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Unwinding ZIRP: A simulation analysis

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ABSTRACT

This paper sets up a zero interest rate policy (ZIRP) experiment, using a two-market agent-based simulation model, in order to analyze the price dynamics of a large and small stock market during the unwinding of a simulated ZIRP. Different unwinding paths are created to determine which path has the most stabilizing impact on both the large and small stock market. Results indicate, that increasing the interest rate every three months create significant financial crises and negative stock market returns. However, increasing the rate every year leads to only modest declines in the large and small stock market. Moreover, the size of the interest rate change plays a much smaller role than the frequency of the rate changes. Rate increases every quarter creates 20% market corrections in the large market during the unwinding phase, as opposed to 4% downside when rate changes occur once a year.

1. Introduction

The US Federal Reserve has maintained a zero interest rate policy (ZIRP) for almost a decade. Never before in the history of US monetary policy has the Federal Reserve Bank (Fed) kept interest rates at zero for so long and then attempt to increase them. Given this new era of monetary policy it is hard to predict how unwinding ZIRP will play out in the global financial markets. It is impossible to learn from past policy mistakes in regard to ZIRP as no US historical data exists. The best the Fed can do is monitor economic data as we go and decide the best policy maneuver. Given the lack of historical data, I attempt to better understand the potential consequences of unwinding ZIRP on the global stock markets using simulation techniques.

More specifically I use an agent-based model to simulate two stock markets, a large developed stock market, meant to mimic the US stock market, and a small stock market, meant to mimic a portfolio of various emerging markets. I use [Feldman's \(2010\)](#) two market agent-base model as the base model. In the base model there exists two markets with two sets of local investment managers and one set of global managers. Each set of local managers invest purely in their local market where as the global managers may invest in both markets. [Feldman \(2010\)](#) finds large global financial crises develop when global investment managers enter the picture and price risk based on behavioral biases as opposed to pricing risk using volatility. There are two reasons why I use Feldman's simulation model. First, the global manager model creates on average one global financial crisis per century allowing a researcher to create a ZIRP experiment.¹ The ZIRP experiment involves adding a central bank that is programmed to decrease the US mimicking market interest rate to zero after a global financial crisis. The central bank keeps the interest rate at zero for ten years before increasing the rate back to the original rate set prior to the global financial crisis. Second, the global manager model can create an environment akin to today's environment, where the interest rate is low in the US market and high in the emerging market portfolio. Therefore, the global manager can hold onto cash in order to earn a high interest rate in the emerging market.

I hypothesize that the global manager linkage may create a dis-allocation as the global managers either use too much leverage,

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¹ A global financial crisis is defined in the simulation as a forty percent decrease from the high in both stock markets simultaneously.

borrowing at the zero interest rate, or flood the smaller market with too much capital trying to earn the higher interest rate. The consequence of unwinding ZIRP could undue this dis-allocation sending both markets into financial crisis. I test this hypothesis by analyzing crash frequency and return moments during the unwinding of ZIRP period. In addition, I explore various central bank unwinding paths in order to search for the unwinding path that creates the most stability across both stock markets. The unwinding paths differ based on the size of the interest rate increase, either twenty-five or fifty basis points (bps) increase, and the frequency of the rate increases, either every quarter, six months, or twelve months.

The main contribution of the paper is the addition of the central bank authority to the (Feldman, 2010) model and the setup of the ZIRP experiment. No central bank is created for the emerging market and therefore its interest rate stays constant throughout the simulation. A central bank is created for the US inspired market which is meant to mimic the Federal Reserve Bank of the United States. When the central bank lowers the rate for the US market this creates a large interest rate spread between the two markets that mimics actual financial markets. In addition, the original 2010 model is altered in that there exists two different interest rates as opposed to one interest rate.

Analysis points out several main results. First, I find increasing the US interest rate by 50 bps, as opposed to 25 bps, made a marginal difference when the rate change occurs every twelve months. In other words, I find that the frequency of the rate changes played a much larger role than the size of the rate change. In addition, increasing the interest rate too soon created significant downside, with US returns falling 20% (35%) on an annual basis during the unwinding period when increasing the rate every quarter by 25 (50) bps. However, US returns only fall by 4% (9%) on an annual basis during the unwinding period when increasing the US rate every year. Lastly, emerging market returns did not experience high crash frequency because of the unwinding of ZIRP. However, returns did decrease overall because of the interest rate intervention. I find no basis to suggest that emerging markets are more prone to a financial crisis because of the ZIRP.

2. Existing literature

Westerhoff (2008) argues that agent-based models may be used as artificial laboratories to improve our understanding of how regulatory policy tools function. Napoletano et al. (2012) also argue that agent-based models (ABMs) allow one to explicitly account for phenomena such as heterogeneous beliefs and the emergence of bubbles and crashes. In addition, they add that ABMs allow one to model elements of real economic structures such that policy-makers can use in order to improve the guidance of policy-making applied to particular contexts. It is for these reasons that I use an ABM to investigate different monetary policies in regard to unwinding ZIRP.

There have been some examples of authors using ABM to better understand monetary policy. Rapaport et al. (2009) use an ABM where heterogeneous countries decide whether to introduce central bank independence taking into account the behavior of their neighbors. Arifovic et al. (2010) use an ABM where the agents can either believe the inflation rate announced by the central bank or employ an adaptive learning scheme to forecast future inflation. Computer simulations of this model show that the central bank learns to sustain an equilibrium with a positive, but fluctuating fraction of believers.

