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COMPLEX MODELLING OF THE STRUCTURAL-FUNCTIONAL RECONFIGURATION OF SUPPLY CHAIN

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The comprehensive approach to modelling the structural and functional reconfiguration of the distribution network (DN) of the supply chain in terms of destructive influences, leading to its structural dynamics, is proposed. Offered for scenarios of structural dynamics of the DN is a generalized dynamic model of the structural and functional reconfiguration, which provided that the constancy intervals of intermediate structural states of the DN reduced to a multicriteria linear programming problem with bilateral constraints.

Keywords: scenario of structural dynamics, structural-functional reconfiguration, supply chain distribution network

1. Introduction

In the design process of adaptive supply chains management (SCM), issues of improving their resiliency and disaster recovery at various destabilizing and destructive effects are focused. The technology of functional reconfiguration (flexible redistribution of the system's goals, objectives and functions between operable, partially operable components) among the ways to ensure the viability and sustainability of SCM is one of the primary values [1-4].

According to the results of the analysis, in the development of advanced SCM, it is expedient to make the transition from the narrow (traditional) interpretation of the processes of reconfiguration of the systems in their broadest interpretation in the framework of the new applied theory of structural dynamics control of complex systems (SDC CS) [4], which is developed. With regard to the current reconfiguration, the CS should be considered not only as a technology of management of structures to compensate for the CS failures, but also as a management technology aimed at improving the CS efficiency. In the framework of the SDC CS, it is proposed to call this reconfiguration as the structural and functional reconfiguration. At the same time, especially important are the formulation and solution of various classes of SDC CS problems, including the tasks of planning and management of processes of receiving, storing, transporting and delivery of certain types of material resources to the consumers at the reorganized SCM structure.

2. The Structural Reconfiguration of Distribution Network of SCM

As an example, let's consider the transport and distribution network (DN) of SCM.

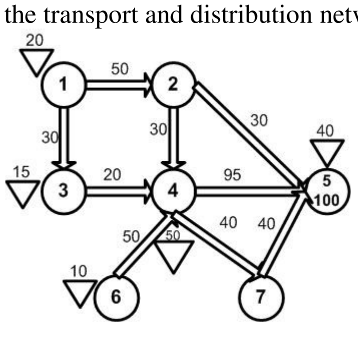

Figure 1. Distribution network of SCM

On Figure 1, triangles characterize the warehouses of the network nodes. The supply of goods is performed through the nodes 1 and 6. The node 4 is the central distribution hub. Node 5 is the regional distribution centre.

The technological structure of distribution network is shown on Figure 2.

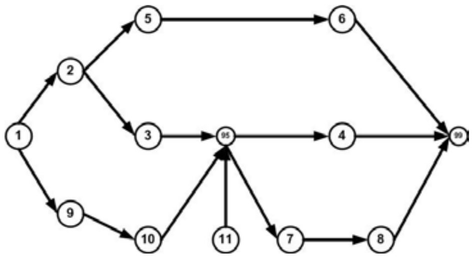

Figure 2. Technological structure of distribution network of SCM

The technological structure of the DN comprises different operations of receiving, warehousing, transportation and delivery of certain types of material resources (raw materials, finished goods) to the consumers in a certain time interval. These technological operations are performed in the nodes of the DN (Fig. 1).

As a result of analysis of the technological structure of the DN, by using techniques from [5, 6], we can identify a set of critical technological operations performed in the nodes of the network.

Let's consider some of the most important critical technological operations of DN:

technological operations of the node 1 – warehousing and delivery of batches to terminals 2 and 3;

technological operations of the node 4 – warehousing and direct delivery of batches from the central node to the regional distribution centre;

technological operations of the node 7 – warehousing and delivery of batches from the central node to the regional distribution centre.

As a result of using the technique of plotting the trajectories of structural reconfiguration as described in [5, 6], we obtain a fragment of scenarios of dynamics of the technological structure of the DN shown on Figure 3. Here, $St_{i_1, i_2, ..., i_k}$ is the structural state of the DN, in which $i_1, i_2, ..., i_k$ are numbers of absent critical technological operations of the corresponding network nodes.

