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SUPPLY CHAIN SIMULATION IN EXTENDSIM ENVIRONMENT

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A stochastic single-product inventory control model for the chain "producer – wholesaler – customer" with two stages in ordering process is considered. The strategy of the first stage executed by customer is based on a model with fixed reorder point and fixed order quantity. The strategy of the second stage is realized by a wholesaler. It is assumed that producer supplies its production to wholesaler according fixed schedule. We have to take into account that the sum of total costs for goods ordering, holding and losses from deficit per time unit should be minimal. The considered models are realized using simulation method with the help of the package ExtendSim 8. The numerical examples of problem solving are presented in the paper.

Keywords: *inventory control, supply chain, wholesaler, customer, simulation, ExtendSim package*

1. Introduction

In practice there are many cases when a wholesaler is involved in ordering process and a situation "producer – wholesaler – customer" takes place. In the given paper we consider a stochastic single-product inventory control model for the chain "producer – wholesaler – customer" [1, 2]. We have to take into account that the sum of total costs for goods ordering, holding and losses from deficit per time unit should be minimal. In proposed criteria total costs are sum of corresponding costs (losses) for all subjects taking part in the ordering process, in our case for customers and wholesaler.

The wholesaler and customers can use different ordering strategies. Two models of ordering process used by customers are considered in the study [3]. The first model is a model with fixed reorder point and fixed order quantity, the second model is the model with fixed period of time between the moments of placing the neighbouring orders. The wholesaler's ordering algorithms are considered in the paper [4]. The simplest algorithm considers the following situation: every customer's order is sent by wholesaler to producer at once and received goods are sent back to customers at once too. In the second variant wholesaler constructs a common order for group of customers taking in account or inventory level for each customer, or time of receiving the customers' orders and quantity of them. Finally we can propose the third variant when the wholesaler has his owner storehouse with definite quantity of goods. There are other variants of ordering policy for wholesaler in practice, which will be considered in our further research.

In practice the suggested models are realized using analytical and simulation approaches. In the given paper the considered task is solved using simulation method.

2. Description of the Model

Let us consider single-product inventory system for the supply chain "producer – wholesaler – customer" with two stages in ordering process (see Figure 1). *First stage* executed by *n* customers. In the moment of time, when the customer's stock level falls till certain level, a new order is sent to wholesaler. *Second stage* executed by wholesaler. A new order is sent to producer in the fixed moments of time. We assume that the wholesaler has his own storehouse. The suggested strategies of each stage are based on the real conditions of the business.

Principal aim of the considered problem is to define the exact ordering strategy for *n* customers and a wholesaler to achieve the minimum expenses for goods holding, ordering and losses from deficit per a time unit [3]. Taking into account stochastic character of the inventory control problem, criteria of optimisation is minimum average total expenses (costs) per time unit, which are calculated by following formula:

$$E = \sum_{i=1}^{n} E_i^{cust} + E^{wh} ,$$

where E_i^{cust} is *i*-th customer's average total expenses for goods holding, ordering and losses from deficit per time unit, $i = 1, 2, ..., n$.

E^{wh} is wholesaler's average total expenses for goods holding, ordering and losses from deficit per time unit.

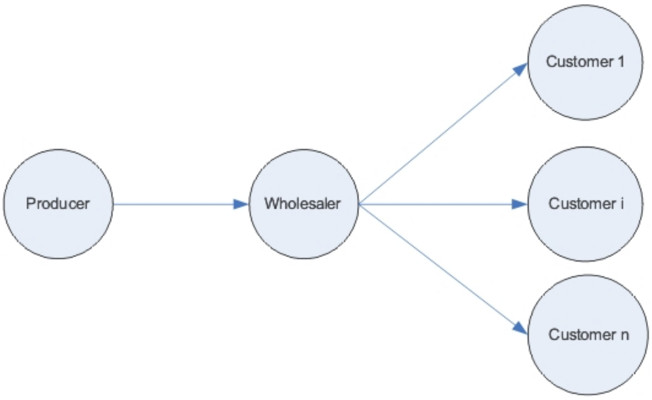

Figure 1. Chain of product ordering.

Let us consider in detail the stages in presented chain of product ordering.

