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AGENT-BASED MODELING AND STRATEGIC GROUP FORMATION: A REFUGEE CASE STUDY

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ABSTRACT

Refugee flight presents a logistics problem for humanitarian aid workers anticipating ebbs and flows of arrivals. These migrations include travel over long distances with little advanced coordination and damaged social networks. The model presented here is based on these two fundamental premises: long distance and strategic en-route coordination. Assuming that over large distances, refugees attempt to construct groups that provide assistance or security as they navigate toward safety, the agent-based model incorporates cooperative game theory to investigate the impact of group formations on egress times. The modeled refugees make decisions based on individual utility functions informed by two factors, speed of the group and group size. Since groups accommodate slower members, they may reform as refugees choose their best available strategies to reach safety. The results indicate a tipping point in average group size as the slowest group members have more of impact in the utility function of the agents.

1 INTRODUCTION

Particularly in its initial stages, flight of refugees results from an instance of social and political chaos. Traditional social groups, including families and neighbors, are often separated and social networks are destroyed through tragedy and violence (Martin 2004). In the following hours, days, weeks, and even months, refugees traverse long distances—most often on foot—to reach humanitarian reception sites promising aid and safety. While this is far from the end of the refugees' journey (Frydenlund 2015), this paper focuses on the dynamics of flight. At refugee reception sites along country borders, humanitarian aid workers must predict the ebb and flow of refugee arrivals. The research here focuses on the dynamics of migration to better understand the arrivals of refugees to reception sites. In particular, the research relies on two premises: 1) refugees traveling long distances by foot and 2) disrupted or destroyed social networks that motivate fleeing refugees to dynamically establish en-route group formations that increase the likelihood that they will safely and effectively reach their destinations. Given the context of forced migration and disrupted social networks, the refugees must assess individual internal utility functions to determine benefits of joining or leaving groups during their journey. Aspects of group formation and reformation parallel ideas from cooperative game theory, which we employ here to evaluate the decision-making process of refugees and analyze the time required for egress under varying assumptions.

The heterogeneity of the decision-making process happening between autonomous individuals in a bounded-rational environment, such as that of a refugee flow, lends itself well to agent-based modeling (ABM) as a tool for investigation (Epstein 2006). The research presented here examines macroscopic-level crowd flow and egress time based on microscopic-level behaviors and decisions, for which ABM is a powerful tool. Similarly, ABM has been used to investigate property markets (Schelling 1971; Gangel et al. 2013) and voting behavior (Wilensky and Rand 2015), each exploring how individual behaviors and interactions produce macro-level effects on wider systems. In the field of crowd flow and pedestrian behavior, recent ABM work has emphasized more exploration of group behavior and how agents acting collectively affect overall crowd egress (Collins et al. 2014; Elzie et al. 2014; Frydenlund et al. 2015).

Social network analysis (SNA) provides an additional means to study group formations. In these methods, however, groups are often simplified, lack a dynamic, spatial component, and focus too much on homophily or popularity to evaluate egress over distances (Wang and Collins 2014). Cooperative game theory, on the other hand, provides a mechanism to explore individual actions and strategies. This approach tends to be limited to a small number of agents due to the computational cost of finding any solution, namely the Core (Gillies 1959), Nucleolus (Schmeidler 1969), or Shapley Value (Shapley 1953).

Adapting cooperative game theory into an ABM context overcomes some of these computational limitations and increases the robustness of the model to account for both spatial, dynamic benefits of ABM while incorporating strategic group formation. Collins and Frydenlund (2016) provide one example of combining cooperative game theory and agent-based modeling in a static spatial environment. This paper applies that approach to the dynamic spatial environment of refugee movement. Other have used cooperative game theory in a multi-agent setting (Shehory and Kraus 1998; Bateni et al. 2010), though these applications could not be considered agent-based simulation. Thus, the majority of work that combines agent-based simulation with game theory considers traditional normal-form game theory and not cooperative game theory. However, there has been an increased recent interest in the subject (Rahwan et al. 2015).

The research presented here marries the concepts of ABM with cooperative game theory to examine the generalized scenario of refugee flight to a hypothetical point of safety. The environment and social interactions are simplified to isolate the effects of dynamics imposed by group formation, disintegration, and reformation over a long distance. The next section first discusses the foundation of the model, namely forced migration, pedestrian group formation, and cooperative game theory. The sections that follow describe the ABM, results, and discussion.