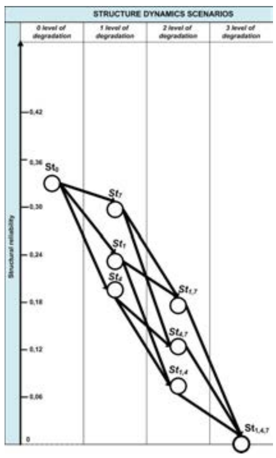

Figure 3. Structural reconfiguration of the DN

Each trajectory of Sc is characterized by a set of structural states $\{St_{i_1, i_2, ..., i_k}\}$ of the DN, which are changed by the DN within a time interval $T = (t_0, t_f]$ as a result of its reconfiguration. In Figure 3, an optimistic ($St_0 \rightarrow St_7 \rightarrow St_{1,7}$) and pessimistic ($St_0 \rightarrow St_4 \rightarrow St_{1,4}$) trajectory of structural reconfiguration of the DN can be distinguished. Optimistic (pessimistic) trajectory is a degradation of the technological structure of the DN, including the structural states of the maximum (minimum) reliability network.

Analysis of the DN SCM functioning revealed the following main features of the problem to be solved:

- Failures of execution of certain technological operations, or the DN nodes in the whole, occur under the influence of various factors (internal, external, objective, subjective, etc.) and are events possible, but improbable, or the probability of which is small and can not be evaluated reasonably in the design process.
- Intermediate structural states of the DN do not change continuously, and are constant within some intervals.
- It is required to consider and analyse the current characteristics of technological operations performed in the nodes of the DN.
- Depending on the current situation, the rapid redistribution of the technological operations between the nodes of the DN should be performed.
- Feature of the estimation and analysis problem for the DN efficiency for a reasonable redistribution of technological operations DN between its nodes lies in its multi-criteria setting, as well as in availability of conditions of essentially unavoidable information and time constraints associated with incompleteness, ambiguity, uncertainty and inconsistency of both the initial data and the information that comes (is formed) in the operation of the DN.

Taking into account the above features of the results (optimistic and pessimistic trajectory), the structural reconfiguration of the SCM DN is shown on Figure 4.

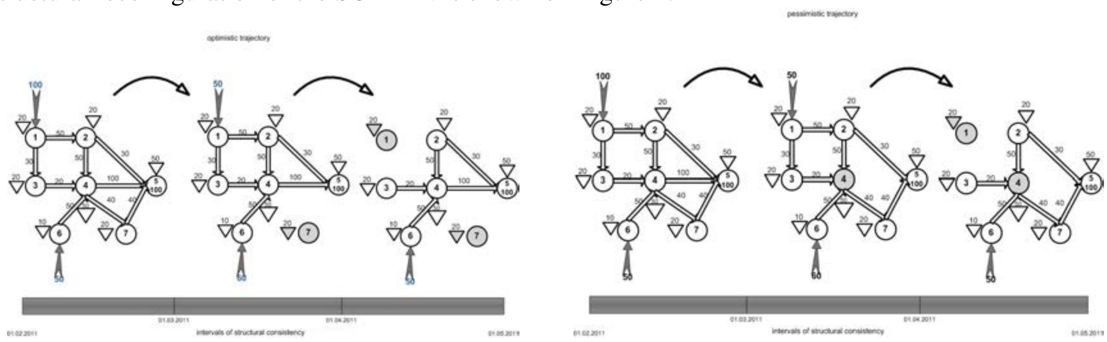

Figure 4. Optimistic and pessimistic trajectories of structural reconfiguration of the DN SCM

3. The Dynamic Model of the Structural and Functional Reconfiguration of Distribution Network of SCM

In a generalized formal setting of the problem of structural and functional reconfiguration, the following additional functional features should be taken into account. When the destructive effects take place, the intensity of transportation between sites, delivery of goods to the consumer, the volume of storage space, the volume of supplies are subject to change. If the amount of supplied products exceeds the capacity of DN in processing, transportation, warehousing, then unclaimed goods are sent back to the warehouses outside DN. This requires an additional cost. It is also necessary to consider fixed transportation and storage costs, variable costs of transportation, warehousing, delivering the products to the consumer. It should be noted that the products are stored in the warehouse for the previous interval of the structure constancy may be used in subsequent intervals. In general, the problem is in finding the structural and functional plan of the DN reconfiguration during the transportation of products supplied from the nodes 1 and 6 into the regional distribution node to meet consumer demand with the minimum total costs.