First stage of ordering process. The demand for goods D_i of customer with number *i* is Poisson process with intensity λ_i. Time L_i of goods delivery from wholesaler to *i*-th customer has normal distribution with parameters μ_i and σ_i. The policy of order forming for *i*-th customer is follows. A new order is placed in the moment of time, when the stock level falls till certain level R_i. The quantity R_i is called as *reorder point*. The order quantity Q_i is constant. We suppose that $Q_i \geq R_i$. Note that order quantity Q_i and reorder point R_i are *control parameters* of the first stage model.

Dynamics of inventory level of product for customer with number *i* during one cycle $T_i = T_i^* + L_i$ (time interval between two subsequent order deliveries for *i*-th customer) is shown on Figure 2. There is a possible situation of deficit, when demand $D_i(L_i)$ during lead time L_i exceeds the value of reorder point R_i. We suppose that in case of deficit the last cannot be covered by expected order.

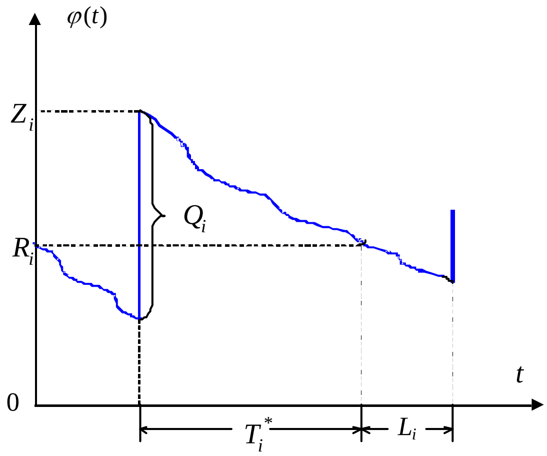

Figure 2. Dynamics of *i*-th customer's inventory level during one cycle

Denote as Z_i the quantity of goods in stock in the time moment immediately after order receiving. We can determine this quantity of goods Z_i as function of demand $D_i(L_i)$ during lead time L_i:

$$Z = \begin{cases} R + Q - D(L), & \text{if} \quad D(L) < R; \\ Q, & \text{if} \quad D(L) < R. \end{cases} \tag{1}$$

Expression (1) is basic. It allows expressing different economical indexes of considered process.

We suppose that the wholesaler has his own storehouse with definite quantity of goods q. If customer's order quantity is less than quantity of production in the stock ($Q_i < q$) the wholesaler performs this order in full volume at once. Otherwise when the order quantity (demand) exceeds the value of stock quantity ($Q_i \geq q$) the customer will receive only part of goods, and there is the situation of deficit of $Q_i - q$ units of products in wholesaler's storehouse.

Ordering cost C_o has two components: constant c_1, which includes cost of the order forming and constant part of expenses of order transportation, and variable component $c_2(Q_i)$, which depends on the order quantity Q_i. We assume that for all customers holding cost is proportional to quantity of goods in stock and holding time with coefficient of proportionality C_H; losses from deficit are proportional to quantity of deficit with coefficient of proportionality C_{SH}.

For customer with number i average total cost during one cycle $E_i^{cust}(T_i)$ is calculated by following formula:

$$E_i^{\text{cust}}(T_i) = C_o(Q_i) + E_H(T_i) + E_{SH}(T_i), \; i = 1, 2, ..., n, \tag{2}$$

where T_i is average cycle time (time interval between two subsequent order deliveries); $C_o(Q_i)$ is ordering cost, which includes costs of the order forming and expenses of order transportation; $E_H(T_i)$ is average holding cost during one cycle; $E_{SH}(T_i)$ is average shortage cost during one cycle, and total cost E_i^{cust} can be found as divided by average cycle time T_i [5]:

$$E_i^{\text{cust}} = \frac{E_i^{\text{cust}}(T_i)}{T_i}, \quad i = 1, 2, ..., n.$$

Note that $E_H(T_i)$ and $E_{SH}(T_i)$ depend on control parameters R_i and Q_i. Analytical formulas for these economical characteristics are presented in the paper by Kopytov and Greenglaz, 2004. For problem solving we have to minimize criteria (2) by R_i and Q_i.

Second stage of ordering process. We assume that producer supplies its production to wholesaler according fixed schedule. In this case we consider ordering process with fixed period of time between the moments of placing neighbouring wholesaler's orders; and order quantity q is determined as difference between fixed stock level S and quantity of goods in the moment of ordering r (see Figure 3):

$$q = S - r. \tag{3}$$

Denote by T time period between the moments of placing neighbouring orders; in considered task T is constant. The time of goods delivery from producer to wholesaler (lead time) L has a normal distribution with a mean μ_L and a standard deviation σ_L. We suppose that lead time essentially less as time of the cycle: $\mu_L + 3\sigma_L << T$.