However, there is no research to date exploring the impact of unwinding ZIRP across multiple markets using ABM. Given the structure of how ABMs work, it makes sense to create this experiment using a global stock market ABM. I use the (Feldman, 2010) ABM which was developed from Friedman and Abraham's (2009) single market model. Friedman & Abraham develop a single market agent-based framework focused on fund managers, where bubbles and crashes occur when investors price risk based on behavioral biases. Friedman & Abraham show the behavioral element in the ABM is the main factor to create bubbles and crashes. Feldman develops the Friedman & Abraham model into a two-market model where global managers may invest across both markets. Feldman finds that when global managers are added to the two-market model, they take on excessive leverage. The reason being is that ability to diversify across two markets lowers their risk perception in a behavioral model and leads to greater leverage taking as they feel safer from diversifying. The result is that large global financial crises can develop as a small shocks leads to a fast deleveraging across both stock markets.

3. Base model

There exists two populations of local managers who buy and sell a riskless asset with constant return r_m and a single risky asset with variable return R_m . The local manager in market one, the large market, earns R_1 and the local manager in market two, the small market, earns R_2 . Each fund manager chooses to allocate u_m that represents the percentage devoted to the risky asset, where m signifies either market one or two. If $u_m < 1$, it means the fund manager is partially invested in the risky asset and partially invested in the risk free asset, $(1-u_m)$. If $u_1 = 1$ the fund manager is fully invested in market one's risky asset and if $u_1 > 1$ the managers is borrowing the risk free asset to invest in market one's risky asset. The manager's net portfolio value is denoted by the variable z_i .

The fundamental value of the risky asset, $V = \frac{e^{g_{m,t}}}{R_s - g_{m,t}}$, is based on future earnings that are discounted at some rate $R_s > g_{m,t}$ where $g_{m,t}$ is the growth rate for market m at time t . The price of the risky assets turn out to be

$$P_m = V \bar{u}_m^\delta \quad (1)$$

The price is equal to, less than or greater than fundamental value whenever the average demand, \bar{u} is equal, less than or greater than 1.0. The average demand, \bar{u}_m is calculated by averaging the most recent allocation, u_i , across all managers weighting by their portfolio size, z_i in market m . The average allocation, \bar{u} represents buying pressure where the intensity is parametrized by δ .

The return on the risk asset is the dividend yield, $(R_s - g_{m,t})\bar{u}_m^{-\delta}$, plus the capital gains rate, $g_{m,t} + \delta\dot{\bar{u}}_m/\bar{u}_m$. The return of the risky asset is

$$R_m = (R_s - g_{m,t})\bar{u}_m^{-\delta} + g_{m,t} + \delta\dot{\bar{u}}_m/\bar{u}_m. \tag{2}$$

The payoff function of manager i in market m is based on the tradeoff between risk and return,

$$\phi^L(u_m) = u(R_m - r_m + \alpha_i) - \frac{1}{2}A_m\sigma_m^2u_m^2. \tag{3}$$

The payoff function is the tradeoff between excess return, $R_m - r_m$, and risk, $A_m\sigma_m^2$ with two different features. First, each manager has their own specific α_i meaning that each manager either outperforms, under performs, or matches the market return by the amount α . The managers' idiosyncratic component α_i is Ornstein-Uhlenbeck which is stochastic and mean reverting in continuous time. Second, risk is defined by the perception of risk, A_m times σ^2 , the variance. In this behavioral world, A is endogenously determined by the behavioral biases of the market participants. The variance of market m at time t , $\sigma_{m,t}^2$, is defined as

$$\sigma_{m,t}^2 = (R_{m,t} - \hat{R}_{m,t})^2 \tag{4}$$

where $\hat{R}_{m,t} = 1 - e^{-\eta}R_{m,t} + e^{-\eta}\hat{R}_{m,t-1}$. The parameter η is a memory parameter. The larger the η the more weight placed on the most recent data and the less weight on data in the past. The perceived risk parameter, A_m , is determined by exploiting the behavioral biases of loss aversion and recency. Loss aversion is exploited by picking up losses, $L_i = \max\{0, -R_{Gi}\}$, where the gross return, $R_{Gi} = (R_1 - r_m + \alpha_i)u_i + r_m$. The next step exploits recency, where the losses are averaged using an exponential average. The exponential average is updated from the previous exponential average loss $\hat{L}_{i(t-1)}$ and current loss, $L_{i(t)}$,

$$\hat{L}_i(t) = e^{-\eta}\hat{L}_{i(t-1)} + (1 - e^{-\eta})L_{i(t)}. \tag{5}$$

I calculate the exponential average of losses using the same parameter η used to calculate variance. The perceived risk index for market one and two is proportional to market-wide losses in each market,

$$A_m = \beta\hat{L}_{T(t,m)}, \tag{6}$$

where the parameter $\beta > 0$ reflects investors' sensitivity to perceived loss, and $\hat{L}_T(t, m)$ is the exponential loss function, $\hat{L}_i(t)$, averaged across managers i in market m weighted by portfolio size z_i at time t . When the risk index is multiplied by the variance, high variance does not necessarily translate into more risk. For example, if volatility is high on the upside (downside) such that losses are low (high), $A_m\sigma^2$, is low (high) because A_m is small (large).

