ENRICHING SIMHEURISTICS WITH PETRI NET MODELS: POTENTIAL APPLICATIONS TO LOGISTICS AND SUPPLY CHAIN MANAGEMENT

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ABSTRACT

Some classic and complex problems in Operations Research consist of simplified versions of real logistic and supply chain management applications. One common and successful, but approximated approach for coping with these problems considers the system of interest isolated from its environment. In such a case, the links to the real world may be reduced to a set of parameters associated with probabilistic distributions. Simheuristics is a solving methodology able to provide efficiently near-optimal solutions for these constrained problems. This paper presents a methodology combining Simheuristics with a Petri net model, describing the environment of a logistic system. An extended version of the capacitated vehicle routing problem with stochastic demands is stated adding a Petri net model. Petri nets are widely used for modeling parallelism and concurrency, providing a realistic description of this environment, which may change the behavior of the isolated system and the scope of the decision-making.

1 INTRODUCTION

The efficient management of supply chain systems is the objective of some classic Operations Research problems, aiming at the coordination of suppliers, producers, warehouses, distributors/logistics, retailers, and customers of goods and services. Supply chain systems often present large complexity, which requires an efficient decision making process for guaranteeing the delivery of the right product in the right quantity, at the right time and the right place. In order to help decision makers, decision support systems should provide high quality suggestions on the configuration of the decision variables of the supply chain system for satisfying the customers' demands with an adequate profit. One successful approach for decision making support in a supply chain consists of stating and solving optimization problems, based on formal descriptions of the system of interest, see Latorre, Jiménez, and Pérez (2013) for an example of application in the food industry. This formal description or model of a complex supply chain system can be difficult to attain.

Moreover, some classic transportation problems, such as the vehicle routing problems, are not easily tackled by Petri net modeling and simulation, since the size of the model rises exponentially with the number of nodes in the logistic system. However, there are already efficient methodologies, such as Simheuristics, for obtaining optimal or quasi-optimal solutions for such problems (Juan et al. 2015). Simheuristics can be considered as a general methodology aimed at applying metaheuristics through simulation to solve stochastic combinatorial optimization problems, most of which belong to the NP-hard category. Real-life uncertainty can be considered in a natural way by integrating simulation into a metaheuristic-driven framework. Simheuristics have proven their efficiency in a range of applications, where it has been used so far, such as the inventory routing problem with stock-outs and stochastic demands (Juan et al. 2014), the arc routing problem with stochastic demands (Gonzalez et al. 2012), the capacitated vehicle routing problem with stochastic demands or CVRPSD (Juan et al. 2011), the capacitated vehicle routing problem (Juan et al. 2010), as well as many other combinatorial optimization problems.

Nevertheless, these classic problems may face simplifying assumptions by considering them isolated from the environment, where the real applications are immersed. The influence of the removed environment is modeled by means of parameters, which can be deterministic, or, more realistically, stochastic in nature. This approximation, based on isolating the system of interest, has led to extraordinary results, both for the large amount and high quality of them, which can be found in the literature by the use of benchmarks as standard problems to be solved. Excellent for testing the qualities of solving methodologies, many benchmarks can be considered as simplifications of real cases in the fields of routing, scheduling, or resource allocation, just to give a few examples.

As a consequence, several efficient algorithms have been developed to find optimal or near-optimal solutions for deterministic versions of classic Operations Research problems. Simheuristics are able to cope successfully with many stochastic problems of combinatorial optimization such as production planning and scheduling, vehicle and inventory routing, and others. Simheuristics are the core of the methodology proposed in this paper, or, in other words, this paper aims at widening the application field of Simheuristics by integrating in this methodology Petri net models that describe the environment of the system of interest. However, real logistic systems rarely can be considered as isolated systems. On the contrary, the behavior of customers and suppliers, as well as other characters in the environment of the logistic system can influence in an important way the dynamics of the system of interest, as well as the objectives of a decision making process. This paper aims at widening the applicability of Simheuristics from a constrained CVRPSD to a wider scope, where a broader chain supply is modeled.

