# A SIMULATION APPROACH FOR MULTI-STAGE SUPPLY CHAIN OPTIMIZATION TO ANALYZE REAL WORLD TRANSPORTATION EFFECTS

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# ABSTRACT

The cost effective management of a supply chain under stochastic influences, e.g. in demand or the replenishment lead time, is a critical issue. In this paper a multi-stage and multi-product supply chain is investigated where each member uses the (s,Q)-policy for inventory management. A bi-objective optimization problem to minimize overall supply chain costs while maximizing service level for retailers is studied. Optimal parameter levels for reorder points and lot sizes are evaluated. In a first step a streamlined analytical solution approach is tested to identify optimal parameter settings. For real applications, this approach neglects the dynamics and interdependencies of the supply chain members. Therefore a simulation-based approach, combining an evolutionary algorithm with simulation, is used for the optimization. The simulation-based approach further enables the modelling of additional real world transportation constraints. The numerical simulation study highlights the potential of simulation-based optimization compared to analytical models for multi-stage multi-product supply chains.

# **1 INTRODUCTION**

The members of a supply chain (SC) – a network of interconnected organizations – are linked by material, information and financial flows. The objective of supply chain management (SCM) is to produce value in form of products and services for the ultimate customer. SCM involves planning, design and control of materials, information and finance along the SC in an effective an efficient manner. Especially in the last years, SCM received a lot of attention in literature as well as practice. One source of complexity in SCM is given by the different stakeholders involved, e.g. manufacturers, suppliers, distributors, transporters and warehouses. Stochastic demand and divergent and convergent flows of materials determine another source of complexity in the SC planning process (Shah 2009; Stadtler 2015). Inside SCM, inventory management is a critical issue for success. The goal of successful SCM is to provide a high service level and simultaneously minimize operating costs such as inventory costs for capital tied up in raw material, work-in-progress and finished goods inventories, respectively. Therefore inventory model parameterization offers an interesting field of research (Axsäter 2015).

A vast amount of literature on inventory models is available discussing the optimal control of inventory systems. In literature, e.g. Silver, Pyke, and Peterson (1998), Axsäter (2015) or Tempelmeier (2011) introduce different types of inventory models. Within inventory management, the (s,Q) policy with

continuous review is a common method discussed in literature and according to Axsäter (2015), singleechelon techniques are used in practice to handle different stocks in a SC. Its replenishment logic orders a lot size O whenever the inventory position drops below the reorder point s. The inventory position consists of stock on hand plus outstanding orders minus backorders. For parameter optimization the objective function is to minimize the costs for inventory, ordering and backorders. Generally, instead of backorders, also a certain service level constraint can be used. Some recent papers that use the (s,Q)policy are Tamjidzad and Mirmohammadi (2015), Kouki, Jemaï, and Minner (2015) or Pérez and Geunes (2014). For simple situations, optimal parameters for reorder point s and lot size O can be determined analytically or heuristically with an iterative approach in order to reach a cost optimum. However, different assumptions are made in the different models, e.g. only single product, single stage or a specific demand distribution. In real world, SCs are multi-stage, multi-product and are facing a lot of stochastic influences. Therefore the determination of optimal parameters for reorder point s and lot size Q for these systems is complex and several assumptions have to be made in order to make the optimization models analytically tractable. Whenever analytical optimization fails, simulation can be used to model such complex inventory systems (Köchel and Nieländer 2005). Simulation-based optimization approaches combine metaheuristic search procedures with simulation (Gosavi 2003) which lead to a powerful optimization approach if the complexity of the investigated systems are too high (see also Beham et al. (2012) and Felberbauer, Schnirzer, and Altendorfer (2015) for recent applications). Cattani, Jacobs, and Schoenfelder (2011) investigate assumptions that are made in standard SC literature and conclude that analytical model results can lead to inefficient solutions if they are applied for determining optimal policies for a real world setting. The authors show that the single-echelon approximation is inappropriate for setting reorder points. For their investigated SC they show that a dual-role warehouse (central warehouse delivers to retailers as well as to end customers) and a centralized control mechanism can reduce inventory costs with respect to a service level constraint. Also the results of Altendorfer, Felberbauer, and Jodlbauer (2016), which discuss hierarchical SC planning effects, indicate that deterministic analytical solution models can lead to significant cost penalties and propose simulation to incorporate complex planning interdependencies. Köchel and Nieländer (2005) used simulation-based optimization for a multi-echelon (s,Q) inventory policy. A decentralized and a centralized control mechanism are compared with different backlogging limits and cost structures. Applying a genetic algorithm, a sequential multi-stage SC with a single product and Poisson demand was optimized. Farahani and Elahipanah (2008) use a multi-objective genetic algorithm to optimize the total cost and the service level of a three-echelon SC. The genetic algorithm is applied to solve a mixed-integer linear programming model for a real size problem with delivery lead time and capacity constraints for a multiperiod, multi-product and multi-channel network. In Beham et al. (2012) the application of the solution heuristics package HeuristicLab for planning parameter optimization is introduced and the results of Felberbauer, Schnirzer, and Altendorfer (2015) for simulation-based planning parameter optimization indicate a good applicability of this tool. Therefore, this solution package is also applied in the current study for SC optimization.

