Proceedings of the 2016 Winter Simulation Conference
T. M. K. Roeder, P. I. Frazier, R. Szechtman, E. Zhou, T. Huschka, and S. E. Chick, eds.

A PRIMER FOR HYBRID MODELING AND SIMULATION

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ABSTRACT

In dealing with complex systems, there is no single "best" possible modeling approach, as each specific system and modeling purpose has subtleties and specific needs. Consequently, in developing models that capture the complexity of real systems, it is useful to combine modeling approaches yielding what is referred to as a hybrid modeling approach. By combining different modeling paradigms, hybrid modeling and simulation provide a more comprehensive and holistic view of the system under investigation and a very powerful approach to understanding complexity. This paper discusses the uses and applications of hybrid modeling, general lessons related to how and when to use such an approach, and relevant tools.

1 INTRODUCTION

The need for, and potential benefits of, combining modeling paradigms to capture the wide range of complexity associated with real systems have been discussed in the literature (Scholl 2001a, b; Martinez-Moyano et al. 2007; Martinez-Moyano et al. 2010; Brailsford et al. 2014). However, the use of multiple modeling approaches to explain complex behavior is not widespread. Although the use of multiple approaches—together or separately—has the potential to yield better understanding and insight into complex problems, it is a more demanding process than using a single-paradigm approach.

Different modeling approaches have different sets of assumptions that simplify the work that the modeler must undertake to complete a modeling project. In a way, the different modeling approaches act as templates for conceptualizing the reality that modelers seek to understand and influence. Using a proven, rigorous template allows the modelers to focus on the aspects of reality that matter most—if and when there is good correspondence between the modeling template and the system under study. Most generally used modeling approaches have evolved to be quite comprehensive in terms of the type of complexity they allow the modeler to address within their set of assumptions. However, because all models are simplifications of reality and all have limitations, no single approach is best for every type of real system under study. Recognizing the limitations of their preferred modeling approaches may be one of the most important tasks that modelers engage in if they truly seek to understand and influence the real systems being modeled.

We can better understand the limitations of different modeling approaches by comparing competing approaches on a pairwise basis. This comparison helps to illuminate the differences and clarify the limitations of both approaches. For example, Brailsford (2014) compares discrete event simulation (DES) and System Dynamics (SD) using seven criteria for selecting a modeling approach: scope, importance of variability, importance of tracking individuals, number of entities, control, relative timescale, and purpose. Brailsford's (2014) comparison shows the possibilities in terms of the types of problems that can be addressed using both approaches. For example, under 'scope,' Brailsford lists DES as being more of an

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operational and tactical approach, and SD as more strategic. However, there may be exceptions to these generalizations that are problem dependent. Also, the relative timescale of both DES and SD might be short (depending on the problem), and the relative importance of variability could be the same in both approaches, as when stochasticity is introduced in SD models.

In addition, Lane (2000), a proponent of the SD approach, presents a descriptive comparison of the same two approaches (SD and DES) that yields a useful framework for modelers. Lane (2000) proposes eight criteria to evaluate both modeling paradigms: perspective, resolution, data sources, problems studied, model elements, human agents, outputs, and how the clients find the model.

Lane (2000) finds the SD approach to be more holistic, in terms of data use, and more transparent, while DES seems to be geared toward understanding details about the system, using mainly numerical data, and is quite opaque for clients. Lane describes DES models as 'dark grey boxes' while SD models are more like 'fuzzy glass boxes.' Through such descriptions, Lane (2000) provides a way for modelers to clarify which modeling approach (DES or SD) might have a greater degree of correspondence with their problem or system of interest. In general, this set of characteristics does not only address correspondence with the complexity of the problem under study, but also compatibility with the stakeholders with whom the modeler is interacting. For example, for some stakeholders, model transparency might be paramount; for others, model opacity can be tolerated—or even desired—as long as model output is useful (for some considerations about model transparency, see Martinez-Moyano 2012). In real, complex systems, the characteristics of the system under study and those of the stakeholders will most likely have various shades of the characteristics expressed by Lane (2000) and Brailsford (2014; Brailsford et al. 2014), making it almost impossible to make the level of complexity of the problem fit perfectly within any one specific approach described in the literature.

Using multiple modeling paradigms should become more widespread, eventually representing standard practice in the modeling and simulation community. The use of multiple modeling paradigms in parallel—tackling the same problem by employing at least two modeling approaches and comparing and contrasting the results obtained with each—is a great start (Morecroft and Robinson 2014). However, the key to generating simulation results not only accurately, but for the correct reasons (i.e., to allow more insightful interventions), is to combine multiple simulation paradigms to tackle complex simulation problems (for an example, see Coyle 1985). The combined use of at least two simulation approaches is known as "hybrid modeling." The two approaches may be any two that, combined, capture a larger spectrum of the complexity of the real system under study. Candidate approaches include SD, DES, agent-based modeling (ABM), ordinary differential equations (ODE), and linear programming (LP)—or, more generally, optimization models that consist of an objective function to optimize and a set of constraints that reflect resource limitations.

The remainder of the paper is organized as follows. First, we describe the use of multiple approaches in parallel using a case study and a generic susceptible-infectious-recovered (SIR) model. Second, we describe the simultaneous use of multiple modeling approaches by discussing the development of two case studies (a model of financial stability and a model of the interaction between energy and conflict). Third, we show two examples of how to incorporate discrete components into SD models and how the introduction of such components changes the resulting behavior. Finally, we discuss general lessons associated with the use of hybrid models.

2 USING MULTIPLE APPROACHES IN PARALLEL

As described before, the need and potential benefits of using more than one modeling paradigm to capture the wide range of complexity of real systems provides the background for developing solutions to this problem (Morecroft and Robinson 2014). We start by using multiple approaches in parallel with the intent to discover insights, derived from different modeling paradigms, into the problems under study. In this section we discuss cases in which we use both the SD and the ABM approaches to learn about the complexity of the real systems we model. For pedagogical purposes, we start with a relatively simple (and well-known) model of disease progression—an SIR model—to then discuss a more complex problem—simulating a proxy war for resources.

