, which sees the empty queue, leaves the system having no effect on it. Thus after the renovation (if it happened) the system never becomes empty.

Discussion

As it was mentioned above, *G/M/n/r* type queues with the general renovation have already been studied in detail in [1, 2], where the authors have proposed quite general analytic methods for the computation of the main stationary performance characteristics. But those results are inapplicable as soon as we introduce refinements in the revovation procedure according to *Option 1* and *Option 2*. Thus there is a need to derive the expressions for the stationary performance characteristics anew but fortunately the methodology from [1–2] still applies here. In the rest of the note we briefly comment on the required steps. The details can be found elsewhere.

Let $N(t)$ be the total number of customers at instant t and $E(t)$ be the elapsed service time of the customer in server in the *M/D/1/N* FIFO queue with the renovation mechanism according to *Option 1* or *Option 2*. For the computation of the stationary queue size moments we need the distribution

$$\lim_{t\to\infty} \mathrm{P}\{N(t)=n\} = P_n,\ 0\le n\le N+1,$$

and for the computation of the loss rate, due to PASTA property of Poisson arrivals, it is sufficient to know

$$\lim_{t\to\infty} \mathrm{P}\{N(t)=n,\ E(t)<x\} = P_n(x), 1\le n\le N+1,\ x\in[0,d].$$

These distributions can be found as follows. At first we find the stationary distribution $\{P_n^+, 0\le n\le N\}$ of the Markov chain $\{\upsilon(t), t\ge 0\}$ embedded at serice completion epochs and counting the total number of customers in the system. Then, using the well-know results for the Markov regenerative processes, we calculate $\{P_n, 0\le n\le N+1\}$ by $P_n=\sum_{i=0}^{N} P_i^+ f_{in}/f^*$, where f_{in} is the mean time spent by the system in the state n, starting from state i, and f^* is the mean time between transitions of the $\{\upsilon(t), t\ge 0\}$. Finally, relations for the functions $P_n(x)$ are found from the results for the classic *M/D/1/N* queue.

The computation of the loss ratio i.e. the probability that the arriving customer is lost is more involved. This is due to the fact that the accepted customer may be lost either after the first service completion or the second, etc. and the chance to be lost varies, depending on the number of new customers that arrived between successive service completions. Yet using the PASTA property of Poisson arrivals and the first step analysis these difficulties can be overcome and eventually one can obtain a recursive procedure for the loss rate computation.

Finally we note that the expressions for the average and standard deviation of consecutive losses are probably the most hard-to-derive quantities in the considered setting. This problem remains unsolved.

Conclusions

Although the renovation mechanism is based on completely different idea than the RED-type AQMs, as our numerical experiments show, it allows one to achieve comparable system performance. Yet the proper choice of the values of its parameters q_i may be difficult. We are unaware of any analytic way of choosing q_i and thus in order to make the choice one has to resort to special search algorithms. Metaheuristics (like particle swarm optimization) are applicable here.

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