2 CROWD MOVEMENT AND REFUGEE APPLICATIONS

The ability to predict forced migration flows presents a serious problem for humanitarian actors attempting to provide aid and safety for refugees. An example of this is the failed predictions of migrations during the response to the Kosovo refugee crises in the 1990s (Suhrke et al. 2000; United States Senate 2000). There is currently no reliable way to computationally predict refugee flows. The research presented here attempts to merge work done in pedestrian evacuation with game theory approaches to explain strategic grouping that occurs as fleeing individuals attempt to compensate for broken traditional social bonds as they move toward safety.

2.1 Refugee Movement

Violence is the most common cause of forced migration, above political regime type and state of the economy (Schmeidl 1997; Moore and Shellman 2007). Decisions made about violence, however, occur at the individual level as people affected by conflict interpret their information environment (Allen and Hiller 1985; Davenport et al. 2003; Moore and Shellman 2004, 2006). Some research suggests that individuals calculate heterogeneous cost/benefit analyses before fleeing, leading to diminished aggregate numbers over time as refugees accumulate in certain parts of the world. In other words, there is a disincentive to continue migrating outwards if a large number of refugees have already left. This is in contrast to previous models that assume refugees all have the same cost/benefit analysis of threat before deciding to flee (Melander and Öberg 2006; Moore and Shellman 2007). While statistical models such as these have been able to isolate certain factors that predict forced migration flows, they do not explain the distribution of arrival rates. Arrival rates are particularly important for humanitarian aid agencies. Being able to predict when and where a refugee flow will occur is a critical part of the ability to supply needed resources and safety for those fleeing violence and/or persecution. Existing statistical and large-n time series models do little to predict the flow rates of refugees (Edwards 2008).

ABMs of refugee movement, while relatively few, do exist (Bailey 2001; Edwards 2008; Sokolowski and Banks 2014; Sokolowski et al. 2014). Like statistical models, often these ABMs focus on the causes rather than the dynamics of flight. Edwards (2008) describes an ABM of refugee decision-making between alternative attracting sites to determine the directional flow of forced migration. Other has been done in this area, namely using data from cross-national datasets to determine cultural pull factors that predict refugee flows (Rüegger and Bohnet 2015). Edwards (2008) argues for the use of computational models in predicting forced migration movements for their ability to capture individual-level decision-making that statistical models often neglect. He notes that what are often considered outliers in large-n models are actually individuals making rational choices about the direction in which they will flee.

2.2 Pedestrian Groups during Egress

Pedestrian modeling generally falls into two broad categories: one in which agents are directed by physics equations and behave as relatively homogeneous particles (Reynolds 1987; Toner and Tu 1998; Treuille et al. 2006). In the other type of modeling paradigm, pedestrians are portrayed as individuals with motivations and push/pull factors related to social dynamics (Matarić 1995; Helbing et al. 2002; Thalmann 2007; Curtis et al. 2011; Wijermans et al. 2013). In both of these categories, agents mainly operate on the individual level without much consideration for groups.

In fact, groups matter when considering egress times (Collins et al. 2014; Elzie et al. 2014). Often groups that stay very close together slow down evacuation time, particularly when groups adjust their speeds for slower members (Collins et al. 2014). Qualitative survey research has also found that individuals adjust their behaviors based on the mobility of slower members of their group (Frydenlund et al. 2014). Previous studies found that group size affects the spread of panic through a crowd, as members of the group emotionally feed off of one another (Elzie et al. 2016). While some research does focus on group awareness or allows some grouping of agents in the model (Granovetter 1978; Raupp Musse and Thalmann 2001; Braun et al. 2003), there is little consideration even in crowd flow literature for the formation and deconstruction of groups dynamically during a simulation. To fill that gap, the rest of this paper discusses the use of cooperative game theory techniques to incorporate dynamic groupings of refugees during flight. This research contributes to both refugee and pedestrian egress work by combining ABM techniques with game theory to better understand social spaces in evacuation contexts.

3 GROUP FORMATION

Two approaches for scientifically investigating group formation are cooperative game theory (Thomas 2003) and social network analysis (Watts 2004a). The latter focuses on formation and structural properties of social networks over time, for example determining the six degrees of separation separating people across large geographic spaces (Watts 2004b). Cooperative game theory focuses almost exclusively on strategic group formation as well as the allocation of resources within those formed groups. This emphasis on the formation of group formation, rather than the composition and structure of networked individuals, is the primary reason cooperative game theory was chosen to apply to the research presented here.