2.1 Case Study: SIR Model

SIR models are widely known and understood models of disease transmission. Figure 1 shows a stock-and-flow diagram of an SIR model built using the SD approach. In this diagram, causal loops and the stocks and flows are explicit, which allows modelers to visually represent material flows in the model and to see how such flows are changed through feedback processes. In general, stock-and-flow diagrams focus on depicting the dynamic complexity of the model, not its detail complexity (for a description of the concepts of dynamic and detail complexities, see Senge 1990).

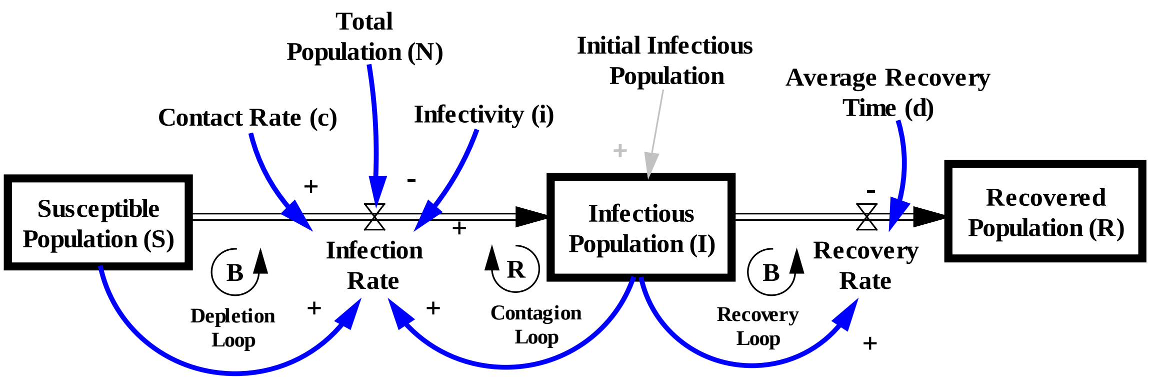


Figure 1: Feedback Structure of the SIR SD Model (from Martinez-Moyano and Macal 2013).

The logic of an AB version of an SIR model, in which each individual in the population is individually represented, is shown in Figure 2.

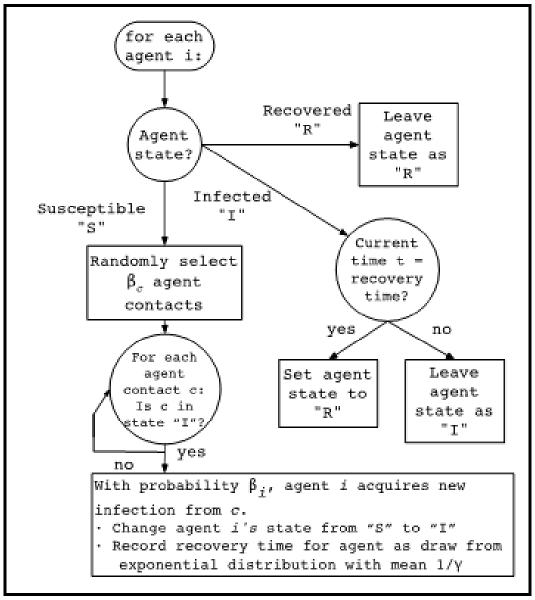
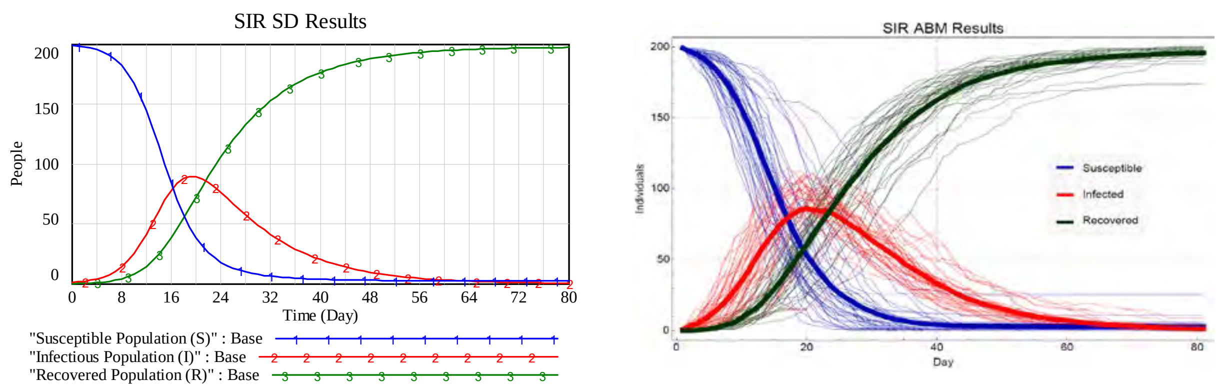


Figure 2: Logic of the ABM version of the SIR Model (from Martinez-Moyano and Macal 2013).

The description is specified in pseudo-code, in which agent types are defined as classes (Macal 2010). In the model, a single agent class covers any individual in the population. The state of an agent at any time is its particular disease state. The agent state is the only information in the model that is endogenously updated, as all other parameters are constants. The number of individuals in the population is defined by the number of agents in each disease state. Based on the agent class definitions, there are N agent instances in various disease states of S, I, and R, where N is the total population size

Figure 3 shows the resulting behavior of the SD SIR model (3a) and the ABM version (in 3b, the thick lines represent the average of the stochastic results). Both results are comparable. In the ABM results, we see results from different runs using varying stochastic input values, while the SD model results shown are a single run without stochasticity reflected in input parameters. The SD model used does not include stochasticity in input parameters. The structure—internal logic—of both models is comparable. In the SD model we can explicitly see the feedback mechanisms that create the endogenous results that are typical of disease transmission (see Figure 1) while in the ABM case, the logic of contagion is presented in more detail (see Figure 2). In both models more detail exists (not shown) in which one can see the details of the parameters and functions used in the model. In general, for modelers without a computer programming background, SD models tend to be more transparent in their representation than their ABM counterparts.