Supposed that during time T wholesaler has received n orders from customers, these orders can be described by vector $\{Q_1, Q_2, ..., Q_n\}$.. There is a possible situation of deficit, when the demand during time between neighbouring moments of orders receiving exceeds the quantity of goods in stock Z in the time moment immediately after order receiving. Analogously model 1 we suppose that in case of deficit the last cannot be covered by expected order.

We denote as S the goods quantity which is needed "ideally" for one period and it equals to the sum

$$S = \bar{D}(T) + S \ , \qquad (4)$$

where $\bar{D}(T)$ is average demand for cycle time; S_0 is some safety stock (emergency stock).

We suppose that "ideally" S gives wholesaler in future the minimum of total expenditure for ordering, holding and loses from deficit per unit of time. So, for the second stage in suggested model time period T and stock level S are *control parameters*.

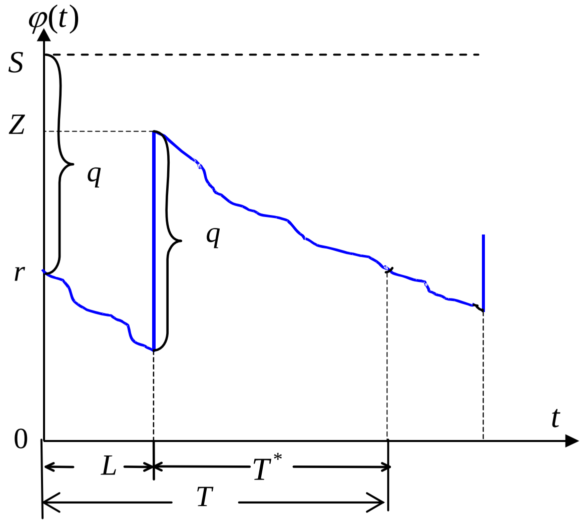

Figure 3. Dynamics of wholesaler's inventory level during one cycle

We suppose that in the moment of time, when a new order has to be placed, may be situation, when the stock level is so big that a new ordering doesn't occur. However for generality of model we'll keep the conception of lead time and quantity of goods at the time moment immediately after order receiving in such case too. It corresponds to real situation when the wholesaler uses the transport means, which depart at the fixed moments of time not depending on existence of the order and which have the random lead time; for example transportation by trailers which depart each 1st and 15th day of each month.

In real situation in the moment of time t the stock level $\varphi(t)$ is equal to S only in two cases:

1) $r = S$ and $D(t) = 0$, where $D(t)$ is the demand for goods during the time t; $0 \le t \le T$;
2) $r < S$ and $D(t) = 0$, where $L \le t \le T$.

Taking into account that in case of deficit it can't be covered by expected order, we can obtain the expression for goods quantity at the moment of time immediately after order receiving:

$$Z = \begin{cases} r + q - D(L), & \text{if} \quad D(L) < r; \\ q, & \text{if} \quad D(L) \ge r. \end{cases} \qquad (5)$$

where $D(L)$ is the demand during lead time L.

Using (4) we have:

$$Z = \begin{cases} S - D(L), & \text{if} \quad D(L) < r; \\ S - r, & \text{if} \quad D(L) \ge r. \end{cases} \qquad (6)$$

The rest r at the end of the period and the goods quantity Z at the moment of time immediately after order receiving take values from interval [0; S]:

- $r = 0$, if in the previous cycle the demand during the time T^* between the order receiving and placing of the new order is more or equal Z ;

  $r = S$, if in the previous cycle Z is equal S and there isn't any demand during the time period T^* .
- $Z = 0$, if the rest r to the moment of ordering is S (i.e. order quantity q is 0) and demand $D(L)$ during the lead time L is more or equal S;

$Z = S$, if the rest *r* to the moment of ordering is 0 or demand $D(L)$ during the lead time *L* is absent.

Finally average total cost for time unit (criteria of optimisation) is expressed by the following formula

$$E^{\mathrm{wh}} = \frac{E_{\mathrm{H}}^{\mathrm{wh}} + E_{\mathrm{SH}}^{\mathrm{wh}} + C_0(q)}{T} . \tag{7}$$

Unlike the stage 1, in the considered model expenditures $E_{\mathrm{H}}^{\mathrm{wh}}$ and $E_{\mathrm{SH}}^{\mathrm{wh}}$ depend on control parameters S and T .

