# DYNAMIC ADJUSTMENT OF REPLENISHMENT PARAMETERS USING OPTIMUM-SEEKING SIMULATION

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#### ABSTRACT

This paper addresses the use of discrete-event simulation and heuristic optimization to dynamically adjust the parameters within a continuous-review reorder point replenishment strategy. This dynamic adjustment helps to manage inventory and service levels in a simple supply chain environment with seasonal demand. A discrete-event simulation model of a capacitated supply chain is developed and a procedure to dynamically adjust the replenishment parameters based on re-optimization during different parts of the seasonal demand cycle is explained. The simulation logic and optimization procedure are described. Further, analysis of the impact on inventory is performed.

# **1** INTRODUCTION

Many companies, such as apparel, pharmaceutical, food, toy and industrial equipment manufacturing, experience seasonal demand patterns (Metters 1998). Seasonal demand patterns are difficult to handle in terms of managing inventory and delivery service level. In high demand seasons, more inventory is required to meet demand if delivery service levels are not to be sacrificed. In low demand seasons, less inventory is necessary to meet demand and excessive holding costs will result if inventories are not reduced. Minimizing inventory while maintaining a desired service level requires adjustment of the decision variables for the replenishment strategy. Continuous-review reorder point (*ROP*) systems are one commonly used replenishment strategy.

The timing of replenishment orders in ROP strategies is based on the current inventory position and the reorder point (Silver et al. 1998). Whenever the inventory position, which includes backorders, falls to the reorder point (OP), a new order is triggered. A fixed batch, or lot size (LS) quantity, is then ordered. Therefore, the decision variables usually associated with ROP strategies are the OP and the LS. If these variables are reset dynamically through time, based on a seasonal demand pattern, there are the additional issues of how often and when to change values. Therefore, the time period (TP) during which different OP and LS values are used becomes a third type of decision variable. Ideally these decision variables should be optimized across a seasonal demand cycle.

There are a few studies that have dealt with seasonal demand patterns but these have not concentrated on the dynamic adjustment of replenishment parameters (Johansen and Riis 1995; Metters 1997; Regasttieri et al. 2007). Dynamic adjustment using analytical models is not feasible, especially in stochastic supply chain scenarios. Simulation is a more appropriate tool for such problems. However, it appears no study using simulation has so far been reported. The present paper describes the use of optimum-seeking simulation to manage inventory under seasonal demand using dynamic replenishment parameter adjust-

ment. Reorder points and lot sizes are changed from one demand region to the next. Furthermore, readjustment is based on interfacing the simulation model with a heuristic optimization engine. Hence the methodology facilitates both dynamic adjustment as well as use of optimal, or near optimal, replenishment parameters. Optimum-seeking simulation is defined as the optimization of decision variables to maximize or minimize performance measures based on the outputs from stochastic simulations (Rogers 2002).

The next section presents the problem formulation. Section 3 describes a capacitated supply chain scenario along with the parameters and assumptions used in the model. Section 4 presents an optimum-seeking simulation model along with the optimization procedure. Section 5 discusses the simulation experiment results along with the near optimal replenishment parameters, with and without dynamic adjustment. Conclusions are presented in the last section.

#### **2 PROBLEM DESCRIPTION**

In this research the customer demand is time-varying and follows a seasonal pattern. The seasonal demand pattern for each product is described by a sinusoidal pattern with four parameters. These are the mean demand rate, amplitude, cycle length and demand pattern lag. The mathematical expression for the expected demand of product type i during any time period t can be defined as follows:

$$D_i(t) = D_i + A_i \left\{ Sin\left(\frac{2\pi}{c}(t - L_i)\right) \right\}$$
(1)

where:

 $\begin{array}{ll} i & -\operatorname{product type} \\ D_i(t) & -\operatorname{expected demand during time period } t \text{ for product type } i \\ D_i & -\operatorname{mean demand rate across a demand cycle for product type } i \\ A_i & -\operatorname{amplitude of the demand pattern for product } i \\ C & -\operatorname{cycle length of the seasonal demand pattern} \\ L_i & -\operatorname{demand pattern lag for product type } i \\ Sin() & -\operatorname{sine function expressed in radians} \end{array}$ 

This equation defines the expected customer demand. The actual demand, discussed in the next section, is defined by a Poisson process with a mean equal to the expected demand defined by Equation (1). In this research two product types are assumed (i = 1, 2).