(a) (b)

Figure 3: (a) Resulting Behavior of the SD SIR Model. (b) Resulting Behavior of the ABM SIR Model.

2.2 Case Study: Simulating a Proxy War for Resources

In previous work, we used the SD and agent-based (AB) approaches to explore the same problem in order to more thoroughly understand the complexity of the system (North and Macal 2009; Martinez-Moyano et al. 2010). North and Macal (2009) successfully replicated, using different modeling approaches and implementation environments, the behavior of the famous “Beer Game” SD implementation by Mosekilde et al. (1991). In reflecting about the process, they concluded that “the model reproduction processes were challenging and time-consuming, but ultimately successful and rewarding” (North and Macal 2009, p. 268) as reproducibility is an important pillar of any scientific enterprise.

Martinez-Moyano et al. (2010) also successfully investigated the same problem using two different modeling approaches and found interesting insights from this activity. In their work, Martinez-Moyano et al. (2010) explored the dynamics of a proxy war for resources using a mixed approach to enhance the understanding of the phenomenon. They present a SD and an AB model of a proxy war for resources motivated by the Tajik Civil War, and compare and contrast the two models and approaches. The AB model was developed following the approach described in North and Macal (2007) and was developed with the Repast Symphony AB modeling toolkit (for details of the AB model see Altaweel, Sallach, and Macal 2012). The SD model was developed following the approach described in the SD modeling literature (see, Forrester 1958, 1961; Richardson and Pugh 1981; Sterman 2000; Martinez-Moyano and Richardson 2013) and was developed using the Vensim platform.

In the Tajik Civil War, four prominent factions with two main alliances and three external actors interacted. The alliance between the Islamic and democratic factions in Tajikistan was governed under the UTO (United Tajik Opposition) after 1992. The focus of attention in the Tajik Civil War began in the spring/summer of 1992 and ended in late December 1992. Soon after the collapse of the Soviet Union and elections held in 1991, Tajikistan was shaken with violence. The new government in early 1992 was in fact represented by the old Soviet-era elite. By the spring and summer of 1992, Tajikistan was in a virtual state of civil war between the ex-Soviet ruling elite and an alliance between Islamic and democratic reform groups. In September 1992, a Leninabad-led government was ousted, and by December 1992, a combined Leninabad-Kulyab coalition, that was supported by Russian and Uzbek military personnel, took over the government (for more details on the case, see Martinez-Moyano et al. 2010).

The model represents the repeated game between two autocratic state actors (agents economically supporting belligerent factions)—Actor A and Actor B—and two Factions (agents fighting for control of resources in a geographical area). Each Actor has the option of selecting between four levels of support for the Faction that they support. Each Actor receives a reward for their support when the Faction they support is in control of the resources. The Actors seek to maximize their net profit while the Factions seek to gain and maintain control of the area. The playing field is a generic geographical area where the two Factions fight for resources by using a stock of arms with different efficiencies which, coupled with a level of support from Actors, determine their probability of staying in power (see Table 1).

Table 1: Probability of Staying in Power (from Martinez-Moyano et al. 2010).

<table>
<tr><td></td><td></td><td colspan="4">Actor B's Level of Support</td></tr>
<tr><td></td><td></td><td>None</td><td>Low</td><td>Med</td><td>High</td></tr>
<tr><td rowspan="4">Actor A's Level of Support</td><td>None</td><td>0.5</td><td>0.25</td><td>0.15</td><td>0.00</td></tr>
<tr><td>Low</td><td>0.75</td><td>0.50</td><td>0.25</td><td>0.15</td></tr>
<tr><td>Med</td><td>0.85</td><td>0.75</td><td>0.50</td><td>0.25</td></tr>
<tr><td>High</td><td>1.00</td><td>0.85</td><td>0.75</td><td>0.50</td></tr>
</table>

The SD model of the proxy war for resources is shown in Figure 3. The proxy war for resources model is a four-stock model with four main feedback mechanisms (R1, R2, R3, and R4 in Figure 4).

Of the four feedback processes, two are central to the observed behavior: the Faction A Support-leads-to-success loop (loop R1) and the Faction B Support-leads-to-success loop (loop R2). These two reinforcing feedback mechanisms link the level of arms that the Faction has with the likelihood of success in the proxy war for resources and with continued support from their respective supportive Actors. The Actors' ability to support the Faction is based on the level of their stock of Capital. If the stock of Capital is depleted, the Actor stops the transfer of resources in support for the Faction. In the model, the Actors decide their level of support by identifying the maximum possible expected revenue given the different combinations of their levels of support, and the level of support of the other Actors in the system.

The results (output) of the two models were compared and analyzed (R^2 between 95.8% and 99.6% were achieved in reproducing the behavior of the main stock variables—arms level and capital). The most important insights that came from the process included that the AB model, contrary to the SD model, was easily scalable to include additional actors and factions, and that the SD model allowed for a transparent identification of assumptions and mechanisms driving the results of the model. The feedback structure responsible for the increase or decrease of resources and support was easier to identify and to explain in the SD implementation of the model. The identification of such structure is key to understanding the way in which the interaction between Actors and Factions builds capability that leads to their success (or failure) in the conflict.