3. Simulation in ExtendSim 8 Environment

For the considered problem solving, the authors have used a simulation model realized in simulation environment ExtendSim8, which is the most powerful and flexible simulation tool for analysing, designing, and operating complex systems in the market. It enables the researcher to test the hypotheses without having to carry them out. ExtendSim has repeatedly proven its being capable of modelling large complex systems [6].

ExtendSim's design facilitates every phase of the simulation project, from creating, validating, and verifying the model, to the construction of the user interface, which allows others to analyse the system. ExtendSim products allow the researcher to build models of any size, to save changes for the existing models and to save the models themselves, print the model worksheets, and to use scripting for the automatic model. ExtendSim is a modular application. Its model is constructed with library-based iconic blocks. Each block describes a calculation or a step in the process

ExtendSim model is created by adding blocks to a model worksheet, connecting them together, and entering the simulation data. Each type of block has its own functionality, helps, icon, and connections. Each instance of a block has its own data.

Blocks perform a number of functions in a simulation model including:

- Simulating the steps in a process (*Queue, Activity*) ;
- Performing a calculation (*Math, Random Number*)
- Interfacing with other application or data storage (*Read, Write*);
- Providing a model utility (*Find and Replace, Count Blocks*);
- Plot model results (*Plotter, Histogram*);
- Tools for interface creation (*Popup, Buttons*).

The source code for all of these blocks is available and can be viewed or modified by the end user. Blocks can be created from existing blocks or created from scratch. Hierarchical blocks contain other blocks (either programmed or hierarchical). This helps to organize the model. Hierarchical blocks can be stored in a library and reused in the same or different models. ExtendSim has tools for creating an interface within the hierarchical block, making it easy for the modeller to expose important parameters and results.

ExtendSim includes a relational database for organizing and centralizing simulation information. The usage of a database in a model allows the modeller to separate the data from the model structure. This database has become a core feature in created models, large and small. In many cases, the ExtendSim model building process begins with the conceptualisation of the database. Once the database design has been completed, the model is built to support the data organization. This approach leads to more scalable and better organized models.

4. Simulation Model

The created simulation model for the supply chain "producer – wholesaler – 3 customers" is shown on Figures 4-7. The main screen of the simulation model is presented on Figure 4. Each zone of model has a numeric label. In zone 1 an executive block, that controls all discreet events in Extend models, is placed. Zones 2 and 3 contain blocks which are responsible for modelling result representation: plotter block is placed in zone 2, and in zone 3 total expenses calculation and data export to Excel spreadsheet are executed. Zone 5 contains block which is intended for scheduled transact generation, lead time and transport activity for goods transportation to the main store are simulated in blocks placed in zone 6. In the main storehouse zone there are placed: block for holding activity simulation, block for order quantity calculation, and an initialisation block that performs queue initialisation tasks before the model starting. In this situation all stocks are initialised before starting to represent a typical situation of goods quantity in warehouse.