In this article an analytical approach is compared to a simulation-based optimization approach. For the analytical case a single-echelon (s,Q) policy is applied to all partners within the SC in order to determine appropriate levels for reorder point s and lot size Q. The optimal solution and parameters derived from the analytical approach are then re-evaluated in the simulation model. This first study leads to insights into how a practitioner's solution of using analytical models and ignoring interdependencies in the SC performs in a complex setting. With the use of an evolutionary algorithm the parameters for reorder point s and lot size Q are optimized and the performance increase of incorporating the dynamics and dependencies between the SC members is evaluated. Note that the simulation-based optimization does not guarantee to find the global optimum solution for the studied SC structure. Nevertheless, the simulation-based optimization technique shows a significant potential to improve the performance in comparison to the iterative analytical approach. Also a detailed analysis of the different optimal

parameters is presented. Furthermore, in a more realistic scenario the influence of the real world effect of having a truck load limit, i.e. the trucks are limited in size, is studied. Finally, in another scenario the truck load limit restriction is extended by modelling a mixed load opportunity where different products can be transported on the same truck. The focus of the paper is twofold. First, from a methodological aspect, the solution and parameter differences of the analytical and the simulation-based optimization approach are discussed. Second, concerning real world assumptions, the cost influence of these assumptions is investigated.

## 2 MODEL DESCRIPTION

The multi-stage and multi-product SC, as illustrated in Figure 1, is modeled in AnyLogic simulation software and consists of a Manufacturer M, a Distribution Center DC and several Retailers  $R_i$ . The objective is to minimize overall costs C while maximizing service level  $\eta$  for Retailers. Overall costs C consist of inventory costs  $c_h$  and also for SC optimization order/setup costs  $c_s$ . Note that no backorder costs are incorporated within the cost function as service level  $\eta$  is maximized in the bi-objective optimization problem. Whereas overall costs C are summarized over all SC members, the service level  $\eta$  is only evaluated for Retailers that deliver their goods to customers, i.e. product availability to the customer.



Figure 1: Multi-stage supply chain.

Each member within the SC uses the (s,Q) policy per product for the inventory management. The parameters that are critical for the SC performance are reorder point *s* and lot size *Q*. Reorder point *s* specifies the decision in time, i.e. when to order, and lot size *Q* the decision on the amount of an order. Higher reorder point *s* reduces the probability of a stock out situation and therefore increases service level  $\eta$ , but also increases inventory costs. Higher lot size *Q* reduces the order costs because fewer orders are necessary, but simultaneously increase inventory costs.