In the simulation managers adjust their exposure to risk by following the slope of the payoff function,

$$\phi_{u_m}^L = (R_m + \alpha_i - r_m) - A_m\sigma_m^2u_m. \tag{7}$$

Inputting the return from Eq. (2) the payoff gradient becomes

$$\phi_{u_m}^L = [(R_s - g)\bar{u}_m^{-\delta} + g_{m,t} + \alpha_i + \delta\dot{\bar{u}}_m/\bar{u}_m - r_m] - A_m\sigma_m^2u_m. \tag{8}$$

That is, each manager continuously adjusts her risk position in proportion to her payoff slope. If ϕ_u is positive (or negative) for managers, they increase (or sell off part of) their risky asset position, and do so more rapidly the steeper the payoff function at their current position. The managers follow a gradient to avoid the market impact of their orders. As the managers choose their u_m this feeds into \bar{u} of the price equation. The price changes thereby changing the market return. The new market return and manager α lead managers to re-adjust their exposure thereby continuing the feedback between the price of the risky asset and the payoff function.

3.1. The ZIRP global manager model

The global managers may invest in both risky assets. Therefore their payoff function becomes

$$\phi^G(u_1, u_2, F) = u_1(R_1 + \alpha_{k,1}) + u_2(R_2 + \alpha_{k,2}) + (1 - u_1 - u_2)r_m - \frac{1}{2}A_m\dot{u}Vu, \tag{9}$$

where u is a vector of weights, u_1 and u_2 , k refers to the global manager, and V is the covariance matrix.

The modeling change between this paper and Feldman (2010) comes into play in the global manager payoff function. In the base model the interest rates for both markets are the same. In this paper the interest rate for each market is different. Therefore, the global manager has the option to borrow at the lower market one interest or earn the higher market two interest rate. The local manager does not have that benefit. The interest rate in the global manager's payoff function, r_m changes in the following fashion:

$$r_m = i_1 \text{ if } (1 - u_1 - u_2) > 1$$

$$r_m = i_2 \text{ if } (1 - u_1 - u_2) \leq 1$$

The global managers' payoff gradient for both markets one and two becomes,

$$\phi_{u_1}^G = R_1 + \alpha_{k,1} - i_1 - A_1\sigma_1^2u_1 - \rho\sigma_1\sigma_2A_1u_2, \tag{10}$$

$$\phi_{u_2}^G = R_2 + \alpha_{k,2} - i_2 - A_2\sigma_2^2u_2 - \rho\sigma_1\sigma_2A_2u_1, \tag{11}$$

where ρ is the correlation coefficient between markets 1 and 2, and the share in the safe asset equals $1 - u_1 - u_2$. A negative share in the safe asset means the manager is borrowing the safe asset in market one to invest in the risky assets.

The correlation coefficient is a function of the covariance which is determined by the following process,

$$cov_t = (R_1 - \hat{R}_1) * (R_2 - \hat{R}_2) \tag{12}$$

Plugging in the returns the payoff gradients become,

$$\phi_{u1}^G = (R_s - g_{1,t})\bar{u}_1^\delta + g_{1,t} + \alpha_{k,1} + \delta \bar{u}_1 / \bar{u}_1 - i_1 - A_1 \sigma_1^2 u_1 - \rho \sigma_1 \sigma_2 A_1 u_2, \tag{13}$$

$$\phi_{x2}^G = (R_s - g_{2,t})\bar{u}_2^\delta + g_{2,t} + \alpha_{k,2} + \delta \bar{u}_2 / \bar{u}_2 - i_2 - A_2 \sigma_2^2 u_2 - \rho \sigma_1 \sigma_2 A_2 u_1. \tag{14}$$

When global managers enter previously closed markets, the perceived risk index in each market is not only defined by local manager's losses but also global manager losses. Global managers losses are defined as $L_k = \max\{0, -R_{GR,k}\}$ where the global managers' gross return equals $R_{GR,k} = (R_1 + \alpha_k^1)u_{k1} + (R_2 + \alpha_k^2)u_{k2} + (1 - u_{1k} - u_{2k})r_m$. The exponential losses for global managers are defined as,

$$\hat{L}_k(t) = e^{-\eta} \hat{L}_{k(t-1)} + (1 - e^{-\eta}) L_{k(t)}. \tag{15}$$

where k signifies the population of global managers. The new $\hat{L}_T(t)$ becomes a function of local and global managers' losses. More specifically, $\hat{L}_T^m(t)$ equals $\hat{L}_i^m(t)$ averaged across local managers i weighted by portfolio size z_i in market m plus $\hat{L}_k(t)$ averaged across all global managers k weighted by portfolio size, z_k .

4. Simulations

I simulate the model using an agent-based modeling Java applet called NetLogo.² Table 1 defines the parameters. Simulations are run for 100 years on a weekly basis. In the simulation I run a baseline configuration using values of $Pop_{US} = 30$, $Pop_g = 10$, $Pop_{EM} = 5$, $i_{US} = 0.05$, $i_{EM} = 0.08$, $\sigma = 0.2$, $\tau = 0.7$, $\eta = 0.7$, $\beta = 2$ and $\delta = 2$. I run additional simulations for robustness testing, varying the global manager and emerging market population sizes otherwise all other parameters are kept constant. I also vary the other parameters one at a time for robustness results shown in the last table. An explanation of the parameters can be found in Table 1. The theory column indicates how an increase in the parameter effects the number of crises in the simulations. The standard deviation of each manager's idiosyncratic shock is 20% consistent which is the long run standard deviation of the S&P 500. The memory rate for the loss index is 0.7 meaning managers remember losses for up to two years. For each configuration I ran 1000 weekly simulations each for 100 years. In addition, the parameter configurations are symmetric for both markets except the interest rates. I define a financial crises as a decline in detrended price of at least 40 percent from its highest point within the last half year. A global crisis period occurs when both markets are experiencing a crisis at the same time.