To formalize the proposed problem, we introduce the following functions describing the features of the supply, storage, transportation and delivery of material resources of various types: $V_i(t)$ – the amount of storage space for each A_i node of the DN; $\psi_{i\rho}(t)$ – the intensity of ρ type product supply to the A_i node of the DN; $\omega_{ij\rho}(t)$ – the intensity of transportation of ρ type goods between the nodes of DN; $\varphi_{i\rho}(t)$ – the intensity of delivery (processing) of ρ type product from the A_i node of the DN to the consumer.

It results from the substantive setting of the problem of manageable reconfiguration of the DN, that the dynamics of change in the amount of ρ type products passing through A_i node the DN can be described by the formula

$$x_{i\rho}^{+}(t) = x_{i\rho}^{-}(t) + y_{i\rho}(t) + z_{i\rho}(t),\ i \in N = \{1,2,...,n\}. \tag{1}$$

In this expression (1), $x_{i\rho}^{+}(t)$ is intensity of ρ type product supply to A_i node of the DN, $x_{i\rho}^{-}(t)$ – the intensity of ρ type product reduce in A_i node, $y_{i\rho}(t)$ – the intensity of accumulation (transportation) of ρ type product in (from) warehouse of A_i node of the DN, $z_{i\rho}(t)$ – the intensity of of unclaimed ρ type product return from A_i node of the DN.

In accordance with the notation introduced in the contensive setting of the problem, the intensity of ρ type product supply to A_i node can be described by the following formula (2):

$$x_{i\rho}^{+}(t) = \psi_{i\rho}(t) + \sum_{j=1}^{n} e_{ji}(t)\cdot\omega_{ji\rho}(t)\cdot u_{ji\rho}(t),\ i \in N, \tag{2}$$

where $u_{ji\rho}(t) \in \{0,1\}$ – management of transportation of ρ type product such in the trajectory of changes of structural states of the DN. Moreover, if $u_{ji\rho}(t) = 1$, then ρ type products are transported from A_j node to A_i node, and if $u_{ji\rho}(t) = 0$, then the products is not transported.

The intensity of reduction of ρ type products in A_i node of the DN can be described by formula (3)

$$x_{i\rho}^{-}(t) = \varphi_{i\rho}(t)\cdot\vartheta_{i\rho}(t) + \sum_{j=1}^{n} e_{ij}(t)\cdot\omega_{ij\rho}(t)\cdot u_{ij\rho}(t),\ i \in N, \tag{3}$$

where $\vartheta_{i\rho}(t) \in \{0,1\}$ – management of ρ type product delivery to the consumer from A_i node. Here, if $\vartheta_{i\rho}(t) = 1$, then ρ type products are delivered to the customer from A_i node of the DN, or if $\vartheta_{i\rho}(t) = 0$, then ρ type products are not delivered.

Equations (1)-(3) describe the dynamics of transportation, delivery, storage and additional operations with the production in the trajectory of structural changes of structural states of the DN. It should be noted that $\psi_{i\rho}(t),\ \omega_{ij\rho}(t),\ \varphi_{i\rho}(t)$ are prescribed functions, $e_{ij}(t)$ describes the structural reconfiguration of the DN as per the relevant scenario in the range of control $T = (t_0, t_f]$.

The functions $y_{i\rho}(t),\ z_{i\rho}(t),\ u_{ij\rho}(t),\ \vartheta_{i\rho}(t)$ are unknown, and $u_{ij\rho}(t),\ \vartheta_{i\rho}(t)$ controls uniquely determine operations of the return of unclaimed goods $z_{i\rho}(t)$, the dynamics of accumulation and transportation of products $y_{i\rho}(t)$ from the warehouse of A_i node of the DN. Consequently, a pair of functions $y_{i\rho}(t),\ z_{i\rho}(t)$ can be considered as a state of dynamic system.