Retailers  $R_i$  (index i specifies the Retailer) are offering *P* different products and customers order a specific product *p* for immediate delivery, i.e. customer required lead time is zero. If the product can't be delivered, the order is backlogged. Note that for simplification reasons all products have the same demand distribution and therefore index *p* is omitted in the remainder. Once the inventory position drops below the reorder point  $s^{R_i}$ , the Retailer orders lot size  $Q^{R_i}$  from the DC. If the order can be fulfilled, the products are packed and transported to the particular Retailer  $R_i$ . The order of lot size  $Q^{R_i}$  for the specific product is packed with random packing time  $t_p^{DC}$  at the DC and transported with random transportation time  $t_t^{R_i}$  to the Retailer  $R_i$ . The replenishment lead time  $L^{R_i}$ , therefore, consists of random packing time as well as random transportation time. Note that the distances between Retailers  $R_i$  and the DC differ which leads to different expectation values for the transportation and packing costs. The replenishment lead time at the DC, i.e.  $c_s^R$  and  $c_s^{DC}$  respectively, consist of transportation and packing costs. The replenishment lead time at the DC,  $L^{DC}$ , also consists of a random packing time at the Manufacturer  $t_p^M$  and a random transportation time to the DC  $t_t^{DC}$ . For Manufacturer M, setup/cleaning costs  $c_s^M$  are modeled whenever

two orders of lot size Q are produced in sequence. The streamlined production system of the Manufacturer M holds one machine, where all the different products can be manufactured and it is assumed that the required raw materials are always available. The replenishment lead time at the Manufacturer  $L^M$  consists of processing and waiting time of the single-stage manufacturing system. In the basic scenario unlimited truck load is assumed.

In a second scenario for the transportation from the DC to the Retailers  $R_i$ , a specific truck load limit *TL* is modeled and the effect of this real world constraint is studied. This scenario considers that trucks which deliver to the Retailers  $R_i$  (usually smaller trucks are used there) are limited in their size. When lot size *Q* is larger than the maximum truck load *TL*, lot size *Q* is split into [Q/TL] trucks.

Finally, in the third scenario a mixed load is modelled. This scenario implements the real world practice that different products are transported simultaneously in the same truck to a Retailer. In detail, mixed load is modeled as follows: if the lot size of the Retailer  $Q^{R_i}$  does not fill the whole truck, the trucks wait at the DC up to a specific truck waiting time  $t_w$  to be filled with further orders. This truck waiting time  $t_w$  of the truck is also an optimization parameter in this scenario.

All SC partners are modeled as agents in AnyLogic simulation software version 7.2. For each partner and each product p, a (s,Q) ordering policy is implemented. Random customer demand is addressed for each Retailer  $R_i$ . Each order within the SC holds information on the requesting SC partner, and the required quantity of the respective product p. The agent of Manufacturer M mimics the functionality of a machine and holds the logic of packing and delivery to the DC. Also for the DC packing and delivery to Retailer  $R_i$  are modeled. Additionally, the DC features the logic of a truck waiting time, which is used within the mixed load scenario.

### **3** SOLUTION METHODS AND MODEL

For the optimization of the bi-objective problem, i.e. minimize overall costs C and maximize service level  $\eta$ , by changing reorder point s and lot size Q, the different optimization methods are introduced.

#### 3.1 Analytical Optimization

Even though there are different multi-stage inventory models available in the literature which also partly cover the restrictions discussed in basic scenario of this paper, their implementation is rather difficult. Therefore, only simple models are applied in practice. Furthermore, some extensions are investigated in this paper, i.e. truck load limit and mixed load, which make the model analytically intractable. The first solution approach uses a single-stage inventory model, still being able to handle stochastic demand and stochastic replenishment processes for each SC member separately. The optimal parameters are calculated for each SC member independently. This seems to be appropriate as this model is simple enough to be practically applicable but still leads to good results in the dynamic and complex situation. As in this paper a continuous review reorder point replenishment policy is discussed with respect to a service level constraint  $\tilde{\eta}$ , the following model according to Axsäter (2015) can be formulated.