5. Experiment

The experiment involves creating a central banking authority for the US market (large stock market). The large market is defined as large because the market contains a greater number of agents, thirty. The smaller market contains five agents in the baseline configuration. The US stock market capitalization is 400 times greater than most larger emerging markets. In this context I assume the second smaller market is a portfolio of emerging markets consisting of markets such as Russia, India, and China. In reality, global managers do not have large exposures to one emerging market but diversify across many. In this respect, the ratio of large market population to small market population mimics the actual relationship.

The interest rate of the US market is static up to the point of the global financial crisis where the rate changes occur. The interest rate of the emerging market is static throughout the simulation. To mimic actual markets, I set the interest rate to the average historical federal funds rate for the US and emerging market portfolio, Russia, India, and Indonesia. I calculate the average federal funds rate for the US market as 5% and 8% for the emerging markets. I use data on the US federal funds rate from 1954 to the end of 2007, prior to the global financial crisis. I use IMF data to calculate benchmark rates for India, Russia, and Indonesia.

The central bank is programmed to decrease the interest rate for the US market once a global financial crisis occurs. Once the 40% threshold is met, the US central bank decreases the 5% interest rate by 50 basis points every quarter until the interest rate becomes zero. The interest rate for the emerging market does not change. Therefore, there is an assumption that exchange rates will change, however, there is no explicit modeling of the exchange rate. Once the US market's interest rate reaches zero, it remains at zero for a period of ten years. I try to approximate for the actual amount of time ZIRP is kept in place. At the end of the ten years, the central bank is programmed to increase the interest rate for the US market back to the 5% rate set prior to the global financial crisis.

In summary there are five ZIRP periods:

1. Period 1: Base.
2. Period 2: Central bank reduces the interest rate to zero.
3. Period 3: Zero interest rate.
4. Period 4: Unwinding ZIRP.

² <http://ccl.northwestern.edu/netlogo/>.

Table 1
Parameters.

Parameter	Definition	Meaning	Theory
r_m	risk-free rate		
dR	discount-rate		
g_s	growth-rate	economic growth	$\uparrow \Rightarrow$ higher growth
σ	standard deviation	variability of a manager's idiosyncratic shock	$\uparrow \Rightarrow$ more crises
η	memory of rate	Weight on current vs. past losses & persistence of idiosyncratic shock	$\uparrow \Rightarrow$ more crises
β	sensitivity to risk	how sensitive managers are to risk	$\uparrow \Rightarrow$ more crises
δ	elasticity	how much changes in demand change price	$\uparrow \Rightarrow$ more crises

5. Period 5: Back to base.

The main focus of the paper is to test which unwinding path creates the greatest stability during Period 4, the unwinding of the US market's interest rate. To do so I create six different unwinding paths. The unwinding paths differ based on the size of the rate increase and the amount of time between rate changes as summarized below.

- 25 point basis increase every three, six, or twelve months.
- 50 point basis increase every three, six, or twelve months.

6. Empirical model

I run four regressions in order to better understand how the different unwinding paths impact simulated stock market returns, crashes, and return correlations. The first regression is an ordinary least square regression run on the Period 4 data only, the unwinding of the interest rate. The dependent variable is the annual stock return for market m and the independent variables are indicator variables that delineate between the various unwinding paths. The purpose of the regression is to test whether the unwinding of the US interest rate creates negative stock market returns,

$$Ret_{m,t-52} = \beta_0 + \beta_1 * 6mth_{25bps} + \beta_2 * 12mth_{25bps} + \beta_3 * 3mth_{50bps} + \beta_4 * 6mth_{50bps} + \beta_5 * 12mth_{50bps} + \epsilon, \quad (1)$$

where $Ret_{m,t-52}$ is the annual return in market m. The error term, ϵ , contains any other factors that my influence the annual return. The focus on this regression is not what are the factors that explain the return but how the return changes when the central bank unwinds the zero interest rate policy compared to the other periods.

The purpose of the next regression is to understand the probability of a market crash during Period 4 relative to the other periods controlling for the different unwinding paths. It is a logit regression where the dependent variable is a binary variable, one zero variable and the independent variables are indicator variables,

$$Crisis_m = \beta_0 + \beta_1 * Period4 * 3mth_{25bps} + \beta_2 * Period4 * 6mth_{25bps} + \beta_3 * Period4 * 12mth_{25bps} + \beta_4 * Period4 * 3mth_{50bps} + \beta_5 * Period4 * 6mth_{50bps} + \beta_6 * Period4 * 12mth_{50bps} + \epsilon, \quad (2)$$

where the dependent variable equals one if a crisis occurred in market m and 0 otherwise. A crisis is determined when the stock market falls forty percent from its recent high in the last six months. The simulation records when the forty percent threshold is hit. I then create a crisis variable in SAS. To do so I look backward by assigning a one to the highest price prior to the crisis up until the lowest price. This is how the $Crisis_m$ variable is created for the logit regression. Period4 is an indicator variable that equals one if unwinding of ZIRP is in the present period. I multiply the Period4 indicator variable by the indicator variables that delineate between the various unwinding paths.