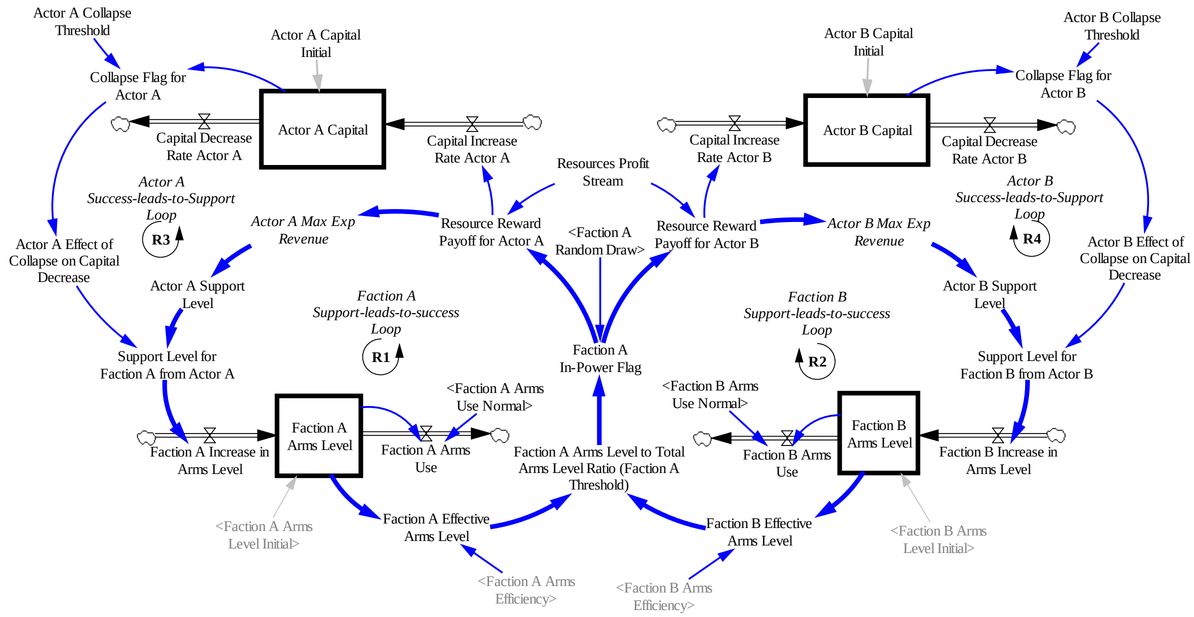


Figure 4: Structure of the System Dynamics Model (from Martinez-Moyano et al. 2010).

3 USING MULTIPLE APPROACHES SIMULTANEOUSLY

As described in the introduction, research has been conducted comparing and contrasting SD and AB models, and some of this research has been geared toward exploring under what circumstances it makes more sense to use each one of them (see Rahmandad and Sterman 2008). Modeling for enhanced understanding of complex systems with policy-oriented implications sometimes requires that several different levels of aggregation be considered and formally included in models. In the SD approach, different levels of aggregation are not usually combined in the same model, and this leaves certain classes of problems outside of the traditional use of this approach. The AB approach, alternatively, provides the ability to capture a greater level of detail of the system under study, but lacks the ability to parsimoniously and clearly link observed behavior to model structure. Many domains, given their natural multilevel complexity, are problem domains in which a combined approach seems to offer advantages (for an example in the Financial Stability domain, see Martinez-Moyano et al. 2007). In this section we present a case in which we integrate the use of the SD and ABM approaches to understand the dynamics of the interplay between energy and conflict.

3.1 Integrated Energy and Conflict Model

In this work, North et al. (2015) describe the integration of an SD model of Energy and an AB model of national stability and conflict. The SD model captures and represents the structure of oil, and oil-related products, of a prototypical country in which its economy is highly dependent on these types of products. The SD approach is helpful to understanding the overall flow of oil and resources (see Figure 5).

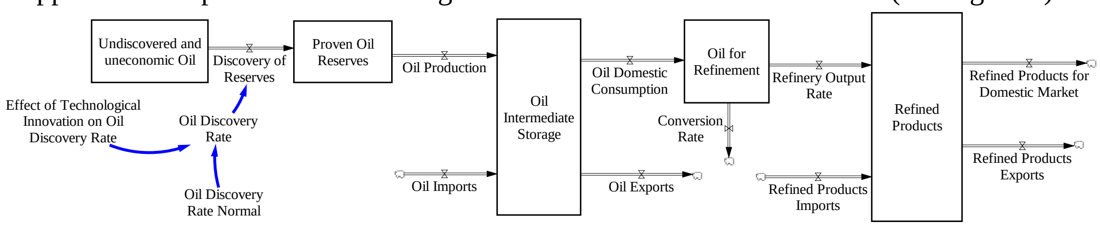


Figure 5: Structure of Flow of Oil and Refined Products.

The flow of oil and oil-related products is a function of decisions and policies established by the government based on the financial need to conduct social programs aimed at keeping the population satisfied and avoiding social unrest. Therefore, in this model, in addition to the technical considerations for oil production, revenue considerations are used (see Figure 6).

The mechanism that determines how much oil is produced is also a function of the availability of the production infrastructure as a consequence of social unrest leading to disruptions. Instability leads to disruptions which lead to decreases in the supply and pressures on the price and revenue. This work is based on literature and current modeling practices (Sterman and Richardson 1985; Martinez-Moyano and Richardson 2013).