The cumulative distribution function of the inventory is defined as (whereby  $f_{DL}(\cdot)$  is the probability density function of the demand within the replenishment lead time), and can be restated for normally distribute demand as:

$$F_{Y}(\theta) = \frac{1}{Q} \int_{s}^{s+Q} \int_{\tau-\theta}^{\infty} f_{DL}(\psi) d\psi d\tau = \frac{1}{Q} \int_{s}^{s+Q} 1 - F_{DL}(\tau-\theta) d\tau = \frac{1}{Q} \int_{s}^{s+Q} 1 - F_{N(0,1)}\left(\frac{\tau-\theta-E[DL]}{\sqrt{Var[DL]}}\right) d\tau$$
(1)

As the service level  $\eta$  can be identified as probability that stock on hand is non zero, the following relationship holds:

$$1 - \eta = F_{Y}(0) = \frac{1}{Q} \int_{s}^{s+Q} 1 - F_{DL}(\tau) d\tau$$
<sup>(2)</sup>

Which provides an implicit statement for the reorder point *s* that can numerically be solved. Note that Wolfram Mathematica is applied to numerically solve this equation.

$$s: \tilde{\eta} = 1 - \frac{1}{Q} \int_{s}^{s+Q} 1 - F_{DL}(\tau) d\tau$$
(3)

As no backorder and lost sales costs occur, the optimal lot size Q according to Axsäter (2015) is the economic order quantity (EOQ) with:

$$Q = \sqrt{2E[D]c_s/c_h} \tag{4}$$

As in this simulation study different distributions for transportation, packaging and production lead time are applied, the demand within the replenishment lead time *DL* is approximated to be normally distributed which is in line with the central limit theorem. To further simplify this approximation, the replenishment lead times are assumed to be deterministic and are linked to the stochastic demand which leads to E[DL] = E[D]L and Var[DL] = Var[D]L. This setting covers the assumptions often made in practice because they are simple and easy to implement. For a set of service level constraints  $\tilde{\eta}$  the optimal parameters for  $\{s^{R_i}, s^{DC}, s^M\}$  and  $\{Q^{R_i}, Q^{DC}, Q^M\}$  are optimized. Note that these parameters are equal for all products because of the identical demand distribution assumption introduced above. The total costs for inventory and setup for all partners in the SC for a single product can be estimated with the following formula:

$$C(\cdot) = c_{h}^{M} \left( \frac{Q^{M}}{2} + s^{M} + E[DL_{DC}] \right) + c_{s}^{M} \frac{E[D]}{Q^{M}} + \sum_{i} c_{h}^{DC} \left( \frac{Q^{DC}}{2} + s^{DC} - E[DL_{DC}] + E[DL_{R_{i}}] \right) + c_{s}^{DC} \frac{E[D]}{Q^{DC}} + \sum_{i} \left( c_{h}^{R} \left( \frac{Q^{R_{i}}}{2} + s^{R_{i}} - E[DL_{R_{i}}] \right) + c_{s}^{R} \frac{E[D]}{Q^{R_{i}}} \right)$$
(5)

Note that in this cost formulation the inventory costs of the Manufacturer apply until the delivery at the DC and the holding costs of the DC apply until the delivery at the Retailer.

The results of the analytical optimization model described above are re-evaluated in the dynamic and stochastic simulation model. In a first step, this allows to compare the solution values of the analytical model and its restrictive assumptions with the realized results of the simulation model.

## 3.2 Simulation-Based Optimization

For simulation-based optimization the simulation model is connected to the open-source heuristic optimization framework HeursticLab. HeursticLab is a framework for metaheuristics, such as evolutionary algorithms, and enables to couple them with a simulation software by use of an external evaluation problem (Affenzeller et al. 2015). As the problem is bi-objective, a Nondominated Sorting Genetic Algorithm II (NSGA-II) is used. The chosen algorithm creates a set of solutions (parameter sets) and send them to the simulation model. After evaluation in the simulation model the solution qualities, i.e. overall costs *C* and service level  $\eta$  are returned to HeuristicLab. In a preliminary study, the following parameterization of the NSGA-II was identified: population size is 100, selected parents are 400, crossover probability is 90%, and a mutation probability is 5%. For the crossover operator MultiRealVectorCrossover is used which randomly selects different crossover methods applicable for solutions encoded as real vectors.