In order to complement the description of the logistic system in a real application, the classic approach can be enriched with a model of the environment of the system itself. Depending on the objectives of interest for the problem, the Petri net model, which complements the application of Simheuristics to the CVRPSD, might encompass more or less elements of the supply chain system. Conceptually, it is possible to develop a Petri net model of a complete supply chain system, with the exception of the logistic stages, which can be formalized as additional constraints of the optimization problem. This optimization problem can be stated for specifying the decision variables of the supply chain and solved by means of Simheuristics.

Based on the existing related work, the present paper proposes the combination of Simheuristics and a Petri net model, aimed at representing realistically a supply chain system or a part of it, with the purpose of developing efficient decision-making support. Section 2 contains a review of related work. Section 3 details the integration of a Petri net model in the methodology of Simheuristics, choosing a particular Petri net formalism, discussing the CVRPSD, and providing the detailed algorithm. Section 4 shows a case study, consisting of two feasible Petri net models of the nodes of the routing problem. Section 5 focuses on the conclusions and the description of the future research lines. The paper finishes with the bibliographical references cited in the text.

2   RELATED WORK

2.1   Decision making support based on Petri net models

The paradigm of Petri nets has been considered in this paper for obtaining a formal description of the environment of a logistic system, seen as a discrete event system. The main reason for this choice can be found in the suitability of this formalism for modeling behaviors with parallelism, concurrency, synchronization, and resource sharing, where subsystems compete for limited resources. See Silva (1993), and David and Alla (2005) for a theoretical introduction, as well as Latorre et al. (2015) for an application example. Silva and Teruel (1998) state that Petri net is a paradigm able for describing a discrete event system in all the stages of its life cycle, including its design process and its operation, among others. Also, an overview of the application of Petri nets for modeling supply chain systems can be found in Zhang, Wu, and Lu (2010). A compromise between level of detail and size and complexity of the model might be critical for the model itself to be useful in practice. Two main approaches have been followed by researchers to achieve the construction of a useful model of a supply chain system:

  a) Developing a low detailed model of a complete supply chain system. In this case, strategic and tactical decisions can be handled, but usually it is not possible to make operational decisions in topics that are related to a particular stage in the supply chain.
  b) Constructing a high detailed model of a subsystem belonging to a supply chain system. Depending on the purpose of the model, it can be developed to describe more accurately certain features of the subsystem. According to this idea, the model might focus on scheduling, inventory, transportation, performance evaluation or structural analysis. This kind of model allows making short-term operational decisions on detailed parts of the supply chain. However, hypothesis on the behavior of the environment of the system of interest should be performed, since the model is developed in isolation from the rest of the supply chain system.

Some examples of the first global approach are given in the following. Thus, Wang et al. (2013) present a model of a holistic supply chain network system, that is to say, a set of supply chains having inter-dependence relations. Hierarchical Petri nets are considered for developing a primary model with the global non-detailed structure of the supply chain system and secondary models detailing the behavior of each firm member of the global system. The model is simulated but not used for decision making support. Another model of a supply chain system is developed by Sarkar, Cortesi, and Chaki (2013) for analyzing the bullwhip effect in the flow of resources of the system by using the formalism of the generalized stochastic Petri nets. The model presents the minimal level of detail for performing the mentioned analysis, therefore, not detailing any logistic system.

A more detailed model is proposed by Dotoli et al. (2009), describing information, financial and material flows of supply chain systems. The formalism of the first order hybrid Petri nets is used for constructing a model for decision making support. The model includes, but not in great detail, five transportation stages of suppliers, manufacturers, distributor, retailers, and customers. The dynamics of the system is analyzed by simulation taking into account the manufacturer production rates and the average firing delays of discrete stochastic transitions. Halfway between modeling of a complete supply chain and a detailed model of a specific subsystem in the global system, Zimmermann, et al. (2007) develops a model of parts of General Motors' North American vehicle supply chain. The concept of model hierarchy is applied for developing a model of a complex system. A general model, made with a stochastic colored Petri net, contains detailed models of subsystems in substitution transitions. A discrete event system is used for the quantitative evaluation of the model and the improvement of the order-to-delivery time, but not for solving a routing problem.