Under these conditions, a model of the structural and functional reconfiguration of the DN will include the following main elements.

The model of the structural and functional reconfiguration of the DN is prescribed by (1)-(3) formulas.

Restrictions for the possible controls and states can be summarized as follows

$$0 \le \sum_{\rho=1}^{p} y_{i\rho}(t) \le V_i\ \ \forall i \in N, t \in (t_0, t_f], \tag{4}$$

$$u_{ij\rho}(t) \in \{0,1\},\ \vartheta_{i\rho}(t) \in \{0,1\}\ \forall i, j \in N,\ \rho \in P, t \in (t_0, t_f], \tag{5}$$

$$z_{i\rho}(t) \geq 0,\ z_{i\rho}(t) \geq 0\ \forall i \in N, \rho \in P, t \in (t_0, t_f]. \tag{6}$$

Boundary conditions are as follows:

$$y_{i\rho}(t_0) = z_{i\rho}(t_0) = 0\ \forall i \in N,\ y_{i\rho}(t_f) \geq 0,\ z_{i\rho}(t_f) \geq 0\ \forall i \in N. \tag{7}$$

Admissible controls $u_{ij\rho}(t)$, $\vartheta_{i\rho}(t)$ that satisfy equations (1)-(3) describing the change of state, restrictions along the trajectory (4)-(6) and boundary conditions (7), can be evaluated according to various indicators of the quality of the DNS functioning. These indicators can be:

$J_1 = \int_{t_0}^{t_f} \sum_{\rho=1}^{p} \sum_{i=1}^{n} v_{i\rho}(t) \cdot z_{i\rho}(t)\ dt$ – costs for the return of unclaimed goods;

$J_2 = \int_{t_0}^{t_f} \sum_{\rho=1}^{p} \sum_{i=1}^{n} c_{\rho}(t) \cdot \varphi_{i\rho}(t) \cdot \vartheta_{i\rho}(t)\ dt$ – the cost of products delivered to the consumer;

$J_3 = \int_{t_0}^{t_f} (\sum_{\rho=1}^{p} \sum_{i=1}^{n} \sum_{j=1}^{n} r_{ij\rho}(t) \cdot \omega_{ij\rho}(t) \cdot u_{ij\rho}(t) + \sum_{\rho=1}^{p} \sum_{i=1}^{n} \pi_{i\rho}(t) \cdot \varphi_{i\rho}(t) \cdot \vartheta_{i\rho}(t) + \sum_{\rho=1}^{p} \sum_{i=1}^{n} \lambda_{i\rho}(t) \cdot y_{i\rho}(t))\, dt + D$ – costs for transportation, delivery and storage of goods,

where $D = \int_{t_0}^{t_f} (\sum_{\rho=1}^{p} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij\rho}(t) \cdot \omega_{ij\rho}(t) \cdot e_{ij}(t) + \sum_{\rho=1}^{p} \sum_{i=1}^{n} d_{i\rho}(t) \cdot \varphi_{i\rho}(t) \cdot e_{ii}(t) + \sum_{i=1}^{n} f_i(t) \cdot V_i(t) \cdot e_{ii}(t))\, dt$ – fixed transportation and storage costs.

This problem of structural and functional reconfiguration of the DN belongs to a class of bilinear time-dependent differential dynamic models and has a number of specific features that distinguish them from classical problems in the theory of optimal control of complex dynamic systems. The first feature of this problem lies in the fact that the right members of differential equations (1)-(3) describing the process of structural and functional reconfiguration of the DN contain discontinuous functions $e_{ij}(t)$ that define the structural dynamics of the DN. Therefore, the problem under consideration can belong to the management problems with intermediate conditions. Another feature of the problems is that these problems belong to a class of multicriteria problems.

However, prescribed interval $T = (t_0, t_f]$ of structural reconfiguration of the DN is divided into a number L of sub-intervals $T = \{(t_0, t_1], (t_1, t_2], ..., (t_{k-1}, t_k], ..., (t_{L-1}, t_L = t_f]\}$ of constancy of the DN structure. In each subinterval, $T_k = (t_{k-1}, t_k]$ is in an unchanged state $St_{i_1, i_2, ..., i_k}$.