## 4 NUMERICAL STUDY

The SC studied consists of 1 Manufacturer M, 1 Distribution Center DC and 6 Retailers R. Each Retailer  $R_i$  offers P = 10 different products with order rate 10 pcs/day and exponential interarrival times. Note that for the service level calculation, customer orders that cannot be fulfilled at the end of the

simulation time reduce the service level  $\eta$ . The inventory costs are  $c_h^M = 0.25$  currency units (CU)/day,  $c_h^{DC} = 0.5$  CU/day, and  $c_h^R = 1$  CU/day (equal for all Retailers). Packing costs at the DC are 100 CU and transportation costs to Retailers are 500 CU resulting in setup costs  $c_s^R$  of 600 CU. Note that in the second extension the transportation costs occur only once if more than one product is loaded on the same truck. Setup costs for the DC  $c_s^{DC}$  are 400 CU for packing and transport. The packing time at the DC  $t_p^{DC}$  and for the Manufacturer  $t_p^M$  are triangular distributed with minimum 1/24 days, maximum 5/24 days and mode 3/24 days. The random transportation times  $t_t=\alpha+T$  are the sum of a fix  $\{\alpha^{DC}, \alpha^{R_i}\}$  value, i.e. the minimum time it takes without any disturbances, and an exponential distributed random part  $\{T^{DC}, T^{R_i}\}$  implementing the possible disturbances in the transportation process. For the transportation time  $t_t^D$  from Manufacturer to the DC  $\alpha^{DC} = 4$  days and  $E[T^{DC}] = 1$  day. The 6 Retailers are clustered according to their distance to the DC, i.e. different distances are assumed. Cluster i=1 includes 3 Retailers (the nearest ones) with transportation time  $t_t^{R_i}$  from the DC to Retailer R<sub>i</sub> having  $\alpha^{R_I} = 0.5$  and  $E[T^{R_I}] = 0.125$  days. Cluster i=2 includes 2 Retailers (medium distance) with  $\alpha^{R_2} = 1$  and  $E[T^{R_2}] = 0.25$  days and cluster 3 is just one Retailer which has the longest distance with  $\alpha^{R_3} = 4$  and  $E[T^{R_3}] = 1$  day. Note that the index *i* here is linked to the Retailer cluster not to the specific Retailer. The setup costs for the Manufacturer  $c_s^M$  are 500 CU. The setup times per lot at the machine are 1.47 hours and processing time per piece is 0.036 hours leading to an utilization of 97.5% with an optimal lot size Q calculated according to Equation (4). For the simulation studies a simulation time of 1250 days is used, whereby 250 days are used as warm up time. Each parameter set is evaluated

For extension 1, the truck load limits investigated are  $TL \in \{75,100,125,150,200\}$  and for extension 2, additionally the truck waiting time t<sub>w</sub> is optimized in a range of 0 to 3 days which has been identified as interesting parameter values in preliminary studies. Note that the truck costs are comprised in the transportation costs, and they do not incur extra costs if they are waiting.

### 4.1 Evaluation Of Analytically Determined *s* and *Q*

In the analytical optimization a set of service level constraints  $\tilde{\eta}$  is evaluated within the interval [0.8,0.998] in order to get optimal parameters for reorder point *s* and lot size *Q* for all different SC partners applying Equation (3) and (4). A broad range of service level constraints  $\tilde{\eta}$  are evaluated in order to address different logistics sector targets.

The replenishment lead time at the Manufacturer  $L^M$  is estimated by the mean production lead time for  $Q^M$  calculated according to Equation (4). Figure 2 compares the estimated costs C from the analytical optimization model with the realized costs C when simulating the optimized parameters.

