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PROPAGATION OF INPUT UNCERTAINTY IN MANUFACTURING PROCESS FLOW SIMULATIONS

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ABSTRACT

We consider a discrete-event stochastic simulation that represents a generic manufacturing process flow and assume the availability of limited knowledge about part inter-arrival times and machine processing times. We investigate how the state-of-the-art simulation methodologies can be utilized to propagate input inter-arrival-time and service-time uncertainty through such a manufacturing simulation. We quantify the impact of limited knowledge on steady-state manufacturing-line performance measures such as utilization, lead time, inventory, and throughput. First, we conduct experiments for single-stage manufacturing systems where we additionally study the impact of yield loss uncertainty on annual throughput predictions. Then, we switch our focus to multi-stage manufacturing systems and investigate whether there may be situations in which it becomes difficult to correctly identify the system bottleneck. We conclude with the identification of the manufacturing system stages that contribute most to the variability in lead time and inventory predictions.

1 INTRODUCTION

This paper considers discrete-part manufacturing lines that are representative for complex production systems and the use of discrete-event stochastic simulation to support their operations management. In industrial manufacturing system design and analysis, the primary role of simulation is to provide strategic delivery predictions that are utilized by stakeholders with responsibilities ranging from daily plant operations to managing vendor decisions and making financial decisions. Therefore, the accuracy of simulation-based predictions is critical to support the decision-making process. A challenge that often arises in the use of simulation to serve this purpose, especially when the facility is under design, is the lack of sufficiently large data sets to characterize the distributions of the manufacturing system's input processes.

In Biller et al. (2018), discrete-event stochastic simulation is used to provide decision support for silicon carbide fabrication design and operational policy selection at the New York Power Electronics Manufacturing Consortium facility. Choosing simulation as the dynamic modeling technique for the silicon carbide project is motivated by the recent advances in simulation software and the resulting flexibility to model production system operations at a level of detail necessary to support the decision-making process. However, there still exist challenges of conducting this semiconductor manufacturing simulation. One of these challenges is the characterization of the input uncertainty in the simulation output analysis. In the industrial application discussed by Biller et al. (2018), 659 different input processes are identified to have limited data in relation to equipment profiles (e.g., loading times, processing times, unloading times, times between failures, and repair times), time of transportation within the facility, and (manual) processing times of the operators. Motivated by this industrial challenge in manufacturing system simulation design and analysis, we focus on first single-stage and then multi-stage production systems in this study. We perform simulation experiments to develop insights on the impact of limited input data on utilization, lead time (i.e., total time a part spends in the system), inventory (i.e., total number of parts in the system) and throughput predictions. The goal is to enhance our understanding of how much risk simulation-based predictions are exposed to due to the lack of full knowledge of the systems' arrival and service processes.

First, we consider a single-stage production system that forms the building block of many complex manufacturing systems. The parts are assumed to arrive at the system at a rate of λ units per hour while the workstation processes a part in an average of $1/\mu$ hours. Both the inter-arrival times between consecutive part arrivals and the machine processing times are independent and exponentially distributed random variables.

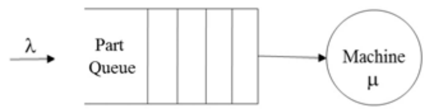

Figure 1: Single-stage production system with part arrival rate λ and part processing rate μ.

We illustrate the single-stage production system in Figure 1 and consider the situation in which the simulation practitioner does not know the true values of the arrival rate λ and the mean processing time μ^{-1}. These unknown parameters are estimated from N inter-arrival time data points and N service-time data points. It is important to note that equal length of history for arrival and service processes is an assumption that can be easily relaxed. We make this assumption only for the purpose of managing the number of experiments conducted in this short paper.

Next, we switch our attention to a multi-stage production system. We specifically consider a ten-station serial production line simulation developed using SAS Simulation Studio (Hughes et al. 2018) and illustrated in Figure 2, where each yellow compound block represents a station whose process step details are provided in Figure 3. In this case, there is a finite amount of service-time data for each of the ten workstations of the manufacturing line. The goal is to investigate the sensitivity of the predictions for machine utilization, lead time, inventory, and annual throughput to the increasing number of workstations.

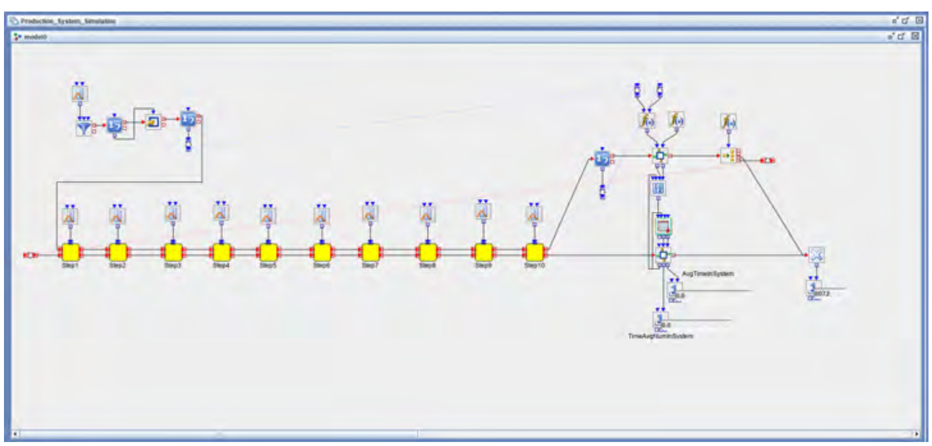

Figure 2: 10-station serial line simulation (Simulation Studio).