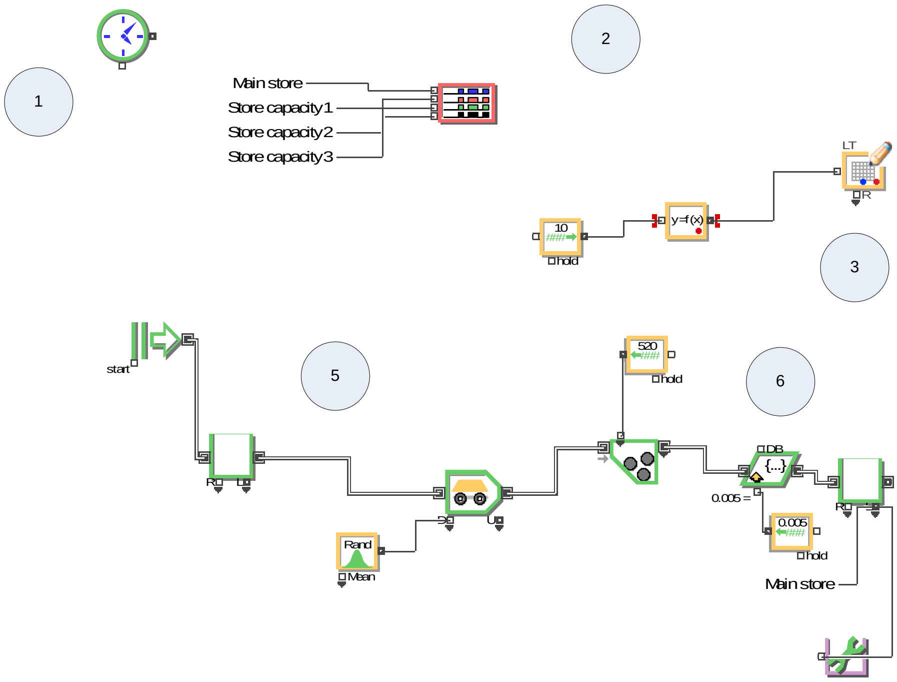

Figure 4. Main screen of the simulation model

After goods delivery to the main storehouse they are transferred to customers' storehouses according their orders. The hierarchical blocks shown on Figure 5 realize the "reorder point" strategy in goods ordering. This hieratical block is made in the way that allows using it in any necessary Extend model that needs such functionality.

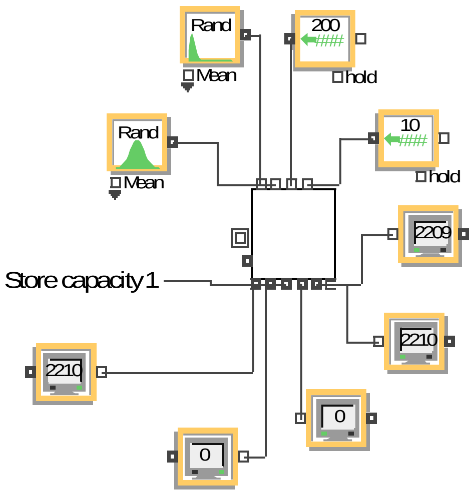

Figure 5. Reorder point store

In this model there are three identical reorder point blocks, for three customers stocks modelling. For this reason, all control parameters and results are realized as input and output connectors. The internal parameters for this type of block are: stochastic lead time of goods delivery and demand for goods, shortage, delivery and holding costs, order quantity and reorder point. Specifying these parameters we can receive appropriate results, such as: quantity of sailed goods, amount of deficit, total costs that include deficit penalty holding and delivery costs. These result parameters are used for total cost calculation.

Order quantity and reorder point are control parameters and must be changed during simulation procedure.

Using output connectors for goods quantity in stocks and plotter block, ExtendSim builds graphical representation of the dynamics of inventory level in all stocks shown on Figure 6.

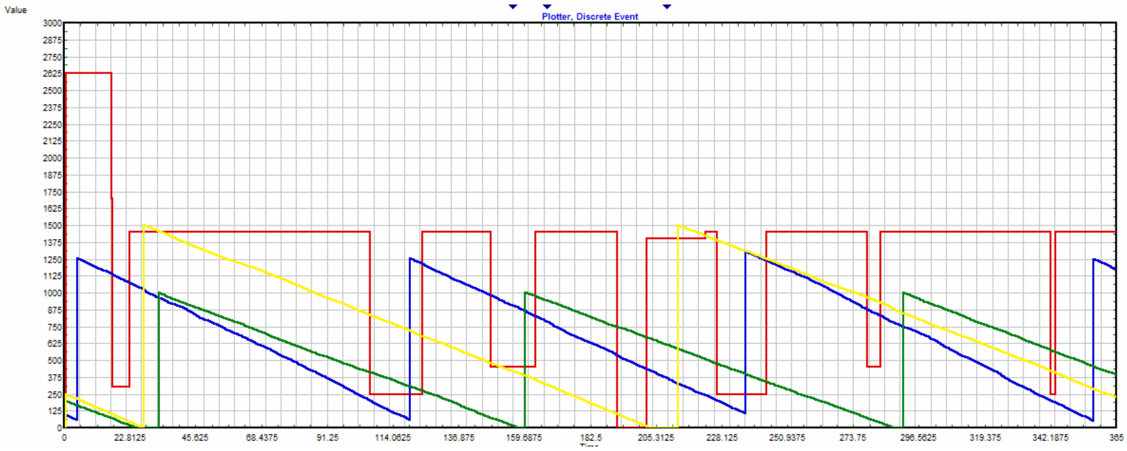

Figure 6. The dynamics of inventory level in all stocks (example of simulation of the inventory control process)

Figure 7 illustrates internal structure of Reorder point hierarchical block. Zone 7 first block is called Gate, that allows or disallows transact entrance to this part of model. Behaviour of this block is controlled by Equation block that collects information about stock level, reorder point and placed order status. Based on calculation of these parameters, Equation block send Boolean value to Gate (0 – close and 1 – open). If transact is allowed for entrance, than it is passed to activity transport block (Zone 8), after appropriate delay to the end store (Zone 9). Blocks of zone 10 are used for expenses calculation. Zone 11 is used for internal communication between hierarchical block together with ExtendSim database.