Figure 2a shows the overall costs *C* of the analytical model that are estimated using Equation (5). The overall costs *C* realized when evaluating the optimal analytical parameters for reorder point *s* and lot size *Q* in the simulation model are as well presented there. Figure 2a as well as 2b show that the observed service level  $\eta$  in the simulation model is lower than the service level that is estimated in the analytical model. The highest service level  $\eta$  that is reached with the analytically optimized parameters and simulating them is approximately 0.95 and the lowest is 0.72. This deviation is linked to some of the simplifying assumptions needed to keep the analytical model practically tractable which is in line with the findings of Cattani, Jacobs, and Schoenfelder (2011). Due to the fact that the realized service level  $\eta$ , which is only evaluated for Retailers, is lower than estimated, there are more out of stock situations than planned. This finding provides the motivation to use simulation optimization to identify good parameter combinations to also fulfill high service levels  $\eta$ . Figure 2c shows the optimal parameters for reorder point with respect to the different Retailer clusters  $s^{R_i}$  and in Figure 2d with respect to the DC  $s^{DC}$  and the Manufacturer  $s^M$ . The results show a non-linear increase in the optimal reorder point *s* with respect to the service level follows from the analytical calculation and can in the further experiments be compared to the simulation-based optimization results. Note that based on Equation (4) it

is clear that lot size  $Q^{R_i}$  is equal for all Retailer clusters and for all products due to their identical demand distribution. Applying a linear regression for the simulation results of Figure 2a, the study shows that an increase of service level  $\eta$  from 90% to 95% leads to a cost increase of 3.6%. Another interesting finding from this analytical results is, that the short and medium distance Retailers have reorder point  $s^{R_i} = s^{R_2} = 0$  for estimated service levels of below  $\eta = 0.93$  and  $\eta = 0.85$ , respectively. The further simulation optimization presented in the next section will show that this low reorder points *s* are not optimal for the complex SC setting discussed.



Figure 2: Overall costs *C*, service level  $\eta$  and optimal parameters for the analytical optimization model with respect to varying service level constraints  $\tilde{\eta}$ .

## 4.2 Simulation-Based Optimization of *s* and *Q*

In the next study, the simulation-based optimization approach, as described in Section 3.2, is used for optimizing the (s,Q) policy parameters of the different SC partners. Figure 3 summarizes the results, comparing the simulation results of the analytically optimized parameters (see Section 4.1) to the simulation-based optimization results.

In detail Figure 3a shows the cost comparison of the optimal analytical parameters and the simulation-based optimization parameters. The simulation-based optimization approach leads to significantly lower costs and also service levels  $\eta$  above 0.95 can be reached. The average cost decrease between  $\eta = 0.95$  and  $\eta = 0.75$  using simulation-based optimization including the real relationships between the SC partners is 11%. A further interesting finding which is in line with previous literature is that for high service level constraints  $\tilde{\eta}$ , overall costs *C* show a non-linear increase (see also Axsäter (2015) for similar analytical results). Figure 3b shows the optimal lot size *Q* with respect to service level  $\eta$ . Exemplary the optimal lot sizes of the DC  $Q^{DC}$  and of Retailer R<sub>3</sub>  $Q^{R_3}$  are depicted. A first interesting result here is that the optimal lot size *Q* from simulation optimization shows only a very slight

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Figure 3: Overall costs C and optimal parameter of reorder point s and lot size Q with respect to the service level  $\eta$ .