2.2   Solving routing problems using Petri net models

In this paper, the CVRPSD is addressed as a particular problem of a more global supply chain. Specific transportation subsystems inside a supply chain are usually addressed by means of detailed models, developed by an appropriate formalism.

Along most of their more than 50 years of existence, Petri nets have been applied extensively to the automation of manufacturing systems (Silva 2013). As a consequence, much research effort has been focused on the routing problems of automatic guided vehicles (AGV) in manufacturing systems. One important challenge of these problems is the avoidance of deadlocks that might arise as a consequence of the existence of limited resources, such as the routes shared by the different AGVs. However, although complex, the routing problem that can be stated on the AGVs of a manufacturing system, usually corresponds to Petri net models that do not contain many nodes (places and transitions), situation that is not common in vehicle routing problems, characterized by a large number of nodes (customers, for instance). A review of routing methodologies for AGVs can be found in Fazlollahtabar and Saidi-Mehrabad (2013), among which a number of them are implemented by Petri net models of the logistic systems.

The problem of simultaneous dispatching and conflict-free routing for bidirectional AGV systems in dynamic environments is addressed by Nishi and Tanaka (2012). The solving methodology implies a resolution of a static problem each time a transportation request arrives, and it is carried out by decomposing the net into subnets representing the AGVs. This decomposition requires the compliance of certain properties; otherwise the problem becomes intractable as its size grows. The solution of the static problem is a sequence of transition firing in the transition-timed Petri net model of the logistic system. In addition, Nakamura et al. (2015) state a routing problem in an unidirectional overhead hoist transport vehicle systems for conveying semiconductor wafers. These authors focus on finding suitable paths and minimizing the total transportation time according to the expected tasks. One of the proposed solving methodologies is based on a Petri net model describing the small number of routes of the system.

Research on routing problems based on Petri net modeling in other sectors than the AGVs in manufacturing systems are less common. However, it is possible to find interesting documents, such as the early work by Richard et al. (1996), who state and solve a the traveling salesman problem by means of a sequential timed Petri net model. The obtained model grows exponentially with the number of nodes (cities) included in the system. The methodology is applied to a scheduling problem in process management in the glass industry. This research line has not prospered, probably due to the existence of other methodologies, more efficient, for solving this same problem.

Aized and Srai (2014) address the last mile distribution system in business-to-customer supply chain aiming at solving a routing problem by means of a Petri net model of the system. A three layered hierarchical colored Petri net model represents the system of interest, which should be specific to a particular geographical location. However, as the authors mention, the presented model is conceptual in nature and has not been applied to a particular application case.

2.3   Additional previous work relevant in the development of the proposed methodology

Some important ingredients, which are necessary for developing the proposed mixed methodology, where Simheuristics are enriched with a Petri net model of the environment of the logistic system, have already been presented by different authors: stochastic Petri nets, simulation, optimization based on simulation, and the construction of hybrid models mixing Petri nets with other mathematical concepts.

The generalized stochastic Petri nets (GSPN) with hierarchical transitions, probabilistic arcs, and fusion places are applied to the development of a methodology for modeling a construction project. This application, proposed by Sawhney (1997), is aimed at carrying out the scheduling of the project. A less constrained formalism, regarding the feasible probability distributions that can be associated to the stochastic parameters, is the non-Markovian stochastic Petri net. This formalism has been considered by

Dersin and Valenzuela (2012) to develop a model of around 2200 places, 2200 transitions and 100 tokens for describing the activity of a rail system supplier. Simulation, based on Monte Carlo experiments, is performed for finding the number of workers needed to maintain a given system. Furthermore, a tutorial on the use of stochastic Petri nets in stochastic simulation can be found in Volovoi (2015).