Cooperative game theory—pre-1990 it was known as n-person Game Theory (Rapoport 1970)—is different from traditional game theory. Incorporating a larger number of agents into a game increases the complexity of the interactions and outcomes, making solution concepts such as the Nash Equilibrium inappropriate (Thomas 2003). New solutions devised to address this problem include imputation, Core

(Gillies 1959), Nucleolus (Schmeidler 1969), and Shapley Value (Shapley 1953). Requirements for being in the Core of a game are detailed below:

Theorem *An allocation $\{x_1, x_2, \ldots, x_n\}$ is in the Core if and only if*

$$\sum_{i=1}^{n} x_i = v(N) \qquad \text{(Efficient)}$$

$$\forall \boldsymbol{S}: \sum_{i \in S} x_i \geq v(S) \qquad \text{(Subgroup power)}$$

Given N agents, each receives a payoff of x_i and the total value of a subgroup S is given by $v(S)$, with $v(N)$ being the value of a super group containing all agents. The key feature of the Core is that every agent must be receiving a greater payoff than it would have as a member of any of the subgroups, i.e., the sum of the current payoffs of the subgroup members is greater than the total payoff they would receive if formed their own subgroup. If this condition did not hold, the agents would be better off forming subgroups and receiving a share of the payoff greater than their current payoff. It is this feature of the Core that we adapt for use in the ABM.

Traditional cooperative game theory does not consider dynamic group formation. It assumes the agents are rational (Homo Economicus) and will instantly form the coalitions that will maximize their payoffs. This is, of course, unrealistic and one of the issues we try to overcome with our heuristic approach. Determining how the groups form, or even $v(N)$, is a no trivial task and requires computationally intensive techniques like linear programming or dynamic programming (Rahwan et al. 2015).

In our model, each agent looks for a subgroup within its group that makes it (and all the other members of the subgroup) better off. We do not implement the Core concept directly because computational costs of resolving the payoff of every subgroup of, say, one hundred agents is immense. Thus, our approach is to allow each agent the chance to consider only one random subgroup at each time-step of the simulation. The subgroup may decide to split from the main group or even join with another group. The authors have previously applied this approach to static agents in a simple scenario (Collins and Frydenlund 2016); the research presented here incorporates the subgroup searching mechanism into a refugee migration model with mobile agents.

4 MODEL

In the model, agents representing refugees move from a left-side origin to a right-side destination representing the flight toward safety. Agents have heterogeneous speed abilities/preferences and groups adjust to accommodate the speed of their slowest members. Agents can perform two basic actions: move and join/leave a group. The strategic element of group formation in the model requires that agents receive benefits for joining or splitting from a group, as would be predicted with cooperative game theory (i.e., the current formed group did not satisfy the requirements for an core (Shapley 1967)). The agents then are *utility-based* (Wilensky and Rand 2015), with utility functions considering protection gained from a group versus the speed of reaching the destination. For instance, an agent may gain safety-related benefits from being in a large group, but that group will take a long time to reach the destination as it accommodates the speed of its slowest member. Splintering of a sub-group may allow some agents to move faster while maintaining some of the benefits of group membership.

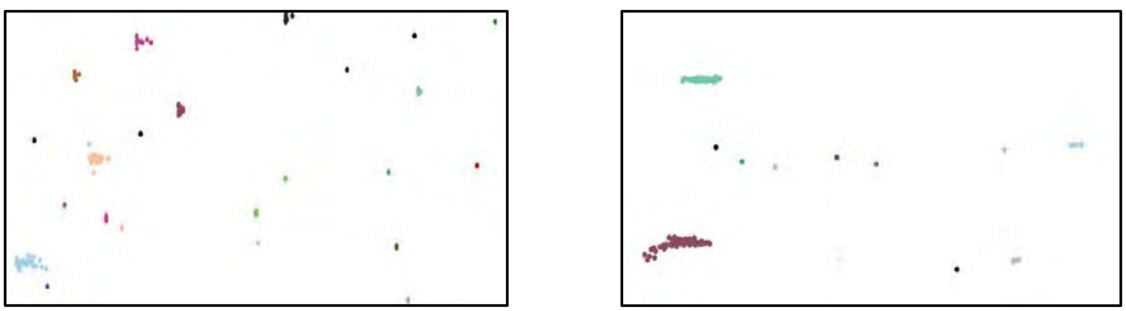
The utility function of the agents is represented with the following equation:

$$V(a) = \lambda \min_{x \in G(a)} s(x) + (1 - \lambda)(|G(a)| - 0.5)/N \quad (1)$$