The next two regressions are ordinary least square regressions that use the running fifty-two week return correlation coefficient as the dependent variable. The logic behind the next two regressions is to first better understand how the different unwinding path scenarios impact the return correlation between both markets. Second, to better understand the relationship between volume and return correlations across the different unwinding paths. These two regressions are run separately for each of the unwinding paths. Therefore, I run six regressions for each unwinding path and compare the estimated coefficients of the following regression,

$$\rho_{m=1,2,t-52} = \beta_0 + \beta_1 * Period2 + \beta_2 * Period3 + \beta_3 * Period4 + \beta_4 * Period5 + \epsilon, \quad (3)$$

where the dependent variable is the running fifty-two week correlation coefficient between market 1 and 2. The intercept is Period1 variable, the base period prior to any central bank intervention. The remaining independent variables are indicators variables that equal one if the current ZIRP period is active and zero otherwise.

The next regression analyzes the interaction between return correlation and volume. I collect the allocation data, u , for each individual agent during the simulations. The difference between today's allocation, u , and last week's allocation is the amount of u traded for that week. I take the absolute value of that difference for each individual agent and then average across all agents for time period t . I use the average weekly volume traded across all agents as the volume variable in the following regression,

Table 2
US market # of crashes per century.

Phase	3-Month	6-Month	12-Month
25 Point Increase			
1	2.97	4.14	3.14
2	0.00	0.94	0.73
3	1.08	1.33	1.52
4	4.77	2.35	1.77
5	0.50	0.31	0.42
50 Point Increase			
1	3.00	3.01	2.17
2	0.50	0.90	0.92
3	1.31	1.79	1.34
4	18.54	3.50	2.06
5	0.56	0.40	0.56

Note: Table 2 displays the number of crashes per century for each ZIRP phase. The number of crashes is adjusted by 100 years in order to compare the number across different phases and ZIRP unwinding frequency. Period 1: Base, Period 2: Decrease in rate after financial crisis, Period 3: Zero interest rate period, Period 4: Unwinding of ZIRP, Period 5: Return to base period.

$$\rho_{m=1,2,t-52} = \beta_0 + \beta_1 * \text{Period2} * \text{volume}_t + \beta_2 * \text{Period3} * \text{volume}_t + \beta_3 * \text{Period4} * \text{volume}_t + \beta_4 * \text{Period5} * \text{volume}_t + \epsilon, \quad (4)$$

where the dependent variable is again the fifty-two week return correlation coefficient. The only difference between these two regressions is that I add an interaction term with the ZIRP period indicator variables, volume. The logic is to understand how volume and return correlations interact throughout the unwinding process.

7. Results

I first calculate crash frequency and return statistics to examine whether the unwinding of ZIRP creates additional financial crises and to study which unwinding path creates the greatest stability. Table 2 displays the number of crashes per century. I calculate the frequency of crashes for each ZIRP period and then adjust the number to equate to the number of crashes per century. I make this adjustment in order to compare the number of crashes across different ZIRP periods and unwinding paths as each period has a different number of total weeks.

Results from Table 2 indicate that unwinding too quickly can create a significant amount of crashes. There are approximately five (eighteen) crashes per century during Period 4 when the 25 (50) bps rate increase occurs every three months. In contrary there are two 1.77 (2.06) crashes per century when the 25 (50) bps rate increase occurs every year for the US market. In comparison, there occurs one to two crashes per century in the base model without any interest rate intervention. Slightly more crashes per century are created when the central bank changes the interest rate every twelve months in comparison to the no interest rate intervention base model.

Table 3 displays the crash statistics for the emerging market. There exists less of a noticeable impact from the US market's monetary policy on the number of crashes in the emerging market. Overall, the crash statistics indicate that increasing the rate every year as opposed to every quarter decreases crash frequency slightly during Period 4. However, I do not find that unwinding the US interest creates disruptions in the smaller emerging markets as some have predicted.

Table 3
EM market # of crashes per century.

Phase	3-Month	6-Month	12-Month
25 Point Increase			
1	2.07	4.16	3.30
2	0.00	8.20	9.46
3	1.08	3.72	3.12
4	4.77	4.06	3.72
5	0.50	1.67	1.43
50 Point Increase			
1	3.91	5.32	1.77
2	6.72	8.28	6.49
3	2.77	5.24	1.44
4	5.41	2.25	2.88
5	1.83	3.22	1.76

Note: Table 3 displays the number of crashes per century for each ZIRP phase. The number of crashes is adjusted by 100 years in order to compare the number across different phases and ZIRP unwinding frequency. Period 1: Base, Period 2: Decrease in rate after financial crisis, Period 3: Zero interest rate period, Period 4: Unwinding of ZIRP, Period 5: Return to base period.

Table 4
US market mean annual return.

Phase	3-Month	6-Month	12-Month
25 Point Increase			
1	-0.95%	-2.53%	-1.45%
2	48.02%	39.66%	41.55%
3	5.79%	9.10%	6.76%
4	-20.43%	-10.16%	-4.92%
5	0.31%	0.13%	0.28%
50 Point Increase			
1	0.71%	-3.38%	-1.68%
2	50.47%	32.55%	41.12%
3	8.02%	10.19%	7.46%
4	-36.32%	-18.37%	-8.96%
5	0.11%	0.18%	0.12%

Note: Table 4 displays the US simulated (large market) annual return for each ZIRP phase. Period 1: Base, Period 2: Decrease in rate after financial crisis, Period 3: Zero interest rate period, Period 4: Unwinding of ZIRP, Period 5: Return to base period.