Simulation of colored Petri nets for assessing boarding interactions in aircrafts is addressed by Mujica and Flores (2015), while Bodenstein and Zimmermann (2015) applies the formalism of the stochastic colored Petri nets to the development of a heuristic-based optimization process. A formalism for decision making support, by simulation-based optimization, in the design and operation of discrete event systems, such as logistic systems and supply chains, is described in Latorre et al. (2014). As a conclusion of this section, it can be said that the approach provided by this paper, consisting in the combination of Simheuristics and Petri nets, cannot be found in the literature reviewed.

3 INTEGRATION OF A PETRI NET MODEL IN A SIMHEURISTIC ALGORITHM

3.1 The chosen Petri net formalism

Simulating the evolution of a Petri net implies defining the initial conditions of the Petri net (specially the initial marking) and allowing the state of the net to evolve until a certain stop criterion is verified. The introduction of timing in an autonomous Petri net allows the Petri net not only to model the logical behavior of the system but also to apply it to performance analysis, scheduling, or real-time control problems. As pointed out by Silva (1993), time can be associated to the places (p-timed Petri nets) or to the transitions (t-timed Petri nets). The latter association will be considered in the models of this paper, since transitions represent activities that change the state of the system and time would represent the duration of these activities.

A stochastic Petri net is a timed Petri net, where the firing delays present stochastic values. Generalized stochastic Petri nets may contain immediate transitions, which do not have any associated firing rate; thus, it will fire as soon as it is enabled (David and Alla 2005). In this paper, where the probabilistic behavior of the model is relevant and justifies the application of the methodology of Simheuristics, the formalism, belonging to the Petri net paradigm, which will be chosen, is not the stochastic or generalized stochastic Petri nets, but non-Markovian generalized stochastic Petri nets. This decision has been made since the restrictive nature of the exponential distribution of probability associated to every undeterministic process of the system or its environment, would reduce significantly the application field and the realistic approach to reality that is aimed with the proposed methodology.

3.2 The Capacitated Vehicle Routing Problem with Stochastic Demands (CVRPSD)

The last sections addressed the use of the Petri nets paradigm for modeling a discrete event system, the simulation of a Petri net for assessing the quality of a given configuration of a complex model according to the goals of the decision-makers, and the eventual statement of an optimization problem for decision making. These ingredients should be complemented with a solving methodology, able to provide in an automatic way with a set of feasible solutions that may satisfy the decision-maker.

Simheuristics have already been applied successfully to the CVRPSD (Juan et al. 2011). In order to achieve this objective, an instance of a given VRPSD is transformed into a small set of capacitated vehicle routing problem (CVRP) instances by assigning different values to the level of safety stocks that routed vehicles must employ to deal with unexpected demands. Monte Carlo simulation (MCS) obtains estimates of the probability that no vehicle runs out of load before completing its delivering route and the expected costs associated with corrective routing actions (recourse actions) after a vehicle runs out of load before completing its route. Total costs of different routing alternatives are obtained, providing a quality parameter that characterizes every feasible solution to the CVRPSD, easing, in this way, the task of a decision-maker.

In order to improve the model of the logistic system given by the statement of the CVRPSD, a Petri net model of its environment can be integrated in the methodology. The Petri net model will transform the stochastic demands in deterministic purchase orders. However, the Petri net model itself may contain stochastic parameters, describing processes not modeled explicitly in the Petri net. For example, the customers may be manufacturing facilities, distributors, or retailers, who should, in turn, satisfy a stochastic demand. However, this partial model of the environment of the Petri net, may model features not included in the conventional CVRPSD, such as waiting time of the vehicles due to non-availability of unloading resources, such as a forklift or staff. A more detailed description of the integration of a Petri net model in Simheuristics will be given in the following section.