decrease over the whole range of service levels  $\eta$  for both, DC and R<sub>3</sub>. This implies that in general also a constant optimal lot size Q, which is found in the analytical derivation, might be a good solution. However, the specific value of the lot sizes Q shows that for the DC the optimal lot size  $Q^{DC}$  is twice as big as the lot size  $Q^{DC}$  from the analytical model. This implies that the interrelations between the SC partners especially influence the upstream optimal parameters. Figure 3c and Figure 3d show the results of the optimal reorder point s with respect to the service level n for the DC  $s^{DC}$  and for Retailer R<sub>3</sub>  $s^{R_3}$ respectively. Both of these results indicate that the interrelated parameter optimization leads to rather different results in comparison to the analytical model treating all SC partners independently. An interesting result here is that a shift of inventory, i.e. reorder point s is higher, from the DC to the Retailer can be observed for low service levels  $\eta$ . Furthermore both reorder point s values show a non-linear increase when the service level  $\eta$  is near 100%. The reorder point for the Manufacturer  $s^{M}$  (not presented in Figure 3) is nearly zero according to the simulation-based optimization results, i.e.  $s^M \approx 0$ . This shows that the optimal solution for this SC setting is that the Manufacturer produces on a make-to-order basis, where a production order is issued when the DC states a replenishment order. The results show that in this system setting it is more cost effective to also include the production lead time of the Manufacturer in the replenishment lead time of the DC and in general to keep more stock at the Retailers to buffer against replenishment disturbances. The optimal parameters found with simulation-based optimization have only low variance and are consistent over the whole range of service levels  $\eta$ . This verifies that the simulationbased optimization applying evolutionary algorithms is well applicable for the studied SC optimization.

## 4.3 Extension 1: Real World Effect Truck Load Limit

Because in real SC settings the delivery to the Retailers is often performed in smaller trucks, the first extension discussed is a limit on the possible truck load from DC to the Retailers. The results shown in Figure 4a compare the overall costs *C* reached with the analytically optimized parameters and the simulation-based optimization parameters for the most restrictive case of a truck load limit TL = 75 pcs.



Figure 4: Overall costs C with respect to the service level  $\eta$  for the real world scenario with specific truck load limits.

Comparing these results with Figure 3a shows that with this restriction the cost reduction potential of simulation-based optimization additionally increases significantly. Between  $\eta = 0.75$  and  $\eta = 0.95$  the new cost reduction potential is 23%. Figure 4b compares the truck load limits TL = 75 and TL = 100 pcs with the scenario without a truck load limit. It is obvious that with lower truck load limit of TL = 75 pcs costs are higher, e.g. with service level  $\eta = 0.90$  overall costs *C* are 6% higher than without a truck load limit. With a truck load limit of TL = 100 pcs there is nearly no cost difference to the scenario without truck load limit TL for high service levels  $\eta > 0.95$ . For lower service levels  $\eta$  a slight cost reduction potential can in general be observed. For the further truck load limits that were investigated (not shown in Figure 4), no significant cost changes could be observed because optimal parameters for Retailer lot size  $Q^{R_i}$  were between 110 and 150 and therefore the truck load limit is independent of this parameter. With truck load limit TL = 75 and TL = 100 pcs the optimal Retailer lot size  $Q^{R_i}$  was always near to the truck load limit TL, i.e. Retailers' should order in full truck load quantities.

## 4.4 Extension 2: Real World Effect Mixed Load

The final model extension is the scenario of a truck load limit *TL* in combination with mixed load. If the truck load limit in this scenario is below the optimal Retailer lot size  $Q^{R_i}$  (compare section 4.3) the mixed load extension does not have any effect. In this case Retailers are still ordering in lot size  $Q^{R_i}$  which is equal to the truck load limit *TL*. For truck load limits *TL* that are higher, results show a different behavior. Figure 5 depicts this behavior for a specific truck load limit of *TL* = 200 pcs. In detail Figure 5a shows the overall cost comparison of the simulation-based optimization with truck load limit *TL* = 200 pcs (which is equal to having no truck load limit) and the scenario with mixed load opportunity and truck load limit of *TL* = 200 pcs.