This assumption allows us to reduce the multicriteria problem of optimal control with discontinuous right members to a multicriteria static model of structural and functional reconfiguration of the DN.

4. The Static Model of the Structural and Functional Reconfiguration of Distribution Network of SCM

In order to trace the dynamic model to its static interpretation, we will carry out integration of formulas (1)-(3) describing dynamics of change of the system condition, in intervals of constancy of structure $T_k = (t_{k-1}, t_k],\ k = 1,...,L$ as functions $e_{ij}(t)$ are constant in intervals of the structure constancy. At the same time, control functions $u_{ij\rho}(t)$, $\vartheta_{i\rho}(t)$ are transformed into the appropriate variables $u_{ij\rho k} = \int_{t_{k-1}}^{t_k} u_{ij\rho}(t)\ dt, \vartheta_{i\rho k} = \int_{t_{k-1}}^{t_k} \vartheta_{i\rho}(t)\ dt$ that characterize the time of transfer of ρ type products from A_i node to A_j node of the DN and time of ρ type products delivery from A_i node in interval $T_k = (t_{k-1}, t_k],\ k = 1,..., L$.

Then expressions (1)-(3) characterizing the flows of heterogeneous products passing through A_i node can be represented as follows

$$I_{i\rho k} + y_{i\rho(k-1)} + \sum_{j\in N_i^-} \omega_{ji\rho k} \cdot u_{ji\rho k} = \varphi_{i\rho k} \cdot \vartheta_{i\rho k} + \sum_{j\in N_i^+} \omega_{ij\rho k} \cdot u_{ij\rho k} + y_{i\rho k} + z_{i\rho k}, \qquad (8)$$

here $I_{i\rho k} = \int_{t_{k-1}}^{t_k} \psi_{i\rho}(t)\, dt$ – the amount of ρ type products received by A_i node from outside in T_k interval; $y_{i\rho(k-1)}$ – the amount of ρ type products stored in a warehouse of A_i node in T_{k-1} interval; $\omega_{ij\rho k}$ – the intensity of ρ type product transportation from A_i to A_j node in T_k interval; $\varphi_{i\rho k}$ – the intensity of ρ type product delivery to the consumer from A_i node in T_k interval; $y_{i\rho k}$ – the amount of ρ type product stored in a warehouse of A_i node in T_k interval; $z_{i\rho k}$ – the amount of unclaimed ρ type products in A_i node in T_k interval; $N_i^- = \{ j \in N | e_{ji}(k) = 1\}$ – set of numbers of the DN nodes, from which products can be transported to A_i node in T_k time interval; $N_i^- = \{ j \in N | e_{ji}(k) = 1\}$ – a set of numbers of the DN nodes, to which the products can be transported from A_i node in T_k time interval.

The physical meaning of the expression (8) consists of the following: all ρ type products received by A_i node either from outside or from other nodes in T_k interval of constancy of the DN structure, or are in the warehouse in previous interval T_{k-1} of the structure constancy should either be delivered to the consumer, or be transported to the other nodes or be unclaimed.

We introduce the following variables: $x_{ij\rho k} = \omega_{ij\rho k} \cdot u_{ij\rho k}$ – the amount of ρ type products transported from A_i node to A_j node in T_k interval; $g_{i\rho k} = \varphi_{i\rho k} \cdot \vartheta_{i\rho k}$ – the amount of ρ type products delivered to the consumer from A_i node in T_k interval. Then expression (8) can be written as follows:

$$(\sum_{j\in N_i^+} x_{ij\rho k} - \sum_{j\in N_i^-} x_{ji\rho k}) + (y_{i\rho k} - y_{i\rho(k-1)}) + g_{i\rho k} + z_{i\rho k} = I_{i\rho k}. \qquad (9)$$

In expression (9), the quantities $x_{ij\rho k}$, $y_{i\rho k}$, $y_{i\rho(k-1)}$, $g_{i\rho k}$, $z_{i\rho k}$ are the unknown variables, which, according to (4)-(7), must satisfy the following constraints:

$$0 \le x_{ij\rho k} \le \omega_{ij\rho k} \cdot (t_k - t_{k-1})\,;\; 0 \le \sum_{\rho=1}^{p} y_{i\rho k} \le V_i\,;\; 0 \le g_{i\rho k} \le \varphi_{i\rho k} \cdot (t_k - t_{k-1})\,;\; z_{i\rho k} \ge 0\,. \qquad (10)$$