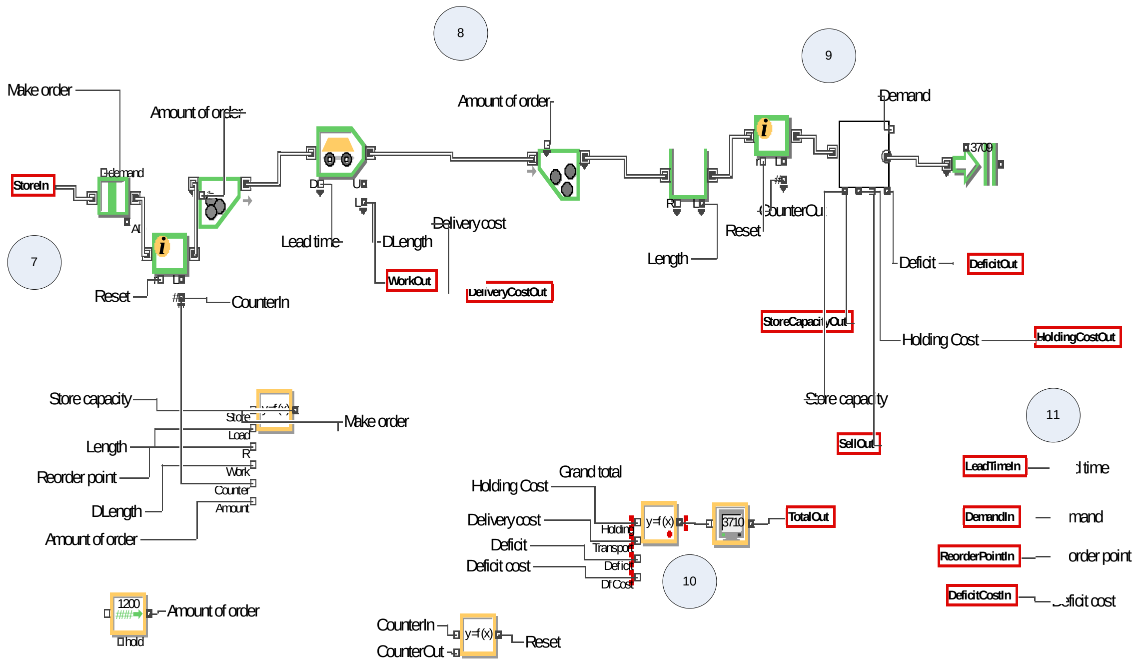

Figure 7. Reorder point hierarchical block (internal)

Structure of the final hierarchical block is shown on Figure 8. In zone 12 transacts are arriving to storehouse, where they assigned with holding cost value. Zone 13 is designed for deficit modelling with dummy supplier and appropriate attribute assigning. Zone 14 represents a market place where goods are sent to customers.