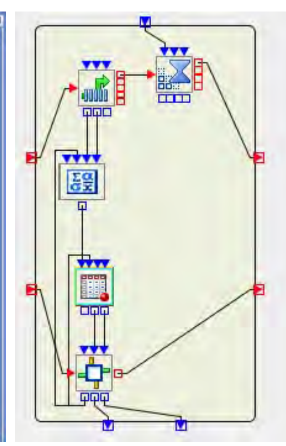

Figure 3: A step.

Independent of the structure of the manufacturing line, there are three main sources of uncertainty to account for in the simulation output analysis (Biller and Corlu 2015): Stochastic uncertainty (i.e., the uncertainty that is due to the dependence of the simulation output on random input processes) (Helton 1997), parameter uncertainty (i.e., the uncertainty that is due to the estimation of the input-model parameters from limited data), and model uncertainty (i.e., the uncertainty due to the selection of a single model from a set of alternative models) (Raftery et al. 1996). Because we assume exponentially distributed inter-arrival times and exponentially distributed service times for each workstation of the production line, we do not consider model uncertainty in this paper. We further assume unlimited computing budget so as to approximate the impact of stochastic uncertainty on simulation outputs to the value of zero. The goal is to obtain accurate quantification for the effect of input parameter uncertainty on single-stage and multi-stage production system performance measurements.

Today, accounting for input parameter uncertainty in discrete-event stochastic simulations is a well-studied problem of interest to considerable numbers of simulation researchers and practitioners. Song and Nelson (2017) present the most recent review of a variety of methods that have been used to capture input parameter uncertainty in stochastic simulations when input models are fit to finite samples of real-world data. In this paper, we propagate input parameter uncertainty via direct simulation; i.e., we perform simulations for different collections of input distributions and study the empirical quantiles of the output performance measures to assess the impact of input parameter uncertainty on system performance. However, the distinguishing feature of our work is the selection of a multi-stage production line as the area of application and to address the issues of yield loss and bottleneck management for the first time under input uncertainty.

The remainder of the paper is organized as follows: We discuss the representation of input parameter uncertainty surrounding inter-arrival time and service-time distributions estimated from limited historical data in Section 2. We also describe how input uncertainty representation is reflected in the 95% confidence intervals constructed for machine utilization, part lead time, system inventory, and annual throughput. We present our numerical findings for single-stage production systems in Section 3. We conclude with a multi-stage manufacturing simulation, representative for industrial manufacturing system simulations (Biller et al. 2017) in Section 4 and with future research in Section 5.

2 REPRESENTATION OF INPUT PARAMETER UNCERTAINTY

We use $\theta = 1/\lambda$ to represent the mean inter-arrival time parameter and $\beta = 1/\mu$ for the mean service-time parameter. We consider the measurement of each of inter-arrival-time and service-time variables in terms of weeks per part in Section 3 and in terms of minutes per part in Section 4. It is critical to account for the opinions of the experts in the input uncertainty characterization, for example, when the objective is to solve a strategic equipment portfolio selection problem (Biller et al. 2017). However, we want the results of this paper to only reflect the impact of limited arrival and service histories on the performance assessment. Therefore, we choose to follow a frequentist approach and capture the input uncertainty using the sampling distributions of maximum likelihood estimates of mean inter-arrival-time and service-time parameters θ and β. Notice that our uncertainty characterizations build on mean inter-arrival time θ and mean service time β instead of the arrival rate λ and the service rate μ. The reason is that the maximum likelihood estimates of θ and β are unbiased while the maximum likelihood estimates of λ and μ are biased.

We denote the length-N history available for the inter-arrival time random variable A by the time series a_n, $n = 1,2,\ldots,N$. Similarly, we define s_n, $n = 1,2,\ldots,N$ for the historical data set of the service-time random variable S. It is well known that both

$$\bar{a}_N = \frac{1}{N}\sum_{n=1}^{N} a_n \ \text{ and } \ \bar{s}_N = \frac{1}{N}\sum_{n=1}^{N} s_n$$

are unbiased maximum likelihood estimators of θ and β, respectively, attaining the minimum variance (Rohatgi and Saleh 2001). Since the random variables A_n, $n = 1,2,\ldots,N$ are independent and identically distributed, each with an exponential distribution having a true mean parameter of θ, it can be shown that the sampling distribution of $\bar{a}_N$ is of type gamma with a shape parameter of N and a scale parameter of θ/N. Similarly, the random variables S_n, $n = 1,2,\ldots,N$ are independent and exponentially distributed, each with a true mean parameter of β; therefore, the sampling distribution of $\bar{s}_N$ is of type gamma with a shape parameter of N and scale parameter of β/N.

Table 1: Standard deviations ($\tau(\cdot)$) and coefficients of variation ($\vartheta(\cdot)$) of $\bar{a}_N$, $\bar{s}_N$ and $\bar{p}_N$ as a function of $N \in \{10, 30, 50, 10000\}$ for the true inter-arrival time parameter $\theta = 0.0625$, the true service-time parameter $\beta = 0.05$, and the true yield-loss parameter $p = 10\%$.