Figure 1 shows the general pattern for a two-product example where the seasonal demand patterns are half a cycle out of phase. The number of periods in each cycle is 8784, which is typical of the number of hours in one year (366 days). Since the products are out of phase with each other, the load on the manufacturing facility will be fairly constant. However, the amount of each product produced in each part of the cycle will likely vary. This means the optimal lot sizes (*LS*) will vary. Since this affects the replenishment lead times due to capacity constraints, the optimal reorder points (*OP*) will also vary in each part of the cycle.

As shown in Figure 1, a 3-level adjustment of the replenishment decision variables is being considered. The seasonal demand cycle is divided into three regions, with region Y occurring twice per cycle. For product type 1, region X is a peak region, region Z is a trough region and region Y is a transient region. Similarly for product type 2, region X is a trough region, region Z is a peak region and region Y is a transient region. The lot sizes (LS) and order points (OP) for product type i in these three regions are designated as  $LS_i^x$ ,  $LS_i^y$ ,  $LS_i^z$ ,  $OP_i^x$ ,  $OP_i^y$  and  $OP_i^z$ , respectively.

The region in which each set of OP and LS values are used is controlled by another set of decision variables,  $TP_i^{offset}$  and  $TP_i^{prtr}$ . The variable  $TP_i^{offset}$  is the proportion a half cycle (4392 hours) at which the decision variables will change to those appropriate for region X. The value  $TP_i^{prtr}$  is the proportion of a half-cycle for which the variables appropriate for the peak or trough demand regions, X and Z, will remain in effect. In other words, this variable defines the length of region X and Z. Since the demand patterns are considered to be out of phase by half a cycle in this research, the values of  $TP_i^{offset}$  and  $TP_i^{prtr}$  are shown

equal for both product types in Figure 1. Therefore,  $TP_i^{offset}$  and  $TP_i^{prtr}$  completely define the position and lengths of regions X, Y and Z. The challenge is then to determine the optimal decision variables for both replenishment and the regions over which they will be used.



Figure 1: Dynamic adjustment of replenishment parameters under seasonal demand

### **3** SYSTEM DESCRIPTION

The capacitated supply chain used in this research, shown in Figure 2, consisted of customer demand, a finished goods warehouse, a manufacturing plant, a transport system and a replenishment order system. It was assumed that this supply chain involved two types of products. These products were not interchangeable with respect to customer demand but did have the same processing requirement characteristics, making it simpler to concentrate on the behavior under focus in this study. Unlimited raw materials for the manufacturing process were assumed to be immediately available from the supplier.



Figure 2: A capacitated supply chain scenario

The customer demand for each product type was assumed to be Poisson, meaning the interarrival times between customers followed an exponential distribution. However, the mean customer interarrival

time was dictated by the expected demand during the current time period, as stated by Equation (1). Each customer was assumed to require only one unit of either product type 1 or 2. If warehouse finished goods inventory of the required product type was in stock, the customer demand was filled immediately. Otherwise the demand was backordered and filled immediately once inventory became available. There were assumed to be no lost sales. The mean demand rate over the complete demand cycle was set to 20 per time unit for both product types, with the time units assumed to be hours. The demand pattern cycle length was assumed to be 8784 hours, approximately equivalent to one year.

The amplitude of the demand pattern for product 1 was assumed to be 40% of the mean demand rate over a cycle. This means the peak expected demand was 28 units per hour while the trough expected demand was 12 units per hour. For product 2, the amplitude of the demand pattern was assumed to be 20% of the mean demand rate. These relatively high and low levels of seasonality were selected to allow the effects of seasonality to be better analyzed. The demand patterns for the two products were offset by half the cycle length, similar to those shown in Figures 1. In other words, the lag between the patterns was 4392 hours, which is equivalent to one half year. This meant that the aggregate processing time workloads at the manufacturing plant, represented as a single resource, were fairly constant through time.

The replenishment decision variables that change through time are the reorder point (OP) and the reorder quantity, or lot size (LS), for each product type. The replenishment lot size was assumed to be the production batch size at the manufacturer as well. The inventory position was reviewed at each event where the inventory changed. The inventory position was defined as the current finished goods inventory, unfilled orders released (work-in-process) and customer backorders. The unfilled orders released included the orders being transmitted to the manufacturer, the orders waiting in queue or being processed at the manufacturer and the orders being shipped to the warehouse. Customer backorders were treated as negative values when evaluating the inventory position. Anytime the inventory position fell to the reorder point, a new order to the manufacturer was initiated. The orders being transmitted to the manufacturer incurred a delay for processing, randomly generated from a uniform distribution with parameters (0, 4) hours, and an order travel time, randomly generated from a triangular distribution with parameters (4, 8, 12) hours.