Where *s(x)* is the maximum speed of agent *x*; *G(a)* is the current group to which agent *a* belongs. *N* is the total number of agents and λ is the weighting constant between the two parts of the utility function. Speed is normalized between [0.1, 1], hence group size has to be normalized too so they are comparable. The minus half for the group size part of the utility is there to put a bias against small groups (since the group size utility increases linearly); this could have been overcome by using an exponential function but that would introduce normalizing problems between the two terms. All agents are assumed to want to maximize their utilities and all agents within a group will have the same utility.

The utility functions drive an agent's strategy to stay or leave a group. An agent on its own forms a singleton group. The model incorporates a "kicked out" behavior that allows groups to eject members that are too slow and decreasing the rest of the group members' utilities. This may also occur if an agent falls too far behind (here, five patches or more). Subgroups of faster members will also split from a main group where they are significantly faster than the slowest member and λ is sufficiently high ("group split"). Smaller groups in close proximity may join together to form a "super group."

The three group formation mechanisms (kicked out, group split, and super groups) working together result in a variety of group sizes and distributions. Figure 1(a) shows a mixture of group sizes, whereas Figure 1(b) shows two large groups surrounded by singletons. In one extreme, if only speed is considered (λ = 1), the agents all form the own singleton groups and move at their max speeds towards the right. In this case, there is no benefit in joining a group. Conversely, if only group formation is considered (λ = 0), then the agents cluster into one super group. The implementations of the three group formation mechanisms are discussed in turn.



(a) (b)

Figure 1: Agents moving in groups from left to right in the NetLogo model (a) λ = 0.15, (b) λ = 0.1.

4.1 Agent Movement

Once all the agents have updated to which group they belong, all the agents move at the speed of the slowest group member. Visually, agents cluster in their group formations. This clustering was achieved by perturbing the agents' rightward headings towards the vertical center of their group. In the long run, this perturbation would result in agents walking in a straight line. To give an idea of numbers within the graphical representation of group, it members where slightly "fuzzed" so they were not all on top of each other (Everitt and Dunn 2010); the effects of which can be seen in Figure 1(a) and (b). Agents more than five patches away from the group get "kicked out." Spatially grouping agents does affect the model results, which we discuss later.

4.2 Kicked Out

At each time step of the simulation, all agents check to see if they can form "better" (higher utility) groups. The simplest check is to determine whether an agent is "kicked out" or rejected from the group. This is done by comparing the group utility with and without the agent in question (Equation 2). If the without group utility is higher than the current group utility, then the agent is removed from the group.

$$V(\neg a) = \lambda \min_{x \in G(a) \setminus a} s(x) + (1 - \lambda)(|G(a)| - 1.5)/N \tag{2}$$

An astute reader will note that only the slowest agent will be kicked out.

4.3 Group Split

Subgroup utility evaluation is more complicated due to the difficulty of determining the subgroup at each time-step. In traditional cooperative game theory, every possible subgroup combination would be considered, but this is computationally intensive. For example, our simulation considers 153 agents; if all the agents were to form a group then there would be 2^{152} (approximately 10^{45}) possible subgroups that contained the current agent. Instead, we only considered one stochastically generated group at each time-step for each agent. Membership to this group is determined by proximity to the agent; that is, all agents within a Moore neighborhood of patches are considered (with multiple agents allowed per patch) and are given a 50% chance of membership to the subgroup. The choice of using proximity for subgroup formation facilitated visualization of groups. The real-world justification for this proximity assumption is that individuals will only be able to conspire with those close to them.

The second assumption of a 50% chance of membership means subgroup size will be about half of the number of agents in the group present in the agent's Moore neighborhood. This results in a bias toward subgroups of this size. To overcome this issue, different subgroup selection methods could have been employed, for example, the subgroup size could be randomly determined first and agents from the group are used to fill its membership randomly. The second issue with this approach is that agents with higher speeds are more desirable for inclusion in a subgroup and membership could be weighted by speed. Accommodations to these biases will be addressed in future iterations of model, especially given the group split results discussed below.