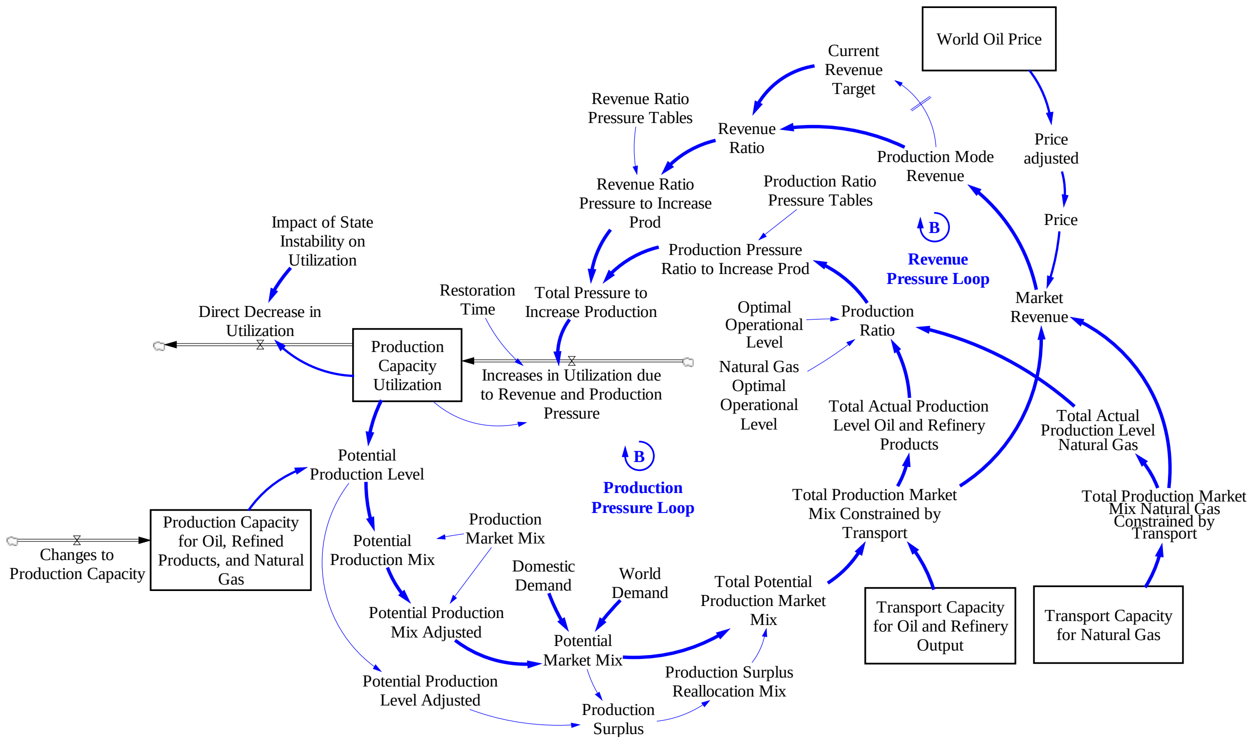


Figure 6: Production Capacity Utilization Feedback Structure (Martinez-Moyano et al. 2012).

The conflict model is based on the earlier exploratory model, named the Virtual Multiscale Strategist (*vmStrat*) developed by Ozik et al. (2012). The actors in this model have four main attributes (or resources): strength, unofficial affect, official affect, and strategy. Strength is the most important attribute and it is a function of the level of resources that are available to the actors. The attributes range continuously from −1.0 to +1.0. Orientations near −1.0 are considered antagonistic, while orientations near +1.0 are favorable. The unofficial affect represents an actor's unexpressed affect—privately—toward another actor. The official affect is the actor's expressed affect—public—which may be different from its unexpressed affect. The value for strategy is a basis for interactions between two (or more) actors. A negative strategic value is likely to give rise to coercive moves while a positive strategic value tends to generate supportive or beneficent action. Values for strategies in the region of 0.0 arise from an instrumental or pragmatic orientation. In addition to the primary attributes, actors have two additional attributes: power and ideational alignment. Power is an extension of the concept of strength. Qualitatively, power is an assessment of the network of support that might be expected from the other actors. Ideational alignment is a measure of the similarity or difference in the way that two actors affect values toward a set of referents. Ideational alignment between two actors is expressed as a value from 0.0 to 1.0, where 0.0 means no alignment and 1.0 means perfect alignment. Importantly, the calculation is based on the perspective of that actor. The outcome of the conflict model is the level of conflict present in the country, which leads to a certain level of disruption to the energy production infrastructure.

The two models—the energy model and the conflict model—are integrated by clearly identifying the variables in both models that need to exchange information as the two models progress in their simulations. The conflict model provides to the energy model values for instability for each appropriate actor and the energy model uses these values in its calculations related to the level of disruption in the country and how this affects the availability of the oil production infrastructure and the revenue produced (see Figure 7). The energy model, in turn, provides revenue values which are used in the conflict model to adjust the actors' strength and power values, which lead to the identification of future instability in the simulation. One advantage to this method of integration is modularity. With this type of integration, the modeler has the ability to consider the two models independently and thus, if needed, to add new elements to the data sent between them or alter the method by which this information is passed without requiring other substantial code revisions.

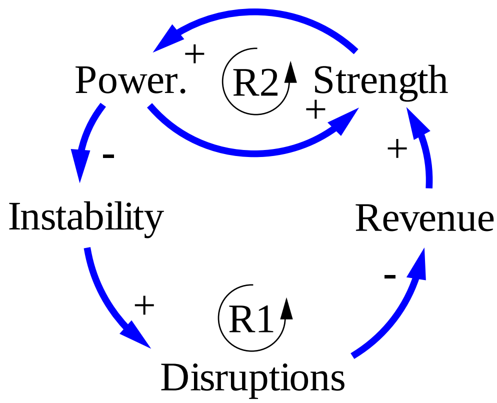


Figure 7: Model Integration.

4 INTRODUCING ELEMENTS OF DES IN SD MODELS: TWO SIMPLE EXAMPLES

4.1 Example 1: Delaying the Outflow of an Accumulation Process

SD models are representations of complexity using continuous processes. Mathematically, the basic structure of SD models is a system of coupled, nonlinear, first-order differential (or integral) equations (Richardson 1996, p. 657) that can be written in the form:

$$\frac{dx}{dt} = \dot{x}(t) = f[x(t), u(t)];\ x(t_0)$$

given

$x(t) = n^{th}$ Order vector of system states (or levels)

$u(t) =$ Vector of exogenous inputs

$x(t_0) =$ Initial value for state vector at $t = t_0$

$f(\_) =$ Nonlinear vector function

$\frac{dx}{dt} = \dot{x}(t) =$ Time derivative of the state vector

For example, consider a simple workforce model in which the accumulated level of staff at any point in time is a function of hiring and attrition. Hiring is a function of the availability of candidates and attrition is a function of the average tenure time that the staff has in the organization (see Figure 8).