Once orders were received at the manufacturer they were processed in first-come-first-serve (*FCFS*) priority. The manufacturer was considered to have only one processing stage. A setup was required between each order (or batch), regardless of the sequence of the product types being produced. This setup time was assumed to follow a Gamma distribution with a mean of 0.25 hours and standard deviation of 0.125 hours. Each unit in the order had a deterministic processing time of 0.015 hours. These values were the same for both product types. The total batch processing time was the setup time plus the processing time per unit times the lot size (*LS*). Once an order was completed at the manufacturer it had to wait for a transporter to ship it to the warehouse. Transporters were released from the manufacturer at fixed intervals of 4 hours and could carry one or multiple lot-size orders of any product type. The downstream travel time distribution was again triangular with parameters (4, 8, 12) hours. Once the orders were received at the warehouse, they were added to finished goods inventory and deducted from the count of unfilled orders released.

# 4 OPTIMUM-SEEKING SIMULATION MODEL

This section describes the structure and logic of the discrete-event simulation model, along with the optimization procedure. The logic used for the dynamic adjustment of replenishment parameters is described but not the logic for the reorder point replenishment strategy itself.

### 4.1 Structure and Logic of simulation model

*Arena 12.0*® was used to develop the discrete-event simulation model for the supply chain scenario described above (Kelton, Sadowski, and Sturrock 2007). This simulation model was highly parametric, flexible and easy to understand. It permitted the reorder point replenishment strategies to be implemented

with and without dynamic adjustment of the replenishment parameters. It could also be interfaced with an optimization engine known as OptQuest<sup>®</sup>. Figure 3 shows a snapshot of the simulation model prepared in *Arena*<sup>®</sup>. The number on each module represents the steps in the following procedure for dynamic adjustment. Note that the delays described pertain only to the logic of changing the replenishment strategy parameters and not all the other events in the simulation models.



Figure 3: Snapshot of the dynamic adjustment simulation steps in Arena®

- *Step 1:* Create one entity using the Create module in *Arena*®.
- *Step 2:* Entity is duplicated for implementing the logic for two product types. The original entity will implement the logic for product type 1 and duplicate entity will implement the dynamic logic for product type 2.
- Step 3: Set  $LS_i$  and  $OP_i$  values equal to  $LS_i^Y$  and  $OP_i^Y$  (i.e. region Y values respectively). Two dimensional array variables were defined in *Arena*® for the *LS* and *OP* values in each region. Rows define the product type and columns define the variable value for each region. *vLotSizeArray* (1,1) and *vOrderPointArray*(1,1) set the *LS*<sub>1</sub> and *OP*<sub>1</sub> values equal to  $LS_1^Y$  and  $OP_1^Y$  values respectively. Similarly *vLotSizeArray* (2,1) and *vOrderPointArray*(2,1) set the values for product type 2. The Assign module was used for this purpose.
- Step 4: Delay for time period equal to  $TP_i^{offset} * half of sinusoidal cycle length for product type$ i. A Delay module was used for each product type. The Arena® variable vOffsetTo-Peak(1) and vOffsetToPeak(2) were used to control this timing for product 1 and 2 respectively.
- Step 5: Set the  $LS_i$  and  $OP_i$  values equal to  $LS_i^X$  and  $OP_i^X$  (i.e. region X values respectively). The Assign module was used to set these values. vLotSizeArray (1,2) and vOrderPointAr-

ray(1,2) will set the  $LS_1$  and  $OP_1$  values equal to  $LS_i^X$  and  $OP_i^X$  respectively. Similarly vLotSizeArray (2,2) and vOrderPointArray(2,2) will set the values for product type 2.