The results indicate that for a broad range of service levels  $\eta < 0.97$  the mixed load opportunity leads to a significant cost reduction potential, i.e. 10% between  $\eta = 0.75$  and  $\eta = 0.95$ . The cost increase for the mixed load opportunity for high service levels  $\eta$  indicates that the evolutionary algorithm has some problems there with the additional degree of freedom in the optimization. Figure 5b shows the results for reorder point  $s^{R_i}$  with respect to the service level  $\eta$  for the three different Retailers. Compared to the other

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Figure 5: Overall costs *C*, optimal parameters for reorder point *s*, lot size *Q* and truck waiting time  $t_w$  with respect to the service level  $\eta$  for real world scenario mixed load with truck load limit TL = 200.

scenarios, i.e. the basic scenario and the scenario with a truck load limit, the reorder point s is higher with the mixed load opportunity. This is due to the fact that the trucks are waiting a specific truck waiting time  $t_w$  before they depart which leads to an increase in replenishment lead time and additional variance in this value. The results for the optimal lot size  $Q^{R_i}$  are shown in Figure 5c with respect to the different Retailers. As in the mixed load scenario different products can be transported with the same truck to one Retailer, the results shown in Figure 5c are very interesting. The solution found with the simulation-based optimization indicates that the optimal lot size  $Q^{R_i}$  (for all products of one Retailer) is always a fraction of the truck load limit 200 pcs. Looking at Retailer R<sub>3</sub>, the optimal lot size  $Q^{R_3}$  is always 1/4 of the truck load limit. This means that a higher order rate is generated for each product and therefore lower inventory costs result. However, not each single order is transported, but up to 4 orders are shipped together which mitigates the higher ordering costs. Note that truck waiting time  $t_w$  limits the time a truck is waiting for further orders to be loaded. For Retailer  $R_2$  which has the medium distance to the DC, the optimal fraction is 1/3 and for Retailer R<sub>1</sub> which has the shortest longest distance to the DC, the optimal fraction is either 1/2 or the full truck load. This indicates a lower distance to the DC decreases the positive effect of the mixed load opportunity. The optimal truck waiting time  $t_w$ , which is also an optimization parameter in this study, is shown in Figure 5d. An interesting finding here is that up to a service level  $\eta < 0.95$  the optimal truck waiting time  $t_w$  is approximately 0.6 days (note that there is one jump of the optimal truck waiting time to 0.9 days for a service level between  $0.95 \le \eta \le 0.97$ ) and for  $\eta > 0.97$  the truck waiting time  $t_w$  significantly decreases. Exactly when the truck waiting time  $t_w$  reduces, the overall costs C of the scenario with mixed load become slightly higher than in the scenario without mixed load. This indicates that the evolutionary algorithm gets stuck in local minimum and also shows the shortfalls of simulationbased optimization to be not able to prove the optimal solution and therefore the limits of the current study. However, in general these results support that simulation-based optimization can be applied to

identify good SC planning parameters and is able to exploit the real world transportation effects which cannot be included in analytical models.

# 5 CONCLUSION

In this paper a multi-stage and multi-product SC is investigated. The supply chain partners involved are a Manufacturer, a Distribution Center and 6 Retailers. Each supply chain member uses an (s,Q) policy for inventory management. For determining optimal parameter settings for lot size Q and reorder point s, two different solution methods are compared. The first method is an analytical optimization model assuming a single-stage, single-product inventory system which is applied independently for all supply chain partners. Optimal parameters are identified for all partners and then re-evaluated in the dynamic and stochastic simulation model. Results show that if analytical optimal parameters are evaluated with simulation, which includes the dynamics and interdependencies between the supply chain members, lower service levels than initially predefined are achieved. The second method is a simulation-based optimization approach, where parameters are optimized with the use of a Nondominated Sorting Genetic Algorithm II (NSGA-II). The numerical results highlight the huge potential of simulation-based optimization for supply chains. Results show that considering the stochastic and dynamic effects within the supply chain lead to an average cost reduction potential of 11% for the service level interval between 0.75 and 0.95. The two real world transportation effects studied, which are truck load limit and mixed load, provide further insights into the optimization of real world multi-stage and multi-product supply chains. The truck load limit is found to constrain the optimal lot size Q only if it is set rather restrictive. Especially the mixed load extension, shows a high potential to reduce the single product lot sizes Q and combines their shipping. This reduces retailer inventory for a certain service level and therefore leads to lower supply chain costs. In further research different demand scenarios, i.e. different demand for single products, as well as the integration of real-time data for the truck waiting time of the truck could be investigated.

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