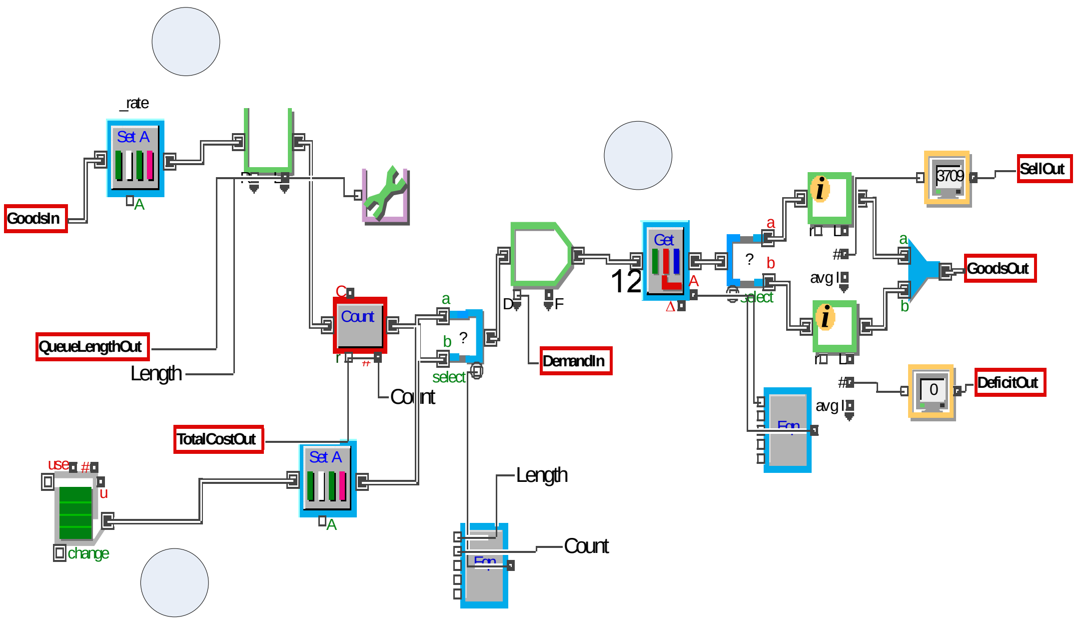

Figure 8. Hierarchical block Customer store.

For end user facilitate specialized user interface was designed. Using this interface user can change control parameters of the model and get final results of simulation. There are several tools for user interface development in ExtendSim. One of them is Notebook window that can be called from any place of model and other is cloning tool that allows clone core control elements from ExtendSim block and place it to Notebook. Example of user interface is shown on Figure 9.

5. Example

Let us consider simulation model for a stochastic single-product inventory control problem for the chain “producer – wholesaler – customer” with two stages in ordering process. Number of customers is equal 3. The customers’ demands for goods $D_i, (i = 1, 2, 3)$ are Poisson processes with intensity λ_i, and time L_i of goods delivery from wholesaler to *i*-th customer has normal distribution with parameters μ_i and σ_i (see Table 1). Ordering costs C_0 (including expenses of order transportation) for each customer are presented in Table 1.

The policy of order forming for *i*-th customer is follows. A new order is placed in the moment of time, when the stock level falls till certain level R_i. The producer supplies its production to wholesaler according fixed schedule, and time period *T* between the moments of placing neighbouring orders is constant and equals 20 days. The time L of goods delivery from producer to wholesaler has a normal distribution with a mean $\mu_L = 3$ and a standard deviation $\sigma_L = 1$. Ordering costs C_0 (including expenses of order transportation) for wholesaler equals to 900 EUR. For customers and for wholesaler holding cost C_H equals to 0,005 EUR per unit per day, losses from deficit C_{SH} equal to 10 EUR per unit. Fixed stock level in wholesaler’s warehouse S is the control parameter of the model. Initial stock in wholesaler’s warehouse is equal 4000 units. The period of simulation is one year and number of realization is 100.

Table 1. Initial data

Customer, i	Lead time, L_i, days	Demand, λ_i units/day	Order quantity, Q_i, units	Initial stock, *Z*, units	Ordering cost, EUR
1	μ_i =11; σ_i =3.5	10	1200	100	100
2	μ_i =12; σ_i =2	8	1000	200	50
3	μ_i =14; σ_i =3.7	8.57	1500	250	35

The example of Notebook’s window with initial data and results is shown on Figure 9.