3.3 Algorithm of Simheuristic for solving the CVRPSD with a Petri net model

The application of Simheuristics for solving an instance of the CVRPSD with a Petri net model of the environment can be performed as stated in the following, where the original algorithm (Juan et al. 2011) has been complemented with the steps that are necessary for integrating the Petri net model:

1. Consider VRPSD instance with a set of n customers and a Petri net model of the environment of the logistic system with stochastic parameters $q_i \geq 0$ (i is a natural number), where each q_i follows a known statistical distribution. Let the vehicle maximum capacity be VMC.
2. Set a value for k ($0 < k \leq 1$), the percentage of the maximum vehicle capacity that will be used during the routing design stage, and calculate VMC* = $k \cdot$ VMC.
3. Define the initial state (marking) of the Petri net model and simulate its evolution.
The initial state can be given by (a) just by assigning random values to the stocks and the expected needs of the customers or (b) performing an initial simulation of the Petri net model, from a deterministic or stochastic initial state, until the point, where a purchase order is created. The initial state of this simulation should be constructed from realistic data that describes the real systems modeled by the Petri net. The result of this initial simulation is obtaining deterministic values for the customer demands d^*_i ($1 \leq k \leq n$).
4. Consider the CVRP(k) defined by a total vehicle capacity of VMC* and by the deterministic demands d^*_j.
5. Solve the CVRP(k) by using any efficient CVRP methodology.
The solution of this CVRP is a feasible VRPSD solution as long as there will be no route failure, i.e., as long as the extra demand that might be originated during execution time in each route does not exceed the vehicle reserve capacity (safety stock) VRC* = $(1 - k) \cdot$ VMC. Notice also that the cost given by this solution, CCVRP(k), can be considered as a base or fixed cost of the VRPSD solution, i.e., the cost of the VRPSD in case that no route failures occur. Chances are that some route failures occur during the execution phase – these chances increase as the value of k gets closer to 1. If so, corrective actions – such as returning to the depot for a reload before resuming distribution – and their corresponding variable costs, CRF(k), will need to be considered. Therefore, for a given value of k, the total costs of the corresponding VRPSD solution will be the sum of the CVRP fixed costs and the variable costs due to the corrective actions, i.e., CVRPSD(k) = CCVRP(k) + CRF(k). Notice that, on average, low values of k (close to 0) will be associated with relatively high fixed costs (more routes will be needed to satisfy total demand) and relatively low variable costs (route failure is less likely to occur). On the contrary, high values of k (close to 1) will have the opposite effect.
6. Using the aprioristic solution with m routes, estimate the expected (average) costs due to possible failures in the jth route, $E[C^j_{RF}(k)]$, for every $j = 1, 2, \ldots, m$.
This can be done by using Monte Carlo simulation, i.e., random values for the stochastic parameters of the Petri net model are generated and the evolution of the Petri net model is simulated from its initial state (defined in step 3), when the purchase is ordered by the customer,

until the vehicle arrives. Whenever a route failure occurs (or just before it happens), a corrective policy is applied and its associated costs are registered (in the experimental section of this paper, every time a route fails we consider the costs of a round-trip from the current customer to the depot; but, since we are using simulation, other alternative policies and costs could also be considered in a natural way). After iterating this process for a number of times, a random sample of observations regarding these variable costs are obtained and an estimate for its expected value can be calculated. Then, the expected total costs due to possible route failures in the aprioristic solution are given by: $E[C_{RF}(k)] = E[C^1_{RF}(k)] + E[C^2_{RF}(k)] + \ldots + E[C^m_{RF}(k)]$.