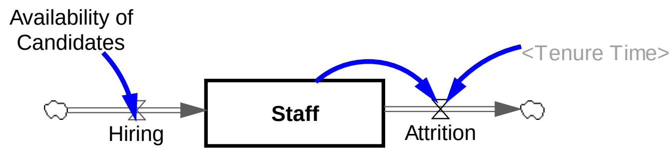


Figure 8: Simple Workforce Model.

In this model, the formulation is as follows:

$$\dot{Staff} = Hiring - Attrition \tag{1}$$

$$Staff_0 = 0 \tag{2}$$

$$Hiring = Availability\_of\_candidates \tag{3}$$

$$Availability\_of\_candidates = \begin{cases} 100 & 6 \le t \le 12 \\ 0 & otherwise \end{cases} \tag{4}$$

$$Attrition = \frac{Staff}{TenureTime} \tag{5}$$

$$TenureTime = 24 \tag{6}$$

Given the continuous formulation of this SD model, the resulting behavior is as follows (see Figure 9).

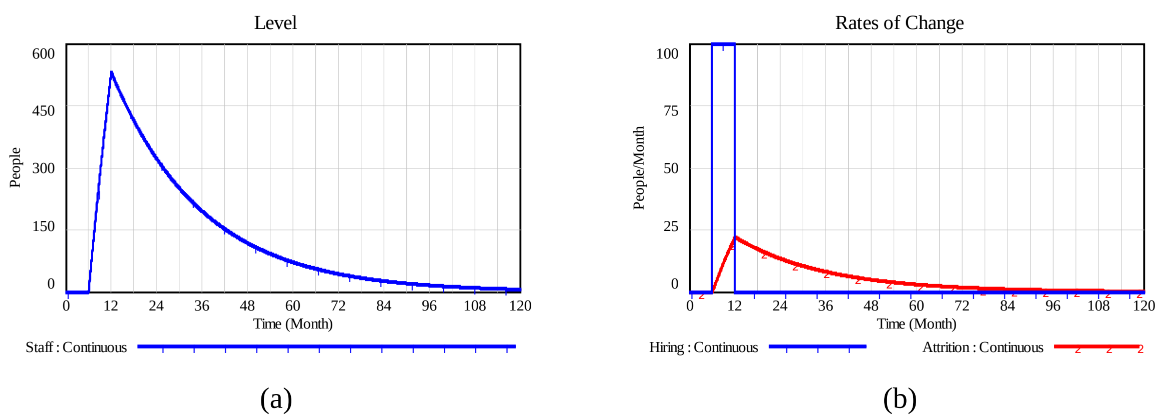


Figure 9: Behavior of Simple Workforce Model.

To incorporate a discrete formulation into the simple SD model, we changed the formulation of the attrition rate to one that is a function of the hiring rate, capturing a process that would represent an outflow that is the same as the inflow, but delayed a certain number of time units (see Figure 10). In the modified formulation, the attrition rate receives information directly from the hiring rate (see bottom arrow in Figure 11). The delay time we use is the same *tenure time* used in the continuous case. This modification represents a process where the people who entered the organization between times 6 and 12 leave it after 24 months following the same distribution.

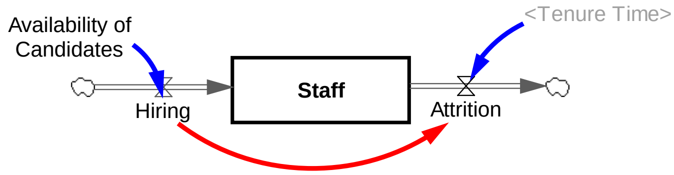
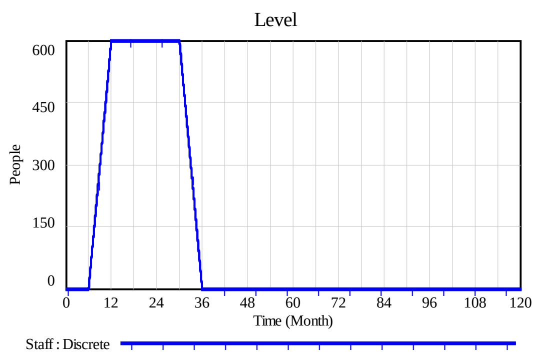


Figure 10: Simple Workforce Model (with modified structure).

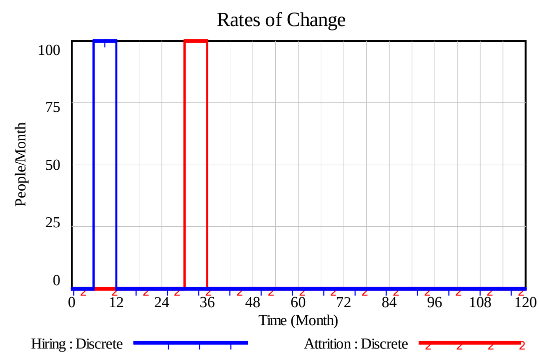
Although the delay formulas within software packages may be relatively complicated (one could potentially consider introducing simpler formulas to represent discrete delays), we chose the "Delay Fixed" function from Vensim because this represents a very intuitive discrete delay process—maintains the input distribution. The formulation for attrition now changes from that in Equation (5) to the one in Equation (7). The last parameter in the *Delay Fixed* function is a parameter that allows the modeler to define an initial condition, in this case, as the inflow changed from 0 to 100, we chose 0.

$$Attrition = DELAY\_FIXED(Hiring, TenureTime, 0) \tag{7}$$

With the new discrete formulation for *attrition*, the resulting behavior is as follows (see Figure 11).