	$\theta = 0.0625$		$\beta = 0.05$		$p = 10\%$	
N	$\tau(\bar{a}_N)$	$\vartheta(\bar{a}_N)$	$\tau(\bar{s}_N)$	$\vartheta(\bar{s}_N)$	$\tau(\bar{p}_N)$	$\vartheta(\bar{p}_N)$
10	0.020	0.32	0.016	0.32	9.487%	0.95
30	0.011	0.18	0.009	0.18	5.477%	0.55
50	0.009	0.14	0.007	0.14	4.243%	0.42
10,000	0.001	0.01	0.001	0.01	0.300%	0.03

First, we investigate the impact of mean inter-arrival-time parameter uncertainty and mean service-time parameter uncertainty on the mean performance measures of single-stage production systems with reliable workstations. Then, we introduce yield loss as an additional event to the simulation. We denote the true but unknown yield loss parameter by p. In this particular case, the data set is composed of p_n, $n = 1, 2, \ldots, N$ where $p_n = 1$ if the part completed its processing is identified as being defective and $p_n = 0$ otherwise. The maximum likelihood estimate $\bar{p}_N$ of p is given by

$$\bar{p}_N = \frac{1}{N} \sum_{n=1}^{N} p_n;$$

the sampling distribution of $\bar{p}_N$ is N^{-1} multiplied by the binomial distribution with parameters N and p.

Table 1 presents standard deviations $\tau(\bar{a}_N)$, $\tau(\bar{s}_N)$, and $\tau(\bar{p}_N)$ of $\bar{a}_N$, $\bar{s}_N$, and $\bar{p}_N$ and their coefficients of variations $\vartheta(\bar{a}_N)$, $\vartheta(\bar{s}_N)$, and $\vartheta(\bar{p}_N)$ as a function of N for the true values $\theta = 0.0625$ weeks per part, $\beta = 0.05$ weeks per part, and $p = 10\%$ yield-loss probability. The goal is to develop an initial insight into how fast the sampling distributions of $\bar{a}_N$, $\bar{s}_N$, and $\bar{p}_N$ approach the true parameter values θ, β, and p with the increasing length of the arrival history, the service history, and the yield loss history, respectively. We further illustrate sampling density functions of $\bar{a}_N$ and $\bar{s}_N$ in Figure 4 and in Figure 5, and the sampling mass function of $\bar{p}_N$ in Figure 6. For each input process, we observe that the input risk due to limited data decreases rather slowly and this is especially the case for yield loss. When there are only 10 observations in the historical data set, the coefficient of variation is 0.32 for each of $\bar{a}_N$ (i.e., $(0.020)/(0.0625)$) and $\bar{s}_N$ (i.e., $(0.016)/(0.05)$) while the coefficient of variation is 0.95 (i.e., $(9.487)/(10)$) for $\bar{p}_N$. When the data length increases from 10 to 50, the coefficient of variation for $\bar{a}_N$ and $\bar{s}_N$ decreases to 0.14 but to 0.42 for $\bar{p}_N$. Our goal is to understand how these observations are reflected in the distributions of the mean steady-state performance measures of the production systems. We achieve this goal in Section 3 for a single-stage production system and in Section 4 for a multi-stage production system.

We propagate the input parameter uncertainty, which is quantified in Table 1 and illustrated in Figures 4, 5, and 6 for arrival, service, and loss events, by using the simulation replication algorithm implemented within the SAS Simulation Studio environment. SAS Simulation Studio is a Java-based application for building and working with discrete-event simulation models (Hughes et al. 2018). Its distinguishing feature of flowing values through the simulation in a flexible manner makes SAS Simulation Studio a good candidate to propagate input uncertainty in manufacturing process flow simulations. Here is the presentation of the simulation replication algorithm for an M-station serial production line:

```
for outer loop simulation replication r from 1 to R do{
   sample a value for theta[r] from Gamma(N,theta/N)
   for process step m from 1 to M do{
      sample process step m processing time beta[m,r] from Gamma(N,beta/N)
    sample process step m yield loss probability p[m,r] from Binomial(N,p)/N }
```

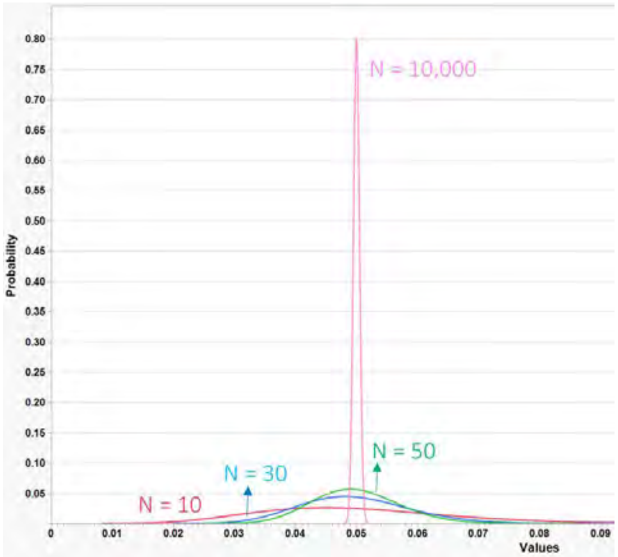

Figure 4: $\bar{a}_N$ sampling function.

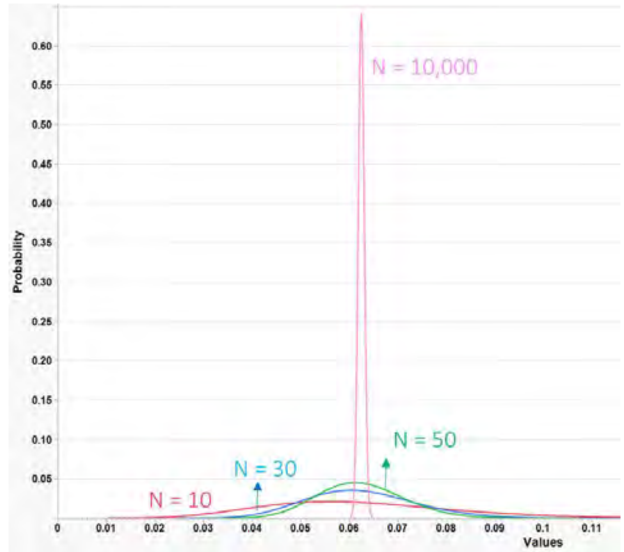

Figure 5: $\bar{s}_N$ sampling function.