7. <u>Using the aprioristic solution with *m* routes, obtain an estimate for the reliability of each route, *Rj* $(1 \leq j \leq m)$.</u>
    In this context, *Rj* is defined as the probability that the *j*th route will not suffer any failure during the distribution phase, i.e., that the *j*th vehicle will not run out of load before attending to all customer demands on its route. This reliability value can be estimated by direct Monte Carlo simulation using the statistical distributions that model the stochastic parameters of the Petri net model and performing an initial simulation of the Petri net model for every estimation or trial, as indicated in step 3, in order to get values for the deterministic values of the demand d^*_i $(1 \leq k \leq n)$. Observe that in each route over-estimated demands could sometimes be compensated by under-estimated demands. To this end, a number of trials can be randomly generated. Each of these trials will provide a random value for the total demand in a given route. Then, the relative frequency of trials in which that total demand has not exceeded VMC can be used as an estimate of the route's reliability. Notice that *Fj* = 1 - *Rj* represents the probability that the *j*th route will fail during the distribution phase.

8. <u>Obtain an estimate for the reliability index associated with the aprioristic solution.</u>
    Under the assumption that customer demands are independent –reasonable hypothesis– this can be attained by simply multiplying the reliabilities of each route. A solution reliability level can be considered as a measure of the feasibility of that solution in the VRPSD context.

9. Depending on the time span of the analysis, it can be considered the possibility of <u>iterating the process</u>, returning to step 3, where the initial conditions are the final marking of the Petri net in the previous operation.

10. Finally, <u>provide a sorted list with the best VRPSD solutions found so far</u> as well as their corresponding properties (fixed costs, expected variable costs, expected total costs and reliability index), as well as any decision variable or measure corresponding to the Petri net model.

As explained in the previous algorithm, the Petri net model is included in three of the steps of the application of Simheuristics. First of all, the Petri net model evolution is simulated (step 3) for the initial estimation of the customer demands d^*_i deterministic values. On the other hand, MCS is applied in two steps (6 and 7) to estimate values for the estimation of expected (average) costs due to possible failures in the *j*th route and the reliability of a feasible solution, respectively. The application of MCS consists of generating random values for the stochastic parameters of the Petri net and simulating its evolution.

4   CASE STUDY

The Petri net model associated to the statement of the CVRPSD can describe a complete or a partial supply chain system, where the logistic system is immersed. In the present case study, two simple non-Markovian Petri net models are presented. They describe the behavior of the customers in the VRP, as it can be seen in Figure 1. On the left side of Figure 1, a model of a manufacturing facility is presented. On the right side, a Petri net of a retailer, such as supermarket, is shown. The differentiation of the diverse resources may be performed by the definition of attributes (colors) associated to the tokens. This feature of the marking allows to assign a different type and amount of resources to each modelled task. An

alternative to this approach consists to unfold the Petri net. In an unfolded Petri net, places associated to different types of resources, such as forklifts or staff, can be represented independently.