Once the subgroup has been determined, it is simply a case of comparing the utility of the subgroup (which is the same for all its members) to that of the original group. If it is larger, then the subgroup splits from the main group to form a new group (leaving the original group with a new utility).

4.4 Super Group

Groups may band together for greater protection. Mathematically, this means the agent receives a better utility from having a larger group size than the loss from reduced speed. For two groups to join together, utility must increase for both. Note that by the utility function (Equation 1), all agents in a group have the same utility. A super group check occurs when a group is near another; if there are no nearby groups, this check is ignored.

4.5 Simulation Experiment

The model was developed in NetLogo (Wilensky 1999). The simulations used 153 agents with heterogeneous speeds starting in their own singleton groups. The agents start at the left and flee to the right ("safety"). Two distributions for speed were considered: Uniform [0.1, 1] and a truncated normal distribution $N(0.5, 0.2^2) \subseteq [0.1, 1]$. These distributions produced indistinguishable results, so only the Uniform results are presented here. A lower speed of 0.1 was used to reduce simulation time.

The effects the weight, λ, were evaluated across experimental runs. The weight parameter was varied from 0 to 1 in 0.025 increments, and each parameter was used in 30 simulation runs. A total of 1,230 runs were completed for the two distribution types. The runs were completed on an Intel i7 3.20 Ghz machine with 8 cores and 16 GB RAM in less than a day.

5 RESULTS

The results focused on the effects of the weight parameter λ, but also included how the total evacuation time (all agents reaching the right-hand side of the model) was effected by the formation of groups within the simulation runs. Though we are cautious of using the term emergent, due to warnings from others (Epstein 2006), there does appear to be a tipping point in the formation of large groups in favor of smaller ones.

Figure 2 below shows the relationship between utility weighting (λ = 0, entirely group size weighted; λ = 1, entirely speed-weighted) and the time-steps taken for every agent to reach safety. The graph shows an almost constant number of time-steps to reach safety except at lower values of λ. At these lower λ, we would expect the agents to favor forming groups over of speed. Recall that agents expend movement to migrate toward group centers, perhaps explaining the increase in time-steps where all or most agents prefer to be in groups. Figure 2 indicates that for low λ that biases utilities toward group size, some time is spent spatially forming groups and resulting in longer time required to reach safety.

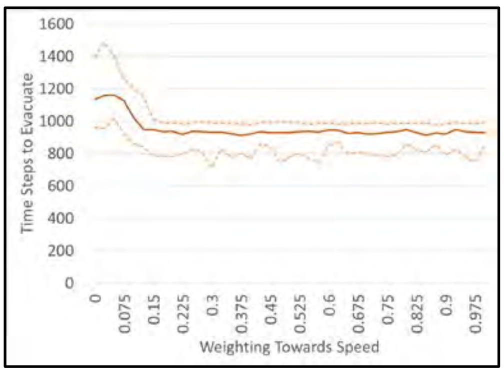


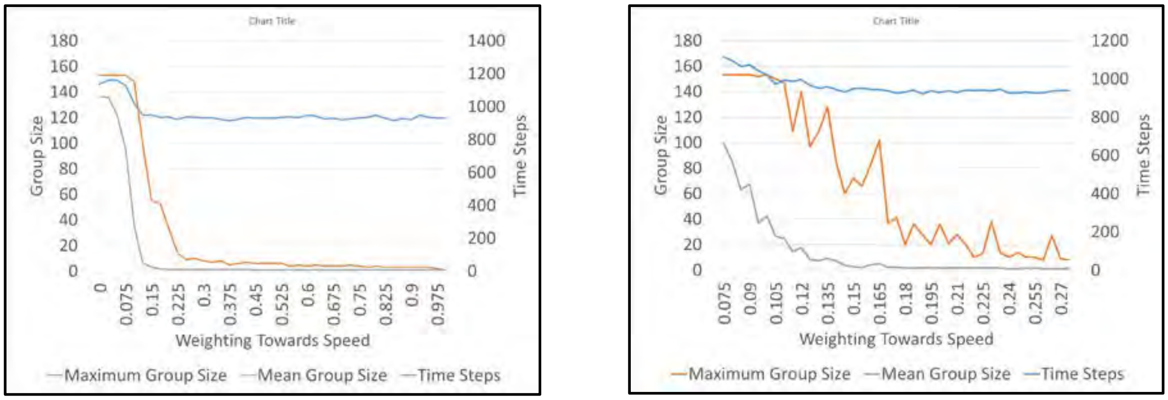
Figure 2: Maximum, average (solid line), and minimum time-steps to reach safety.