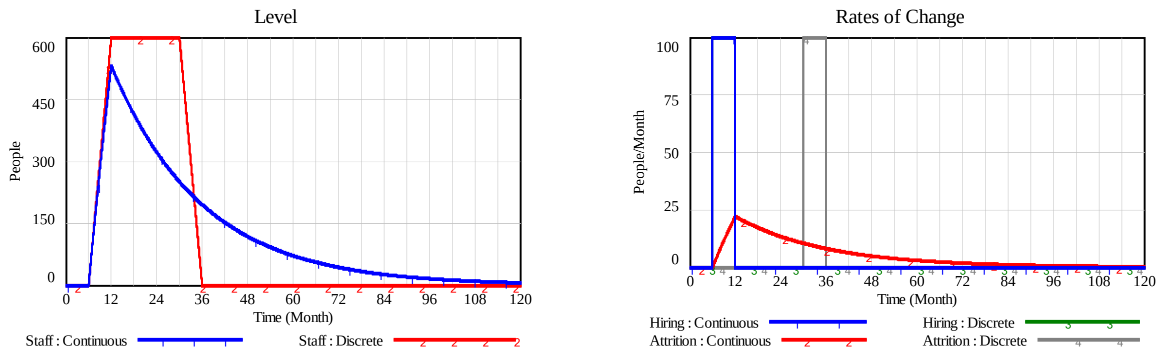
(a)



(b)

Figure 11: Behavior of Simple Workforce Model (with discrete formulation).

The resulting behavior changes in an important manner. For example, the maximum number of people in the organization changes from approximately 530 people (at time 12) to 600 individuals from time 12 to 30 in the simulation (see Figure 12a). The maximum number of individuals in the organization is lower in the continuous case as attrition starts immediately after the hiring surge. Also, the shape of the decrease changes from a smooth exponential decrease which lasts 108 months (9 years) to be 0, to a linear decrease that lasts 6 months to reach the new level. The implications of this change can be substantial in a larger model in which staff size influences other aspects of organizational performance (e.g., overall organization output, payroll, general expenses, etc.). In addition, the most important aspect to consider here is the way in which the real system behaves (i.e., if the mechanism of attrition in the real system is better captured by a continuous or a discrete formulation). With respect to the behavior change of the rates, Figure 12b shows the critical difference in the *attrition* flow, as the *hiring* flow remains the same in both formulations.



(a) (b)

Figure 12: Comparison of Continuous Versus Discrete Behavior of Simple Workforce Model.

4.2 Example 2: Continuous and Discrete Delays in Aging Chains

Aging chains in SD models capture the process by which elements along the chain change in their characteristics over time. Population models are a typical example in which aging chains are used. In population models, however, continuous formulations may capture the characteristics of the real system or may introduce spurious behavior. Consider a simple population model with three age cohorts (see Figure 13).

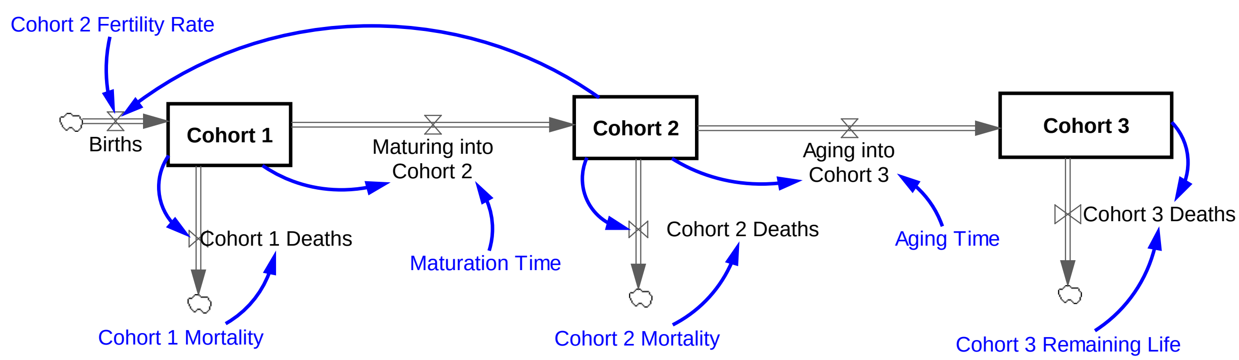


Figure 13: Simple Population Model.

In this model, the rates of change that determine the accumulations for *Cohort 2* and *Cohort 3* are formulated using a continuous approach (see equations 8 and 9). Also, all *death rates* are formulated continuously.

$$MaturingRate = \frac{Cohort1}{MaturationTime} \tag{8}$$

$$AgingRate = \frac{Cohort2}{AgingTime} \tag{9}$$

With this formulation, cohorts change immediately after the beginning of the simulation. If we introduce a discrete formulation for the *maturing* and *aging* rates, recognizing that the cohorts change not all at once but in a conveyor-like process (i.e., some members of the cohort reach maturity first), we add realism to the formulation, as the real system is more likely to behave in such a manner. In the discrete formulation we use a "flat" distribution within the conveyor (i.e., all elements within it are uniformly distributed) and a leakage rate equal to the *death rates*. The resulting behavior is shown in Figure 14.