Table 4 displays the US simulated annual return for each ZIRP period across different unwinding paths. Results indicate that the US market annual returns are high for both Period 2 and 3, when the rate is falling and held at zero, as seen in the actual US market. However, the average annual return declines by twenty percent (thirty-six percent), in Period 4 when the 25 (50) bps rate increase occurs every quarter. There is significant downside potential when the central bank unwinds too quickly. In contrary, the US market declines five percent (nine percent) on an annual basis in Period 4 when the 25 bps (50 bps) rate change occurs once a year. Period 4 is longer when the US interest rate change occurs once per year, however, the downside potential is significantly reduced. The 50 bps rate increase plays a bigger role when the rate change is every three months but not as much when the rate change is once per year. These results indicate that the Fed can increase the federal funds rate at 50 bps as long as they allow sufficient time in between rate changes.

Results from Table 5 indicate that the emerging market returns are essentially negative for all periods beside the zero interest rate period, where there is a slight positive return. In comparison, the average return, using the same parameter configuration, is between three and five percent for the smaller market when there is no interest rate intervention. In other words, the zero interest rate policy has negative return effects on the smaller emerging markets. However, I do not find emerging market crashes as some market participants have anticipated.

Table 6 displays the standard deviation for the US market. As in actual markets, the volatility declines significantly during Period 3, the zero interest rate period. Volatility picks up in Period 4 as expected. However, not as expected, volatility is lower in Period 5, which is the return to the base period. One would think that the volatility of Period 5 should be equal to Period 1. However, this is not the case. The simulation results indicate there could be a new normal of lower volatility after the unwinding of the ZIRP.

Table 7 displays the mean volume. Volume is calculated in the same way as explained above in regression 4, the average amount of u traded across all agents. The unwinding path plays little role in the mean volume. The mean volume and standard deviation is pretty stable across all ZIRP periods and unwinding paths. However, there is a slight uptick in volume in Period 2 when the interest rate is declining and a downtick in volume in Period 5, when the unwinding ends. I also analyze the volatility of volume and I find the dispersion of stock traded is most volatile during Period 1 and Period 4. This is to be expected as these are the periods where the global financial crises occur the most.

Table 8 displays the regression results from running Regression 1 on Period 4 data for the US market. Results indicate there is a

Table 5
Emerging market mean annual return.

Phase	3-Month	6-Month	12-Month
25 Point Increase			
1	-0.29%	-0.97%	-0.67%
2	-1.52%	0.27%	-4.65%
3	3.99%	4.47%	4.76%
4	-2.44%	-1.17%	-0.83%
5	-0.34%	-0.23%	-0.40%
50 Point Increase			
1	0.22%	-0.79%	-1.09%
2	-1.12%	-2.70%	0.06%
3	4.89%	3.14%	4.51%
4	-2.68%	-1.31%	-2.01%
5	-0.33%	-0.04%	-0.14%

Note: Table 5 displays the EM simulated (small market) annual return for each ZIRP phase. Period 1: Base, Period 2: Decrease in rate after financial crisis, Period 3: Zero interest rate period, Period 4: Unwinding of ZIRP, Period 5: Return to base period.

Table 6
US market standard deviation.

Phase	3-Month	6-Month	12-Month
25 Point Increase			
1	21.03%	21.03%	21.14%
2	22.54%	22.54%	23.67%
3	5.72%	5.72%	7.42%
4	20.68%	20.68%	29.00%
5	14.47%	14.47%	14.51%
50 Point Increase			
1	20.84%	20.83%	21.65%
2	23.29%	25.79%	26.05%
3	6.76%	8.06%	9.99%
4	21.48%	25.24%	24.16%
5	14.77%	14.99%	15.03%

Note: Table 6 displays the annual US simulated market standard deviation for each ZIRP phase. Period 1: Base, Period 2: Decrease in rate after financial crisis, Period 3: Zero interest rate period, Period 4: Unwinding of ZIRP, Period 5: Return to base period.

Table 7
Mean volume.

Phase	3-Month	6-Month	12-Month
25 Point Increase			
1	0.18	0.20	0.19
2	0.21	0.33	0.24
3	0.18	0.36	0.19
4	0.20	0.20	0.20
5	0.18	0.18	0.18
50 Point Increase			
1	0.19	0.19	0.19
2	0.22	0.22	0.26
3	0.19	0.19	0.24
4	0.20	0.20	0.21
5	0.18	0.18	0.18

Note: Table 7 displays the average mean volume across all agents. Period 1: Base, Period 2: Decrease in rate after financial crisis, Period 3: Zero interest rate period, Period 4: Unwinding of ZIRP, Period 5: Return to base period.

Table 8
Regression 1a: Return vs. Unwinding scenario: US market.