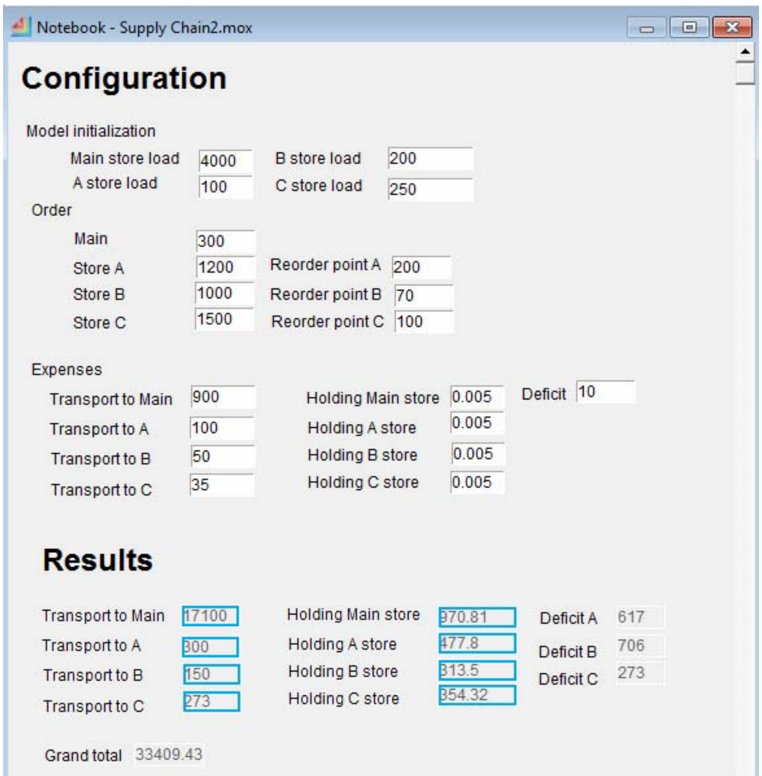

Figure 9. Notebook's window with initial data and results

The results of calculation are presented in graphical form shown on Figure 10. For the given steps (from 600 to 2000 with step 50) of control parameter *store* (stock level *S* in wholesaler's warehouse) changing the optimal result *E*=20035 EUR is achieved at the point *S*=1400 units.

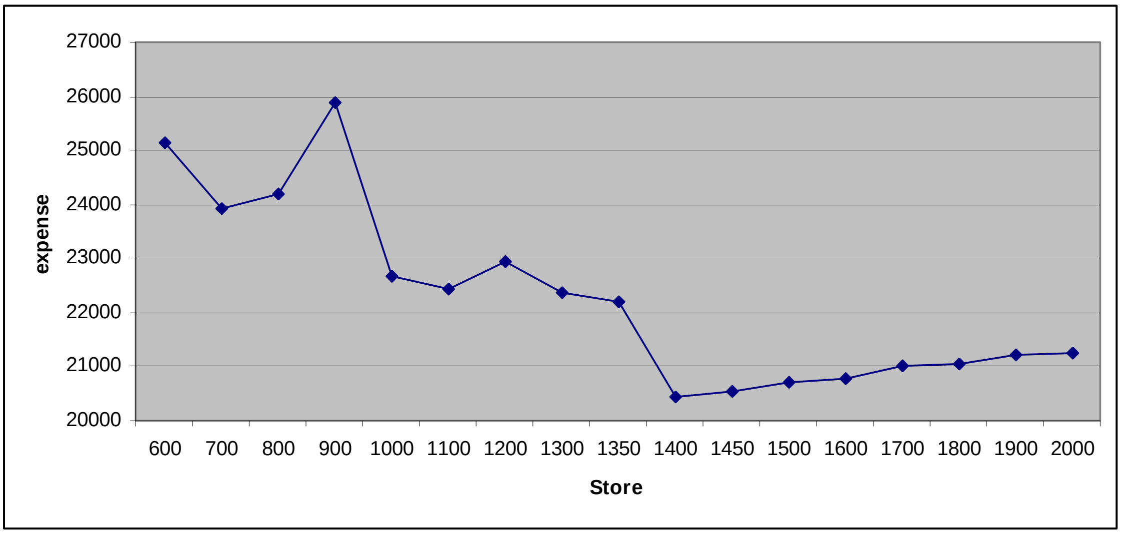

Figure 10. Average total expenses for the goods holding, ordering and losses from the deficit per year

6. Conclusions

Principal aim of the proposed supply chain model is to define the exact order quantity to achieve minimum expenses for holding, ordering goods and losses from deficit per time unit. Suggested model based on different principles of ordering, gives the closely related results near optimum solution.

The main advantages of the considered simulation method of inventory control problems solving are as follows:

- the clearness of the presentation of results; firstly, it touches the case of analysis of total expenses dependence on one control parameter with fixing others;
- the possibility of finding optimum solution of an inventory problem in the case when realization of analytical model is rather difficult;
- descriptive user interface and ability to control any necessary parameter.

In given research authors use the simulation modelling in supply chain inventory control system with two strategies of order placing. The future plan is investigation of different variants of wholesaler‘s ordering policy.

Further guidelines of the current research are the following: to investigate multi-product inventory control problem with certain constraints.

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