The Petri net model of the system may contain decision variables. Non-controllable parameters may also be included and can be modeled by stochastic variables. These decision variables, which are additional to the already existing ones increase the freedom degrees of the problem. In addition, the goals of the classic decision making (minimize the delivery time, minimize the number of vehicles, minimize the probability of route failure) can be increased with the ones provided by the Petri net (maximize the benefit, maximize the utilization rate of the manufacturing resources, operators and machines.
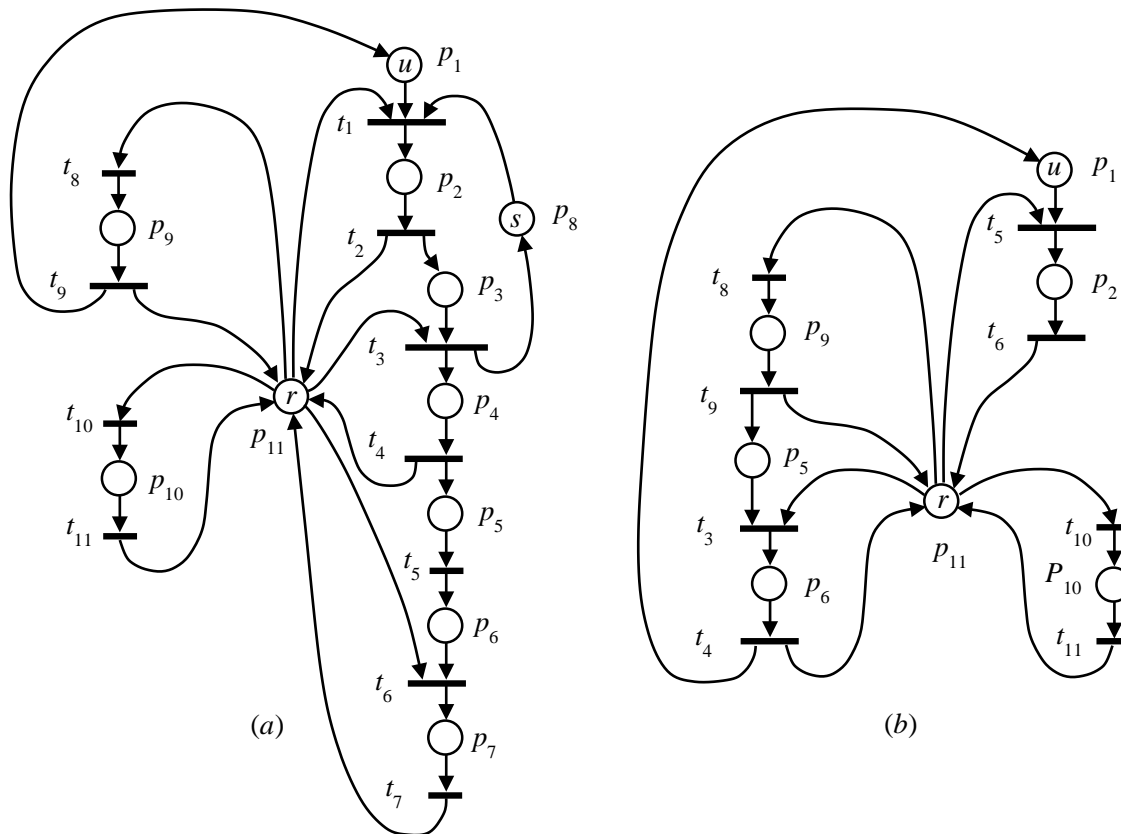
Figure 1: Petri net models of customers: (a) manufacturing facility and (b) retailer.

The meaning of the nodes of the Petri net *a* (*b*) of Figure 1, for a manufacturing facility (retailer) is:

- p_1. Stock of products (idem).
- p_2. Manufacturing process (cashier).
- p_3. Buffer of finished products.
- p_4. Storage process in the warehouse.
- p_5. The finished products are stored in the warehouse, waiting to be sold (the unloaded products are stored in the warehouse, waiting to be placed on the shelves of the retailer).
- p_6. The finished products have already been sold, but they are waiting for delivery (the shelves of the retailer are replenished with the products stored in the warehouse).
- p_7. The sold finished products are delivered.
- p_8. Size of the output buffer of the manufacturing line.

- p_9. Unloading of the vehicle transporting the products from the depot to the customer, belonging to the CVRPSD or from other system, which can be modeled or not in this problem (idem).
- p_{10}. Other non-productive activities, such as preventive maintenance, conveying of materials, cleaning, etc. (idem).
- p_{11}. Resources, such as staff, forklifts, etc. (idem).
- t_1. A raw material is introduced in the manufacturing line.
- t_2. A finished product is placed in the manufacturing buffer.
- t_3. A finished product is taken from the manufacturing buffer for storage (a delivered product is taken to the warehouse).
- t_4. A final product is stored in the warehouse (a delivered product is stored in the warehouse).
- t_5. A final product is sold (products are selected by a customer). The firing rate of this transition may be associated to a stochastic distribution. As a consequence, this parameter can receive random values in the MCS experiments described in the algorithm.
- t_6. A final product is taken from the warehouse (products sold are taken away by the customer).
- t_7. A final product is delivered.
- t_8. A vehicle arrives with raw materials. This transition synchronizes the Petri net model with Simheuristics, since a feasible routing solution of Simheuristics provides the delivery time required for a vehicle to arrive to this customer, enabling this transition. If there are available resources in p_{11}, then the vehicle can be immediately unloaded. Otherwise, the vehicle should wait, perhaps increasing the makespan of the complete routing solution of the CVRPSD.
- t_9. The unloading process in the present customer has finished and the vehicle can continue its route. This transition also presents synchronization with Simheuristics.
- t_{10}. A resource is occupied by a non-productive task.
- t_{11}. A resource occupied by a non-productive task is freed, being available for another task.