Variable	Coefficient
Intercept	-0.205** (0.002)
6mth _{25bps}	0.108** (0.002)
12mth _{25bps}	0.154** (0.002)
3mth _{50bps}	-0.201** (0.003)
6mth _{50bps}	0.015** (0.003)
12mth _{50bps}	0.118** (0.002)

Note: Regression uses only Period 4 simulated data. Standard errors are in parenthesis. The dependent variable, $Ret_{i,t-52}$, is the future 52-week return. [†]significant at 10%; *significant at 5%; **significant at 1%.

$$Ret_{i,t-52} = \beta_0 + \beta_1 * 6mth_{25bps} + \beta_2 * 12mth_{25bps} + \beta_3 * 3mth_{50bps} + \beta_4 * 6mth_{50bps} + \beta_5 * 12mth_{50bps} + \epsilon.$$

twenty percent decline in returns when increasing the interest rate 25 bps every quarter, intercept. The annual returns become less negative when the frequency in the rate increase is slower. T-test results indicate that there is not a statistical significant difference at the five percent level between the 50 bps and 25 bps for the every twelve month regimes. This indicates, the Fed can increase at 50 bps, especially later in the unwinding period, as long as they don't increase too fast. Table 9 displays the regression results from running regression one but on the emerging market data. Results from Table 9 indicate negative returns for the emerging market during Period 4, however, not as negative as the returns in the US market. The emerging market returns do suffer because of the zero interest rate policy and the unwinding, however, not as bad as expected. There are no major emerging market crises that take place during the unwinding period.

Table 9
Regression 1b: Return vs. Unwinding scenario: Emerging market .

Variable	Coefficient
Intercept	-0.034** (0.003)
6mth _{25bps}	-0.004** (0.003)
12mth _{25bps}	-0.004** (0.002)
3mth _{50bps}	-0.018** (0.004)
6mth _{50bps}	-0.017** (0.003)
12mth _{50bps}	-0.009** (0.003)

Note: Regression uses only Period 4 simulated data. Standard errors are in parenthesis. The dependent variable, $Ret_{i,t-52}$, is the future 52-week return. †significant at 10%; *significant at 5%; **significant at 1%.
 $Ret_{i,t-52} = \beta_0 + \beta_1 * 6mth_{25bps} + \beta_2 * 12mth_{25bps} + \beta_3 * 3mth_{50bps} + \beta_4 * 6mth_{50bps} + \beta_5 * 12mth_{50bps} + \epsilon$.

Table 10
Regression 2: Logit regression: US market.

Variable	Coefficient
Intercept	-6.082** (0.009)
Period4*3mth _{25bps}	1.722** (0.067)
Period4*6mth _{25bps}	1.727** (0.041)
Period4*12mth _{25bps}	1.606** (0.032)
Period4*3mth _{50bps}	3.270** (0.039)
Period4*6mth _{50bps}	2.263** (0.046)
Period4*12mth _{50bps}	2.092** (0.034)

Note: All simulated data is used. Standard errors are in parenthesis. The dependent variable, $Crisis_{i,t}$, is the future 52-week return. †significant at 10%; *significant at 5%; **significant at 1%.

$$Crisis_{i,t} = \beta_0 + \beta_1 * Period4 * 6mth_{25bps} + \beta_2 * Period4 * 6mth_{25bps} + \beta_3 * Period4 * 12mth_{25bps} + \beta_4 * Period4 * 3mth_{50bps} + \beta_5 * Period4 * 6mth_{50bps} + \beta_6 * Period4 * 12mth_{50bps} + \epsilon,$$

Table 11
Regression 3: Return correlation vs. ZIRP period indicator variables.

Variable	25bps/3mths	25bps/6mths	25bps/12mths	50bps/3mths	50bps/6mths	50bps/12mths
Intercept	0.142**	0.142**	0.131**	0.152**	0.128**	0.112**
Period2	0.040**	0.026**	0.018**	0.098**	0.207**	0.240**
Period3	0.027**	0.028**	0.110**	0.189**	0.045**	0.053**
Period4	-0.004	0.004	-0.002	0.004	-0.0311**	-0.0139**
Period5	-0.0136**	-0.040**	-0.020**	-0.0364**	-0.010**	-0.019**

Note: All simulated data is used. Regressions run for separately for each unwinding path, 6 paths. Standard errors are in parenthesis. The dependent variable, $\rho_{ij,t-1}$, is the future 52-week return. †significant at 10%; *significant at 5%; **significant at 1%.

$$\rho_{ij,t-52} = \beta_0 + \beta_1 * Period2 + \beta_2 * Period3 + \beta_3 * Period4 + \beta_4 * Period5 + \epsilon$$

Table 12
Regression 4: Return correlation vs. ZIRP period-volume interaction variables.

Variable	25bps/3mths	25bps/6mths	25bps/12mths	50bps/3mths	50bps/6mths	50bps/12mths
Period1Vol	0.141**	0.148**	0.133**	0.151**	0.129**	0.118**
Period2Vol	0.007**	0.007**	0.010**	0.007**	0.008**	0.018**
Period3Vol	0.016**	-0.004**	0.520**	0.087**	0.206**	0.0480**
Period4Vol	0.011**	0.018**	0.013**	0.020**	0.0318**	0.0256**
Period5Vol	0.038**	-0.235**	-0.089**	-0.168**	-0.028**	-0.138**

Note: All simulated data is used. Regressions run for separately for each unwinding path, 6 paths. Standard errors are in parenthesis. The dependent variable, $\rho_{ij,t-52}$, is the 52-week return correlation coefficient. †significant at 10%; *significant at 5%; **significant at 1%.

$$\rho_{ij,t-52} = \beta_0 + \beta_1 * Period2 * volume_{it} + \beta_2 * Period3 * volume_{it} + \beta_3 * Period4 * volume_{it} + \beta_4 * Period5 * volume_{it} + \epsilon$$

Table 13
Robustness testing: US market 25 bps.