- Step 6: Delay for time period equal to  $TP_i^{prtr} * half of sinusoidal cycle length for product type$ *i*.A Delay module was used for each product type. Arena® variables vPeakDuration(1) and vPeakDuration(2) were used to control this timing for product 1 and 2 respectively.
- Step 7: Set the  $LS_i$  and  $OP_i$  values equal to  $LS_i^Y$  and  $OP_i^Y$  (i.e. region Y values). Similar settings as described in step 4 were required to assign the appropriate LS and OP values.
- Step 8: Delay for time equal to  $(1 TP_i^{prtr}) * half of sinusoidal cycle length for product type$ *i*. The Delay module was again used to implement this in the simulation model.
- Step 9: Set  $LS_i$  and  $OP_i$  values equal to  $LS_i^Z$  and  $OP_i^Z$  (i.e. region Z values) using the Assign module. vLotSizeArray (1,3) and vOrderPointArray(1,3) set the  $LS_i$  and  $OP_i$  values equal to  $LS_i^Z$  and  $OP_i^Z$  respectively. Similarly vLotSizeArray (2,3) and vOrderPointArray(2,3) set these values for product type 2.
- Step 10: Delay for time period equal to  $TP_i^{prtr} * half of sinusoidal cycle length for product type i.$
- Step 11: Set  $LS_i$  and  $OP_i$  equal to  $LS_i^Y$  and  $OP_i^Y$  (i.e. region Y values). Similar settings as described in step 4 were required.
- Step 12: Delay for time period equal to  $(1 TP_i^{prtr}) * half of sinusoidal cycle length for product type$ *i*.
- *Step 13:* Repeat this logic for each seasonal cycle.

## 4.2 Optimization model and procedure

This section presents the formulation of the optimization problem with and without dynamic adjustment of the replenishment variables. Secondly, the optimization procedure using *OptQuest*® is described.

### 4.2.1 Optimization model formulation

The optimization problem for the static reorder point strategy, without dynamic adjustment of the replenishment parameters, can be defined as follows:

$$Minimize \ TI \ (LS_i, OP_i) = \frac{1}{(t_{\varepsilon} - t_{s})} \sum_{i=1}^{2} \left\{ \int_{t=t_{s}}^{t_{\varepsilon}} (WIP_{i,t} + FG_{i,t}) dt \right\}$$
(2)

Subject to:

 $\begin{aligned} \beta_i(LS_i, OP_i) &\geq \beta_p \\ L(LS_i) &\leq LS_i \leq U(LS_i) \\ L(OP_i) &\leq OP_i \leq U(OP_i) \\ LS_i \text{ and } OP_i \text{ are integers} \end{aligned}$ 

where,

ΤI	- total inventory in the system
$WIP_i$	- work-in-process inventory for product type <i>i</i>
$FG_i$	- finished goods inventory for product type <i>i</i>
$t_s$	- start of simulation data collection
$t_e$	- end of simulation data collection
$\beta_i$	- actual service level for product type <i>i</i>
$\beta_p$	- pre-defined, or target, service level

$LS_i$	- lot size for product type <i>i</i>
$OP_i$	- order point for product type <i>i</i>
$L(LS_i)$	- lower search bound for lot size for product type <i>i</i>
$U(LS_i)$	- upper search bound for lot size for product type <i>i</i>
$L(OP_i)$	- lower search bound for order point for product type $i$
$U(OP_i)$	- upper search bound for order point for product type $i$

Equation (2) seeks to minimize the time-averaged inventory in the system across the two products types, *i*. This is done subject to meeting a customer delivery service level constraint,  $\beta_p$ . In this research the service level,  $\beta$ , is defined as the proportion of customer orders filled immediately from finished goods inventory stock at the warehouse. This is often also referred to as the fill rate. Since the lot size  $(LS_i)$  and reorder point  $(OP_i)$  decision variables are integer, it is not be possible to obtain the target level constraint,  $\beta_p$ , exactly. Therefore the actual service level observed,  $\beta_i$ , will be slightly higher than the constraint.

Similarly, the optimization problem for the dynamic adjustment strategy, shown in Figure 1, was defined using the following:

$$Min TI\left(LS_i^X, LS_i^Y, LS_i^Z, OP_i^X, OP_i^Y, OP_i^Z, TP_i^{offset}, TP_i^{prtr}\right) = \frac{1}{(t_e - t_s)} \sum_{i=1}^2 \left\{ \int_{t=t_s}^{t_e} (WIP_i + FG_i) dt \right\}$$
(3)