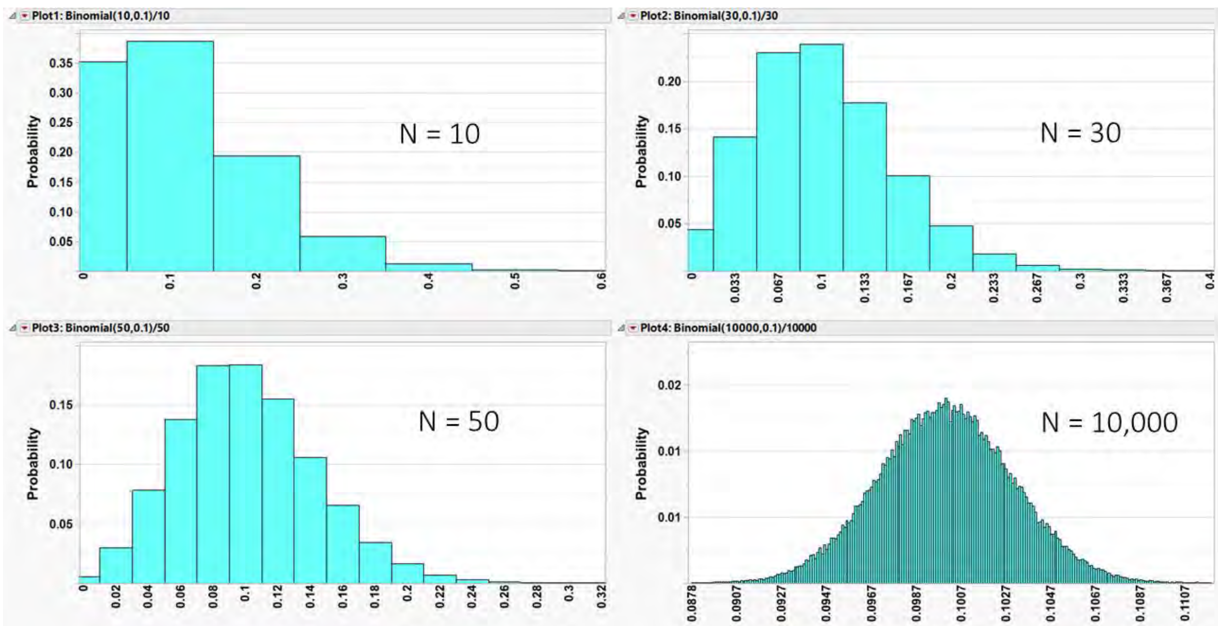

Figure 6: $\bar{p}_N$ sampling functions for maximum likelihood estimate of yield loss p probability.

```
    for inner loop simulation replication j from 1 to J do{
        drive the manufacturing line simulation with sampled parameter values.
        collect simulation output data, generically indicated by Y[r,j]. }
};
```

We implement the simulation replication algorithm for $M = 1$ in Section 3 and for $M = 10$ in Section 4. In each section, we choose a sufficiently large value for J to reflect the impact of infinite computing budget. Because the density functions of machine utilization, lead time, inventory, and throughput may not necessarily be symmetric, we construct a 95% confidence interval for each of these performance measures using 2.5% quantile, 50% quantile, and 97.5% quantile of the simulation output data $\bar{Y}_r := J^{-1}\sum_{j=1}^{J} Y_{rj}$.

It is common practice to construct the confidence intervals of the output performance measures using mean ($\mathbb{E}$) and variance ($\mathbb{V}$) of the simulation output data $\bar{Y}_r$. In fact, characterization of the variance of the mean output performance measure gives us the well-known insight that increasing the number of simulation replications J decreases the amount of stochastic uncertainty in the confidence interval but it has no impact on the portion of the confidence interval length due to input uncertainty. We demonstrate this result for manufacturing lines with reliable workstations by first representing the output random variable Y_{rj} of the jth simulation replication as $Y_{rj} := g(\theta,\beta,p) + \sigma Z_j$ where σ^2 represents the simulation stochastic variance and $Z_j,\ j = 1,2,\ldots,J$ correspond to the independent and identically distributed random variables, each with a mean of zero and a standard deviation of one. Furthermore, $g(\theta,\beta,p)$ is the mean simulation response function with the mean inter-arrival time parameter θ, mean processing time parameter β, and failure-probability parameter p. We are now ready to write down the variance of the mean simulation output variance:

$$\begin{aligned}
\mathbb{V}[\bar{Y}_r] &= \mathbb{V}_{\theta,\beta,p}\left[\mathbb{E}\left[\frac{1}{J}\sum_{j=1}^{J}\left(g(\theta,\beta,p)+\sigma Z_j\right)\Big|\theta,\beta,p\right]\right] + \mathbb{E}_{\theta,\beta,p}\left[\mathbb{V}\left[\frac{1}{J}\sum_{j=1}^{J}\left(g(\theta,\beta,p)+\sigma Z_j\right)\Big|\theta,\beta,p\right]\right] \\
&= \mathbb{V}_{\theta,\beta,p}\left[\frac{1}{J}\sum_{j=1}^{J}\left(g(\theta,\beta,p)\right)\right] + \mathbb{E}_{\theta,\beta,p}\left[\frac{1}{J^2}\sum_{j=1}^{J}\mathbb{V}\left(\sigma Z_j\right)\right] \\
&= \mathbb{V}_{\theta,\beta,p}\left[g(\theta,\beta,p)\right] + \frac{\sigma^2}{J}. \qquad (1)
\end{aligned}$$

As shown by various studies, increasing values of J does not affect the first component of the right-hand side in (1). Our goal is to understand how this term $\mathbb{V}_{\theta,\beta,p}[g(\theta,\beta,p)]$ responds to increasing arrival, service, and yield-loss history under different production system designs. However, we do this by the computation of 2.5%, 50%, and 97.5% quantiles of $\bar{Y}_r$ especially for the accurate computation of machine utilization in congested systems.