If the agents were forming relatively small local groups, then they would not have a long distance to travel to reach the vertical center of their group. Thus, the extra distance required to increase the time-steps required for evacuation must have come from the formation of large groups. Figure 3(a) shows this to be the case by considering the average group size and the maximum group size. Notice that after a weighting value of 0.225, the maximum group size is relatively very small (approximately 10) whereas for very small weighting values, the maximum group size is the number of agents (153) implying that a single large group was formed. The graph indicates an apparent tipping point between a single large group and relatively small groups around λ = 0.2. After that point, time-steps to full evacuation remain relatively constant.

Figure 3(b) shows the results from re-running only the tipping point range (0.075, 0.275), at 0.005 increments, and indicates that this transition might not be as smooth as first thought. These results could be interpreted through an example. Suppose people are moving away from some source of danger, say a gunman, and clumped together for safety. Now imagine the people hear a gunshot that results in the collapse of the group as people run for safety. This visual tipping point in the graph may represent some kind of "panic point" such as this. The results in the graph represent only 30 runs per λ increment, so further research would need to be done to determine the nature of this visual tipping point.

This tipping point is also seen in Figure 4(a), which shows a clustering of results with high evacuations times and high group size and another clustering with relatively low evacuations times and low average group size. In terms of the three group formation mechanisms, some interesting results are shown in Figure

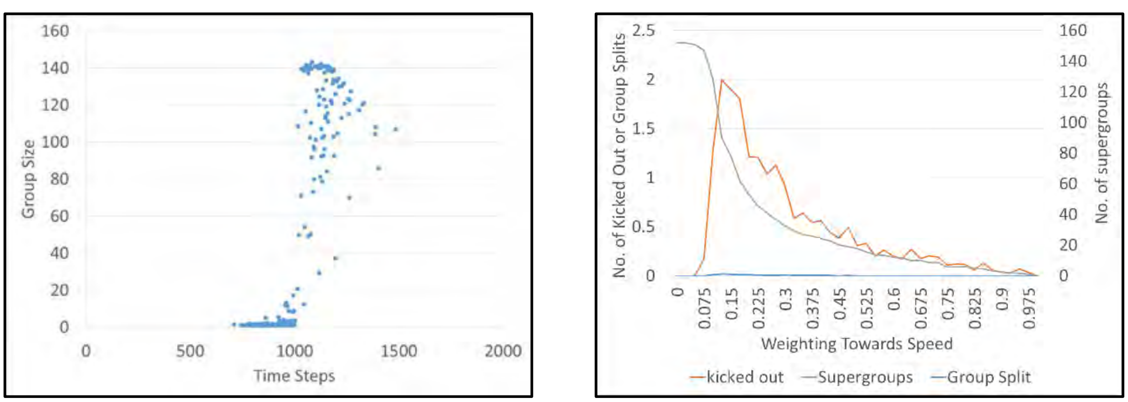
4(b). The number of super groups that are formed steadily decreases as the weighting towards speed increases. This makes sense since agents are less likely to be concerned about group size if they are focusing on speed and do not want to align with slower agents. Remember that a super group could just be when two agents joined together to form a new group, so initial singleton agents quickly form into super groups. The average number of group splits that occur is relative low, but this could be due to the mechanism used to determine the subgroup considered for removal from the group.



(a) (b)

Figure 3: Average of the mean and maximum group sizes (30 runs) with time-steps required for evacuation. (a) Complete range and (b) zoomed in range.

The average number of agents kicked out of a group produces a peak, which may at first glance seem surprising. To understand this phenomenon, consider the extremes. If the agents only care about group size, then no one is ever kicked out the group. If the agents only care about speed, then no groups are formed in the first place so no one can be kicked out of them. In this second case, the agents quickly form groups, with each agent contributing significantly to the group's utility as they overcome the '-0.5' shown in the utility function in equation (1). In a scenario of 10 agents, the group utility would be '-0.05' for one agent, '0.15' for two agents, '0.25' for the third, and 'n/10 – 0.05' for the nth agent. Thus going from a singleton to a pair, in this example, you would see an increase in utility of '0.2,' whereas going from a pair to a trio only gives a reward of '0.05.' Thus, initially the agents will pair up with anyone—even slow agents; but as group sizes increase, these slow individuals look less appealing and are kicked out.