Variable	Annual Mean Return	# of Crashes
$Pop_{EM} = 5, Pop_g = 10$	-21.21%	8.04
$Pop_{EM} = 10, Pop_g = 5$	-22.00%	4.84
$Pop_{EM} = 10, Pop_g = 10$	-21.92%	1.17
$Pop_{EM} = 10, Pop_g = 15$	-16.58%	5.02

Note: All simulated data is used. Results shown across different population sizes for the emerging market and global manager population. Pop_{EM} = population for emerging market and Pop_g = population for the global managers.

Table 14
Robustness results: Parameter changes.

Parameters		25 bps/3 mth	25 bps/6 mth	25 bps/12 mth
variables	value	Crashes / Century	Crashes / Century	Crashes / Century
Baseline	*	4.77	2.35	1.77
gs1	-0.04	1.65	0.86	0.35
	0.04	6.46	3.87	2.89
sigma	0.05	1.56	0.98	0.35
	0.4	5.88	3.76	2.56
eta	0.1	1.10	0.56	0.21
	3	8.88	5.45	4.11
beta	1	3.45	2.10	1.45
	5	5.66	3.30	2.20
alpha	1	1.92	0.97	0.55
	4	20.44	15.63	10.40

Note: Baseline values are as in Table 2. Table shows the number of crashes per century specifically for Period 4, the unwinding period for various parameter configurations.

Table 10 displays the logit regression results from running Regression 2. Results indicate that the log odds of a financial crisis in the US market decrease when the 25 bps rate increase occurs once per year. The log odds of a financial crisis increases when there is a 50 bps rate increase every quarter, as opposed to a 25 bps increase, however it drops significantly when the rate changes occur every twelve months.

Table 11 displays the regression results from running Regression 3. The intercept is the base period, Period1. The motivation is to better understand the how 52-week return correlation changes over the course of the ZIRP and unwinding of the ZIRP. Results indicate that return correlations between the two markets increases during the lowering of the US interest rate and the zero interest rate period. However, the unwinding of ZIRP, there is no change from the base period, and declines back to base period, Period5. In addition, the return correlations increase more for the 50 bps rate changes more so then the 25 bps rate changes which indicates the larger the rate change the more correlated the returns of the US and emerging markets.

Table 12 displays the regression results from running Regression 4. The purpose of regression 4 is to understand how volume and return correlations are related. Results indicate that volume and return correlations are higher in the 50 bps periods, but not significantly. Volume is relatively stable and does not have much an impact on return correlations. However, volumes and return correlations do increase when the rate change is larger. For large rate changes, such as 100 bps, this could present problems.

Table 13 displays the annual mean returns and the number of crashes for each population parameter configuration. The purpose of Table 13 is for robustness testing in order to ensure the population parameters chosen do not change the main results significantly. Results indicate, the mean annual return and crash frequency is relatively similar across the different population configurations. The results shown in Table 13 are for the 25 bps rate change. I find consistent results across parameter configurations for the other rate changing scenarios as well.

Table 14 displays robustness results from changing the baseline parameter configuration. If I change the parameter configuration and get completely different results it signals the conclusions cannot be relied on. To do so I increase only one parameter at a time keeping the other parameters constant. I increase this chosen parameter to a higher and lower level and test to see how it affects the number of crashes per century for the 25 bps unwinding paths for Period 4 only, the unwinding period. The number of crashes per century should decline as the frequency of the rate change is slowed down. Indeed that is the result I find. In addition, changing the parameter to a high number leads to more crashes then the baseline for each path and lowering the parameter leads to less crashes then the baseline for each path. Robustness results confirm the base results.

8. Conclusion

I add a central banking authority to an two-market agent-based model in order to setup an experiment to test how unwinding a zero interest rate policy impacts global stock market price dynamics. The Feldman (2010) model lends itself well to creating this type of experiment. I adapt the Feldman model to create two markets, a US inspired market and an emerging inspired market where the US inspired market interest rate is changed by the central bank after a global financial crisis. In this setup the global manager, who may invest across both markets, can decide to borrow at a zero interest rate or hold cash in the higher interest rate market.

The main purpose of the paper is to test which unwinding path of the zero interest rate policy creates the greatest stability across both stock markets. I find that no major dis-allocations occur as some argue will happen when the central bank increases the US inspired rate very slowly, once per year. Also, I find that increasing the rate by 50 bps can be done as long there is a good amount of time between rate changes. In real markets I propose that this may be more beneficial as the rate gets higher, around 2%, and a 50 bps is not as big of an increase as when the rate is close to zero percent.

In summary I find the frequency variable of how long to wait in between rate changes plays the greatest role, in keeping returns, volatility, crashes, and correlations stable. The size of the rate change also plays a role but not as large, especially when the rate change occurs slowly such as every twelve months.

Even though simulations cannot take into account all the realities of the global stock markets, it still provides an interesting experiment to help guide policy makers, especially when no historical data exists. Future research can upgrade these models to make them more sophisticated. For example, adding in exchange rate modeling, using more than two markets, etc. Simulations are best used in conjunction with real data. I find some of the results obtained are similar to what we have seen in actual markets. For example, US returns increase during the lowering of the interest rate and zero interest rate periods, US volatility decreases significantly during the zero interest rate period, and emerging market returns also decline throughout the simulations. Hopefully this provides confidence in the results which aim to forecast what stock market returns will look like in the future.

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I retain sole responsibility for remaining idiosyncrasies and errors.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at [10.1016/j.frl.2017.09.024](https://doi.org/10.1016/j.frl.2017.09.024).

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