3 SINGLE-STAGE PRODUCTION SYSTEM

The goal of this section is to quantify the impact of input parameter uncertainty on four primary performance measures of a single-stage production system. We experiment with the three scenarios tabulated in Table 2 under the assumption of full knowledge of arrival and service processes:

Table 2: Single-stage production system: Scenarios and performance under full knowledge.

Scenario Index	λ (parts/week)	μ (parts/week)	Machine (utilization)	Lead Time (weeks)	Inventory (inventory)	Throughput (annual)
1	14	20	70%	0.17	2.33	700
2	16	20	80%	0.25	4.00	800
3	17	20	85%	0.33	5.67	850

Table 3 presents the 95% confidence intervals of the mean performance measures (i.e., their 2.5% quantile, 50% quantile, and 97.5% quantile) as a function of the data history of length $N \in \{10,20,30,50,100,200,300,500,1000,5000,10000\}$ under Scenario 1. Similarly, we present the results from Scenario 2 and Scenario 3 in Table 4 and Table 5, respectively. In each case, we observe that the impact of input uncertainty can be significant on the mean performance measures of single-stage production systems. Specifically, when there are only ten historical observations in each data set, we determine the median utilization as 62%, underestimating the true utilization by an absolute difference of 8% under Scenario 1. When the data length increases to 100, we then determine the median utilization to coincide with 70%. However, the 95% confidence interval for machine utilization is identified as between 53% and 91%. Thus, the length

Table 3: Scenario 1: 95% confidence intervals for mean performance under input uncertainty.

N	Utilization			Lead Time			Inventory			Annual Throughput		
10	24%	62%	98%	0.03	0.12	2.94	0.32	1.62	49.13	402	672	1103
20	37%	66%	97%	0.05	0.14	2.15	0.59	1.91	33.70	459	687	1049
30	41%	68%	97%	0.06	0.15	1.87	0.69	2.08	28.42	496	689	962
50	48%	69%	95%	0.08	0.16	1.10	0.92	2.24	18.85	543	700	903
100	53%	70%	91%	0.09	0.17	0.62	1.15	2.33	10.05	583	701	868
200	58%	70%	86%	0.11	0.17	0.41	1.37	2.33	6.35	615	698	807
300	60%	70%	82%	0.11	0.17	0.30	1.49	2.33	4.57	621	698	787
500	62%	70%	79%	0.12	0.17	0.26	1.64	2.33	3.81	647	702	761
1000	64%	70%	77%	0.13	0.17	0.23	1.77	2.33	3.33	657	699	745
5000	67%	70%	73%	0.15	0.17	0.19	2.06	2.33	2.67	682	701	720
10000	68%	70%	72%	0.16	0.17	0.18	2.14	2.33	2.57	687	700	714

Table 4: Scenario 2: 95% confidence intervals for mean performance under input uncertainty.

N	Utilization			Lead Time			Inventory			Annual Throughput		
10	29%	66%	98%	0.03	0.13	3.00	0.42	1.91	53.67	441	740	1253
20	41%	72%	98%	0.06	0.17	2.79	0.71	2.51	50.64	520	767	1107
30	47%	75%	98%	0.06	0.19	2.70	0.88	2.98	45.38	564	778	1045
50	55%	77%	98%	0.09	0.21	2.34	1.21	3.27	43.12	622	787	998
100	61%	79%	97%	0.11	0.23	1.82	1.54	3.70	34.63	656	795	945
200	66%	80%	95%	0.13	0.25	1.16	1.95	3.90	19.95	699	797	914
300	68%	80%	94%	0.14	0.25	0.98	2.08	3.98	16.79	710	800	894
500	71%	80%	91%	0.16	0.25	0.60	2.40	3.99	10.15	733	802	878
1000	73%	80%	87%	0.18	0.25	0.40	2.70	3.99	6.64	750	802	851
5000	77%	80%	83%	0.21	0.25	0.30	3.34	4.02	4.93	778	801	822
10000	78%	80%	82%	0.22	0.25	0.29	3.53	4.02	4.02	786	800	815

Table 5: Scenario 3: 95% confidence intervals for mean performance under input uncertainty.

N	Utilization			Lead Time			Inventory			Annual Throughput		
10	31%	70%	99%	0.04	0.15	4.18	0.45	2.33	74.16	476	776	1312
20	42%	76%	99%	0.05	0.20	3.47	0.72	3.25	68.76	552	795	1170
30	49%	77%	98%	0.07	0.21	3.20	0.96	3.43	61.61	597	809	1075
50	56%	80%	98%	0.09	0.25	3.16	1.30	4.09	58.65	645	823	1030
100	63%	83%	98%	0.11	0.29	3.613	1.68	4.85	54.03	694	840	993
200	70%	84%	98%	0.15	0.32	2.24	2.36	5.45	42.05	751	849	961
300	73%	85%	97%	0.17	0.32	1.95	2.68	5.49	35.75	764	850	952
500	75%	85%	96%	0.19	0.32	1.30	3.01	5.50	23.07	782	850	927
1000	78%	85%	93%	0.22	0.33	0.73	3.61	5.55	12.83	799	850	904
5000	82%	85%	89%	0.26	0.33	0.45	4.43	5.65	7.74	825	850	876
10000	83%	85%	87%	0.28	0.33	0.40	4.77	5.66	6.92	833	850	868

of the 95% confidence interval continues to be too large – even when when there are 100 observations of inter-arrival times and 100 observations of service times – to effectively inform the plant manager about the machine utilization. Nevertheless, increasing the data length by tenfold decreases the confidence interval half-length from 17% to 6%. We further decrease the 6% half-length to a 3% half-length by increasing the data length from 1,000 observations to 5,000 observations.