In forming the plan of the structural and functional reconfiguration of the DN, a search for the best solution is based on the optimisation of the following generalized indicators of functional capabilities of the system:

total costs for the return of unclaimed products

$$J_1 = \sum_{\rho=1}^{p} \sum_{i=1}^{n} \sum_{k=1}^{L} \nu_{i\rho k} \cdot z_{i\rho k}\,; \qquad (11)$$

the cost of production delivered to the consumer

$$J_2 = \sum_{\rho=1}^{p} \sum_{i=1}^{n} \sum_{k=1}^{L} c_{\rho k} \cdot g_{i\rho k}\,; \qquad (12)$$

the cost of transportation, delivery, and storage

$$J_3 = \sum_{\rho=1}^{p} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{L} r_{ij\rho k} \cdot \omega_{ij\rho k} \cdot u_{ij\rho k} + \sum_{\rho=1}^{p} \sum_{i=1}^{n} \sum_{k=1}^{L} \pi_{i\rho k} \cdot \varphi_{i\rho k} \cdot \vartheta_{i\rho k} + \sum_{\rho=1}^{p} \sum_{i=1}^{n} \sum_{k=1}^{L} \lambda_{i\rho k} \cdot y_{i\rho k} + D\,. \qquad (13)$$

Here $D=\sum_{\rho=1}^{p}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{L}c_{ij\rho k}\cdot\omega_{ij\rho k}\cdot e_{ijk}+\sum_{\rho=1}^{p}\sum_{i=1}^{n}\sum_{k=1}^{L}d_{i\rho k}\cdot\varphi_{i\rho k}\cdot e_{iik}+\sum_{i=1}^{n}\sum_{k=1}^{L}f_{ik}\cdot V_{ik}\cdot e_{iik}$ – fixed transportation and storage costs.

Plan for structural and functional reconfiguration δ can be presented as consisting of four vectors x, y, g, z as follows $\delta=\|x,g,y,z\|=\left\| \|x_{ij\rho k}\|,\|g_{i\rho k}\|,\|y_{i\rho k}\|,\|z_{i\rho k}\| \right\|$, where $\|x_{ij\rho k}\|$ – describes the transportation of products between the nodes of the DN; $\|g_{i\rho k}\|$ – describes delivery of the products to the consumer; $\|y_{i\rho k}\|$ – describes the storage of products in the DN warehouses; $\|z_{i\rho k}\|$ – describes the losses of products.

Then expressions (9), (10) define the set of feasible plans of structural and functional reconfiguration $\Delta_\beta=\{\delta\}$ in Sc trajectory of the DN structural states. At set $\Delta_\beta=\{\delta\}$, rational plans $\Delta^*_\beta\subseteq\Delta_\beta$ of the structural and functional reconfiguration are selected considering preference relations prescribed by the criterion functions (11)-(13) ($J_1(\delta)\to\min$, $J_2(\delta)\to\max$, $J_3(\delta)\to\min$). It follows from the analysis of (9)-(13), that under the proposed formalization, the problem of structural and functional reconfiguration of the SCM DN with intervals of constancy of dynamically reconfigurable structure is reduced to multicriteria linear programming problem of large dimension with bilateral constraints.

5. Conclusions

- By using a dynamic interpretation of the processes of performing the operations of transportation, warehousing, delivery of products to the consumer and return of unclaimed products, the generalized mathematical model of the structural and functional reconfiguration of the DN has been constructed, which belongs to a class of multicriteria differential bilinear time-dependent dynamical models with discontinuous functions in the right members of differential equations defining structural dynamics of the DN.
- The assumption that the time interval of structural reconfiguration of the DN consists of sections of the constancy of the system structure has allowed reducing the multicriteria optimal control problem with discontinuous right members to a static model of structural and functional reconfiguration of the DN, which is a multicriteria linear programming problem of large dimension with bilateral constraints.

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