(a) (b)

Figure 4(a): Time-steps required for evacuation versus the average group size. (b): Average number of each of the three group formation mechanics that occurred during a run.

5.1 Results from Normal Distribution Case

The results presented here draws from a uniform distribution to determine the preferred agents' speeds. Runs were also completed that used a truncated Normal distribution $N(0.5, 0.2^2)$, truncated [0.1, 1]. All of the phenomena discussed in this section were present in the normal distribution case results with approximately the same data locations and scale. There were only two noticeable differences between the two data sets. First, in the Normal equivalent to Figure 2, the line representing minimum time-steps to safety was more turbulent; this was due to a more variable minimum speed of all the agents in a run. Second, the top cluster in the Normal equivalent of Figure 4(a) was more pronounced. This indicates more runs resulted in one single, large group. Again, this is likely due to a more variable minimum speed.

Comparing the two simulations versions relates to what Axtell et al. (1996) calls "relational alignment;" that is, the observed phenomenon of interest are present in both versions. This is one category of determining success in replication of the results between two simulations. Other categories are "numerical identity," which is observing the exact same numerical results, and "distributional equivalence," where the results are shown to be statistically the same. As well as relational alignment, we looked at distributional equivalence by conducting paired t-tests for means on key outputs from the simulations. The results are shown in Table 1, which we accept rely on normality and equivalence of variance assumptions. Both "kicked out" and "group split" are shown to have statistically equal means. It is unclear why the Normal version resulted in a 6% increase in time-steps over the Uniform case. The Normal case may have a larger number of very slow agents due to the truncation method.

Table 1: Results from a paired t-test for means comparing the two variations of the simulation results.

	Uniform	Normal	df	p-value
Time-steps	954.939	1012.071	40	7.60E-22**
Kicked Out	0.472	0.470	40	0.976
Super groups	36.464	43.240	40	2.87E-05**
Group Split	0.157	0.133	40	0.293
**Significant at the 99% confidence level				

The results from this paper give some insights into the impact on time to evacuation based on individuals preference for speed over safety in numbers. However, the parameters and the distributions used in the simulation were arbitrarily determined and, as such, do not provide actual evacuation times. Using this approach for determining quantitative numbers, for real-world scenarios, would require significant improvement to the simulation including accounting for the geopolitical environment. Determining the parameters values of the simulation could be achieve by analyzing arrival rates to a refugee processing center. By inputting the demographic spread into the model, we could try to replicate the same group arrival rate as found in the real world. This analysis could give some insight into what the lambda parameter should be. This approach would be limited due to the complex nature of actual refugee movement situations, e.g., perceived and actual dangers along the route. However, this research is a step in the direction of estimating group arrival times so that aid organizations are able to better prepare for the flow of refugees.

6 CONCLUSIONS

This paper introduces a strategic group formation mechanic into an ABM to investigate the impact on refugee evacuation time. The mechanism is inspired by the methods of cooperative game theory, namely, the Core. Since we are concerned with strategy, the agents each evaluate a utility function based on group speed and group size. Agents possess heterogeneous speeds evaluated drawing from each of the Normal and Uniform distribution. Results do not vary significantly between these two distributions. Weighting toward speed in the utility function produces an apparent tipping point. With low weight placed on speed, agents tend to cluster in one large group; high weight on speed conversely produces mostly singleton groups or pairs.

A tipping point in the data requires more investigation and will be the focus of future work. The tipping point indicates that certain values of weight applied to the utility function quickly change the preference from large groups to smaller, more mobile groups. This may be related to panic behaviors, but more simulation runs are required. Though relational alignment was shown in the results, no empirical validation was conducted on the model due to a complete lack of available data.

Though the approach presented here is inspired by cooperative game theory, it is a heuristic approach and does not find the core of a game. To accomplish this would be computationally expensive, as each subgroup would have to be evaluated at each time-step. Future work on this project will consider an additional weighting factor that favors those with the highest potential utility function. This would adjust the mechanism for determining possible subgroup formations. Additionally, future versions will allow faster agents to move slightly faster than slowest agent in their group. This will provide insight into the further effects of mobility within a group. Finally, we plan to identify alternative case studies to which this research could apply in order to explore the generalizability of the model proposed here.

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