Another striking observation is the impact of limited input data on average inventory predictions. When there are only 10 observations of each of inter-arrival time and service time, we find that the 95% confidence interval for average system inventory is between 0.32 parts and 49.13 parts with a median of 1.62 parts. This observation is concerning because focusing on the median will lead to the underestimation of the need for the maximum buffer size. When there are 100 observations in each of the data sets, we identify the upper bound of the 95% confidence interval as 10 parts. Therefore, in the presence of very limited data, it would be ill-advised to make any buffer investment decisions. This observation also holds for annual throughput in which case the range of the 95% confidence interval is approximately the true unknown mean of the annual throughput; i.e., the lower bound, median and upper bound of the confidence interval are calculated as 402, 672, and $1,103$. In the case of having 100 observations in each of the data sets, we then identify the lower and upper bounds of the 95% confidence interval as 583 and 868.

The severity of input uncertainty on machine utilization, average inventory, and annual throughput increases further with the higher (true) system utilization. We illustrate the corresponding results presented in Tables 3, 4, and 5 in Figures 7, 8, and 9, respectively, where we also observe the need of a longer history for the median of the performance measures to converge to their true values.

Table 4 presents the results identified under the true utilization of 80% with reliable workstations. Next, we relax this assumption and compute median annual throughput and its confidence interval at true

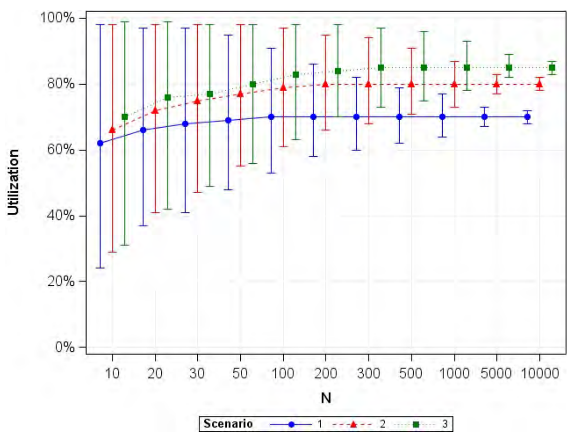

Figure 7: Machine Utilization.

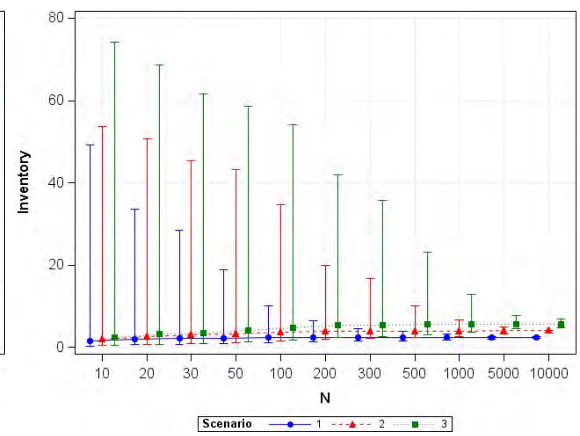

Figure 8: Total Inventory.

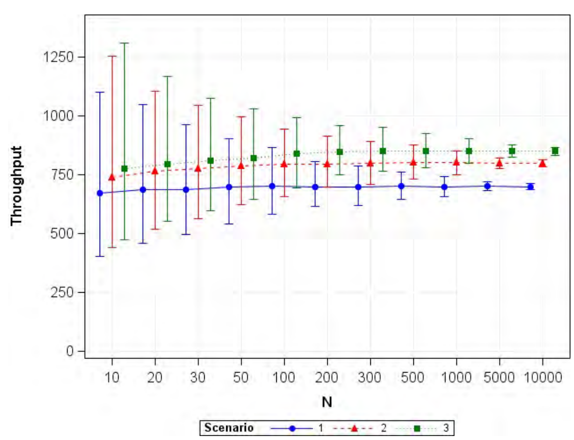

Figure 9: Annual Throughput.

yield loss probabilities of 5%, 10%, and 20%. We present our findings in Table 6 and illustrate them in Figure 10. The relative comparison of the medians calculated under 0% and 5% (or 10% or 20%) true yield probabilities result in a reduction of 5% (or 10% or 20%) in annual throughput. We observe the input uncertainty to exhibit its impact on annual throughput at lower values of *N* and especially at the lower bound of the 95% confidence interval. In the presence of 5% (or 10% or 20%) true yield loss probability, the lower bound of the 95% confidence interval is found to be 8% (or 15% or 34%) lower than its counterpart at 0% yield loss probability. In each case, however, we find this observation to disappear and that all relative differences converge to their corresponding yield loss probabilities as soon as the number of historical data points for each input process reaches 200.

Table 6: Scenario 2: Impact of yield loss uncertainty on annual throughput.