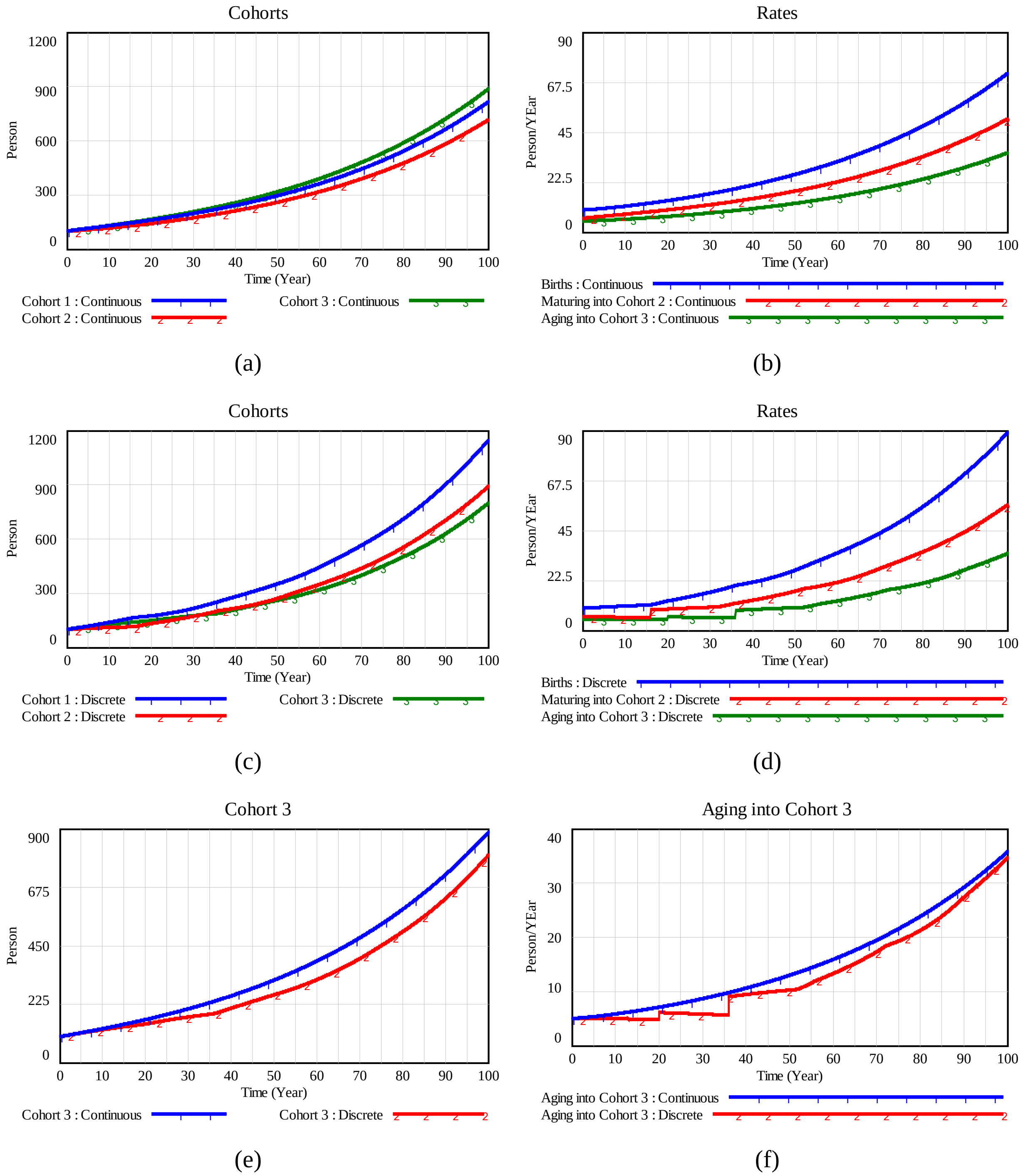


Figure 14: Comparison of Continuous versus Discrete Behavior of Simple Population Model.

In Figures 14a and 14b, we show the behavior obtained using the continuous formulation while in Figures 14c and 14d the discrete formulation results are shown. It is interesting to note that in the continuous formulation results, *Cohort 3* accumulates the highest number of people at the end of the simulated time (*Cohort 1*=814.8, *Cohort 2*=714.9, and *Cohort 3*=886.3), while with the discrete formulation it is *Cohort 1* that comes up on top (*Cohort 1*=1145, *Cohort 2*=890.3, and *Cohort 3*=797.7). Rate values also change at the end of the simulated time. With the continuous and/or discrete formulations the results are (at time 100): *Cohort 1*=71.46/89.03, *Cohort 2*=50.93/56.51, and *Cohort 3*=35.74/34.61. Also, as shown in Figures 14b and 14d, the shape of the behavior curves are clearly different as the discrete results (Figure 14d) are not smooth and show discontinuous changes.

In the case of *Cohort 3*, the resulting behavior is of interest, as the results throughout the simulation are always lower in the discrete case (see Figures 14e and 14f). For *Cohort 2*, comparatively, the discrete formulation yields a higher behavior for the accumulation from time 60 until time 100. Finally, in the case of Cohort 1, the results are consistently higher in the case of the discrete formulation. These changes in behavior are of interest as, similar to the simple workforce model results, behavior results would have an impact in a larger model in which the population model is embedded (e.g., services provided to the different age cohorts, engagement of the age cohorts in economic activity, health care infrastructure implications, etc.)

5 CONCLUSION

We have presented the concept of hybrid modeling and some of its implications. We have presented hybrid modeling in the context of modeling in general and linked it to the ideas of the use of multiple modeling approaches, both in parallel and simultaneously. Although the use of multiple modeling approaches in parallel is not technically recognized as part of the hybrid modeling process, we think it is a good first step into broadening how modelers approach system conceptualization and formulation leading to a hybrid modeling approach. Hybrid modeling—the use of multiple modeling approaches simultaneously—has the potential to make models more realistic and capable of exploring complex systems in a more insightful manner. Hybrid modeling also poses added cognitive demands on modelers and, therefore, makes the modeling enterprise more challenging.

ACKNOWLEDGMENTS

This work is supported by the U.S. Department of Energy under Contract No. DE-AC02-06CH11357.

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