Notice that the initial marking is given by:

- s. Size of the manufacturing buffer.
- r. Number of resources.
- u. Number of raw materials at the beginning of the simulation.

5   CONCLUSIONS

A development of Simheuristics, successful methodology to solve the CVRPSD, has been implemented by integrating a Petri net model of the environment of the logistic system. The scope of this model can be as large as a complete supply chain system or as small as a simple model of the customers in the logistic system. The objectives of the problem would determine or, at least, influence the size of the model. The inclusion of a Petri net model in the statement of the CVRPSD may imply the following advantages:

a) A higher level of detail in the model of the logistic system and its environment. As a consequence it might be expected that the Petri net model would represent the behavior of the system in the problem statement with higher accuracy.
b) Possibility to give solution to a wider range of decision variables, belonging to the environment of the logistic system, such as the management of the depot or the customers' facilities.
c) Possibility to include in the objective function of the optimization problem, not only performance parameters from the CVRPSD but also from the components of the supply chain described in the Petri net model. For example the yield or performance of the customers.

d) Depending on the particular problem, it might happen that finding the statistical distributions of the stochastic parameters of the Petri net model is easier or more precise than the distribution for the stochastic demands in the classic CVRPSD.

On the contrary, the requirement of simulating the Petri net model repeatedly along the application of Simheuristics may slow down the process to obtain a solution. For this reason, it might be interesting to find a compromise between the size of the associated model and the expected speed in obtaining good solutions to the routing problem. Furthermore, an example of application has been presented in this paper, where the classic statement of a CVRPSD has been enriched with a Petri net model of the environment of the logistic system. This approach of mixing Simheuristics and Petri nets, does not constrain only to solve the CVRPSD. In fact, the characteristics of the model and the implementation in Simheuristics can be modified from this example in order to better adapt the statement and solving process of the detailed CVRPSD to other application cases.

Of course, one of the elements of the problem that can be changed is the Petri net model of the environment. In fact, in a different application case the environment might be different or the level of detail of the required model can be other than the one considered in this example. Other elements of the problem that can be modified are the parameters of the simulation. A particularly challenging case is described in the following. It is possible to simulate the behavior of the Petri net model, associated to every feasible solution of the CVRPSD, for a time span longer than the active period of the vehicles in a conventional statement of the routing problem. It is even possible to perform a series of sequential instances of CVRPSD, while the dynamics of the Petri net continues active, with the purpose of evaluating the behavior of the system in a significant period of time for tactic management.

Future developments of this technique might explore different approaches for integrating Petri net models in logistic systems to state combinatorial optimization problems with stochastic parameters to be solved by Simheuristics, for instance:

a) Model with a Petri net the logistic system itself for example for avoiding collisions/conflicts/ deadlocks or for considering failures in the vehicle, traffic jams, etc. (things that can be modeled alternatively by stochastic parameters). This approach might present as practical limit the number of alternative feasible routes: the model might be too large to lead to an efficient methodology.
b) Complement the logistic system with its environment, as in the present case-study, but widening the scope of the Petri net model, by including more elements of the supply chain.
c) Develop stochastic models of Petri net in no matter which field and apply Simheuristics as a solving methodology for optimization problems with stochastic parameters.

ACKNOWLEDGMENTS

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