N	0% Yield Loss			5% Yield Loss			10% Yield Loss			20% Yield Loss		
10	441	740	1253	419	700	1218	387	659	1156	300	580	1070
20	520	767	1107	492	728	1066	456	678	1034	385	609	931
30	564	778	1045	527	739	1028	499	690	949	426	619	882
50	622	787	998	587	748	957	540	708	934	460	629	829
100	656	795	945	634	762	915	590	720	857	516	636	779
200	699	797	914	660	761	872	627	720	833	553	640	741
300	710	800	894	673	760	853	638	721	813	563	641	719
500	733	802	878	698	761	835	659	720	791	576	640	705
1000	750	802	851	714	759	809	675	719	769	595	642	688
5000	778	801	822	739	760	783	699	720	743	620	640	659
10000	786	800	815	745	760	776	706	720	734	627	640	655

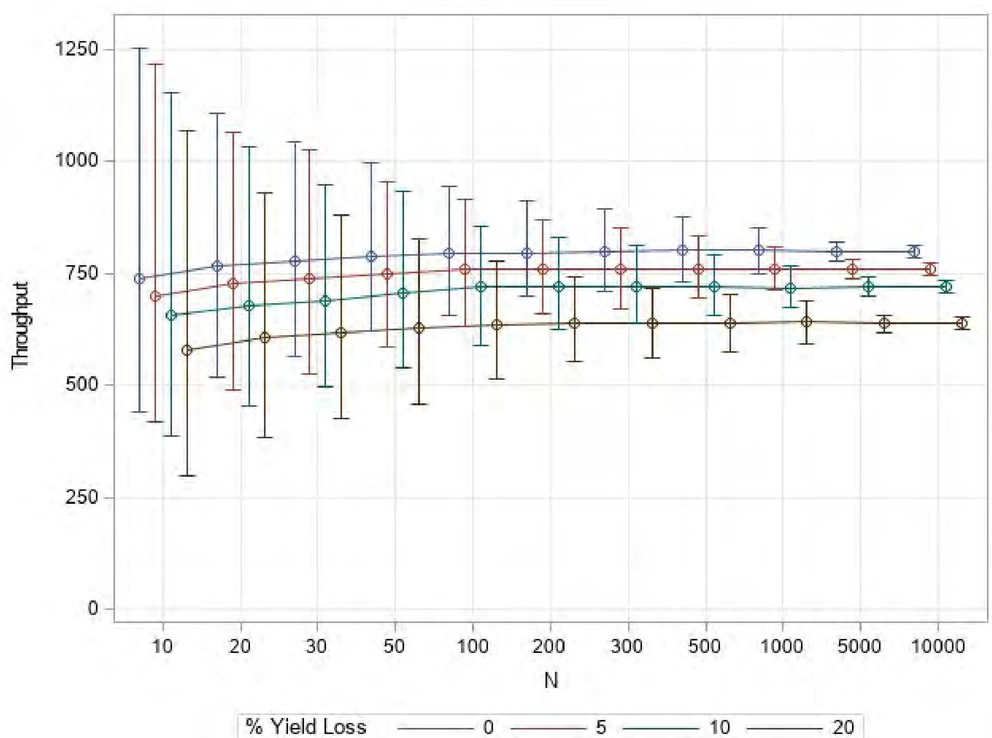

Figure 10: Illustration of the impact of yield loss uncertainty on annual throughput.

4 MULTI-STAGE PRODUCTION SYSTEM

In this section, we consider a serial manufacturing line motivated by an application discussed in Biller et al. (2017) and illustrated in Figure 2 and Figure 3 of Section 1. We continue to make the assumption of exponentially distributed part inter-arrival times and exponentially distributed processing times for each workstation of the production line. We impose no constraints on the sizes of the buffers between consecutive workstations. We aim to understand the inventory storage needs under the given operational assumptions. Furthermore, we consider each workstation to be reliable; i.e., 0% yield loss probability. Table 7 assumes 200 as the true mean inter-arrival time (i.e., $\theta = 200$ minutes per part) and presents the design of this manufacturing line under full knowledge of arrival and service processes. In this table, we also provide steady-state average performance measures utilization, lead time, and inventory, and present each at the workstation level. It becomes immediately evident that the bottleneck is Machine 5 with an average utilization of 85% where each part is expected to spend a total of 1,133 minutes. Furthermore, there are, on average, 5.67 parts in this workstation. The second busiest workstation of the production line is Machine 1 with an expected utilization of 80% where each part is expected to spend 800 minutes. In addition, there are, on average, 4 parts in the first station of the manufacturing line.

Table 7: Multi-stage production system: Process flow and performance under full knowledge.

Process Step i	Resource Type at Station i	β_i (minutes/part)	Station Utilization	Lead Time (minutes)	Inventory (parts)
1	Machine 1	160	80.0%	800.00	4.00
2	Operator 1	0.80	0.40%	0.80	0.00
3	Machine 2	0.87	0.44%	0.87	0.00
4	Operator 2	0.80	0.40%	0.80	0.00
5	Machine 3	0.85	0.43%	0.85	0.00
6	Machine 4	45	22.5%	58.06	0.29
7	Machine 5	170	85.0%	1133.33	5.67
8	Machine 6	45	22.5%	58.06	0.29
9	Machine 7	55	27.5%	75.86	0.38
10	Machine 8	67.5	33.8%	101.89	0.51

We assume the availability of only 50 observations for the inter-arrival times and for each of the processing times in the 10-station serial production line characterized in Table 7. We first present the 95% confidence interval for utilization of each workstation in Table 8. Then, we illustrate these confidence intervals associated with all stages of the production line in Figure 11. We immediately notice that it is not clear anymore whether Machine 5 is the bottleneck of the manufacturing line. In fact, it is inconclusive whether Machine 1 (i.e., process step 1) or Machine 5 (i.e., process step 7) is the bottleneck. In this particular case, there may be value in designing the data collection plan in a way to reveal system bottlenecks in a fast and accurate manner.

Table 8: Impact of arrival and service process uncertainty on workstation utilization.

	Process Step									
Utilization	1	2	3	4	5	6	7	8	9	10
2.5% Quantile	50.8%	0.3%	0.3%	0.3%	0.3%	14.6%	54.8%	14.5%	18.1%	22.6%
50% Quantile	75.3%	0.4%	0.4%	0.4%	0.4%	21.5%	79.4%	21.5%	25.9%	32.0%
97.5% Quantile	97.6%	0.5%	0.6%	0.5%	0.6%	30.1%	98.9%	31.2%	37.7%	45.1%

Next, we present the 95% confidence intervals for steady-state annual throughput, total lead time, and overall inventory in the system and compare them to those provided by Table 7:

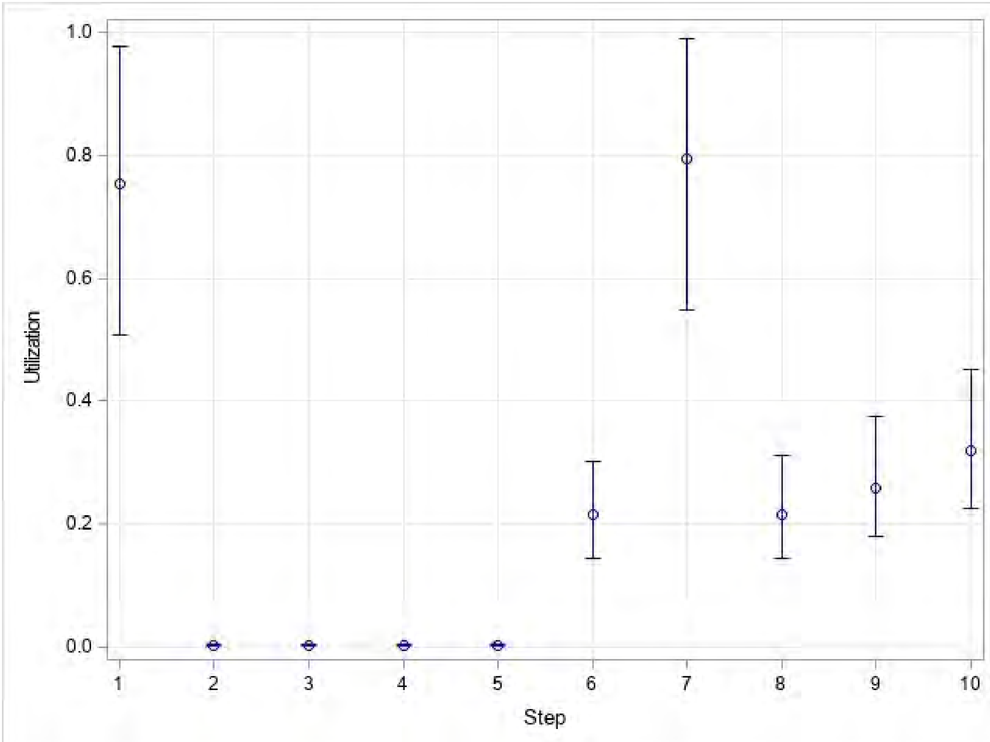

Figure 11: Illustration of the impact of input process uncertainty on workstation utilizations.

Table 9: Impact of lacking full arrival and service process knowledge on system performance.

	Full Knowledge			Data Length $N = 50$		
	Throughput	Lead Time	Inventory	Throughput	Lead Time	Inventory
2.5% Quantile	600	$2,230.55$	11.15	451.82	953.20	3.81
50% Quantile	600	$2,230.55$	11.15	575.49	$1,920.07$	9.24
97.5% Quantile	600	$2,230.55$	11.15	718.35	$24,883.87$	142.89

It has already been observed in the single-stage production system simulation that the input uncertainty may cause high variability in the confidence intervals constructed for mean performance measures such as annual throughput, lead time, and inventory. We continue to make this observation in Table 9 but with even more pronounced impact of arrival and service process uncertainty on lead time variability and inventory variability. Under the full knowledge of the interarrival-time and service-time distributions, the part lead time is expected to be $2,230.55$ minutes and the average inventory is identified as 11.15 parts. When we lack this knowledge and assume the availability of only 50 historical observations for each input process of the simulation, we are 95% confident that the mean part lead time falls between 953.20 minutes and $24,883.87$ minutes and the average system inventory falls between 3.81 parts and 142.89 parts. Thus, this limited level of knowledge about inputs causes simulation to fall short of delivering accurate predictions about the performance of the manufacturing line described in this section. Nevertheless, it would be critical to understand which operational stages of the manufacturing line would contribute to this high level of variability in lead time and inventory predictions. Without conducting any further experiments, Table 7 would suggest those stages to be Machine 1 and Machine 5. We, however, quantify the contribution of each resource to the lead time variability. Consequently, we find that 22% of simulation's average lead time variance is due to Machine 1 and remaining 78% of the variability is due to Machine 5. Similarly, 25%

of the simulation's average inventory variance is due to Machine 1 and remaining 75% of the variability is due to Machine 5. These types of quantification would be beneficial in attempts towards improving the accuracy of simulation-based predictions.

5 FUTURE RESEARCH

Section 4 decomposes the manufacturing line lead time and inventory variability into different stages of production. The resulting quantification would be guiding the collection of new data and information in an effective manner. However, each stage is exposed to both arrival uncertainty and service uncertainty and additionally to yield loss uncertainty in the case of unreliable machines. This makes it difficult to further trace the simulation output variability to the input processes themselves. Solving this problem for multi-stage production systems is the objective of future research.

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