Subject to:

$$\begin{split} & \beta_i \left( LS_i^X, LS_i^Y, LS_i^Z, OP_i^X, OP_i^Y, OP_i^Z, TP_i^{offset}, TP_i^{prtr} \right) \geq \beta_p \\ & L \left( LS_i^X \right) \leq LS_i^X \leq U \left( LS_i^X \right) \\ & L \left( LS_i^Y \right) \leq LS_i^Y \leq U \left( LS_i^Y \right) \\ & L \left( LS_i^Z \right) \leq LS_i^Z \leq U \left( LS_i^Z \right) \\ & L \left( OP_i^X \right) \leq OP_i^X \leq U \left( OP_i^X \right) \\ & L \left( OP_i^Y \right) \leq OP_i^Y \leq U \left( OP_i^Y \right) \\ & L \left( OP_i^Z \right) \leq OP_i^Z \leq U \left( OP_i^Z \right) \\ & 0 \leq TP_i^{offset} \leq 0.5 \\ & 0 \leq TP_i^{prtr} \leq 1 \end{split}$$

where  $LS_i^X$ ,  $LS_i^Y$  and  $LS_i^Z$  are the lot sizes and  $OP_i^X$ ,  $OP_i^Y$  and  $OP_i^Z$  are the reorder points for region X, Y and Z respectively. These decision variables, as well as the lower and upper bounds for the search, L() and U(), are again integer values.  $TP_i^{offset}$  and  $TP_i^{prtr}$  are the decision variables defining where regions X, Y and Z start and end.

#### 4.2.2 Optimization procedure in OptQuest®

*OptQuest*® is a heuristic search procedure designed to find optimal parameters, or decision variables, with respect to a defined objective function. *OptQuest*® can be integrated with *Arena*® to perform simulation optimization. Once the simulation model is created and the optimization problem is formulated, it is easy to perform simulation optimization. Key issues involved in setting up the optimization model are the selection of control parameters, the objective function and constraints. The procedure to set up the optimization problem with dynamic adjustment is shown in Figure 4 and described below:

Step 1: Set decision the variables, such as  $LS_i^X, LS_i^Y, LS_i^Z, OP_i^X, OP_i^Y, OP_i^Z, TP_i^{offset}, TP_i^{prtr}$ . *vLotSizeArray* (...), *vOrderPointArray* (...), *vOffsetToPeak(.)*, *vPeakDuration(.)*. Figure 4 shows the variables under their *OptQuest*® control name.

- Step 2: *OptQuest*® requires the specification of lower, suggested, and upper values for the variables that are to be optimized. These facilitate constraining the search space and making the search more efficient. Define the lower bounds, upper bounds, step of increment, lot size, and order points as integers. As well, the timing variables should be set to change in discrete steps.
- Step 3: Set the response variables, such as inventory and service levels.
- Step 4: Define the constraints such as:

 $\beta_1 \geq \beta_p$  and  $\beta_2 \geq \beta_p$ 

In Figure 4, *SL1* and *SL2* are the service levels for product 1 and 2 respectively.

- Step 5: Set the objective function, which is to minimize the total inventory.
- Step 6: Run the optimum-seeking simulation. Since the procedure is heuristic, the solution may not be optimal but should be near optimal.

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	vi otSizeArray[1 1]	58	51			
	vLotSizeArray[1,1]	54	52			
	vLotSizeArray[1,3]	38	39			
	vLotSizeArray[2,1]	38	39			
	vLotSizeArray[2,2]	40	46			
	vLotSizeArray[2,3]	50	47			
	vOffsetToPeak[1]	0.100000	0.080000			
	vOffsetToPeak[2]	0.100000	0.140000			
	vOrderPointArray[1,1	J 488	498			
	vOrderPointArray[1,2	1 410	412			
	vOrderPointArray[2,1	1 470	465			
	vOrderPointArray[2,2	443	433			
	vOrderPointArray[2,3	507	507			
	vPeakDuration[1]	0.800000	0.810000			
	vPeakDuration[2]	0.800000	0.780000			
	Cons	traints				
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Figure 4: A snapshot of the optimization model setup in OptQuest®

# 5 **RESULTS AND DISCUSSION**

This section first discusses the adjustment of optimal lot sizes and order points. Further, the behavior of inventory levels with and without the dynamic adjustment of replenishment parameters are discussed. Finished goods, work-in-process and total inventory are examined.

# 5.1 Replenishment parameter adjustment

The optimal reorder points and lot sizes, based on minimizing total inventory subject to a target service level of 85% deliveries from stock, were determined. The optimal reorder points without the dynamic adjustment were found to be 586 and 497 for product types 1 and 2, respectively. The optimal lot sizes were found to be 46 and 60 for product types 1 and 2, respectively. For the dynamic strategy, the decision variables were reset four times during each demand cycle for each product. The optimal reorder points for product type 1 were found to be 605, 491 and 358 for the regions of high, transient and low demand respectively. The following shows the order points and switching points.

 $OP_1 = \left\{ \begin{array}{ll} 491, & 0 \leq t < 395.28, 3908.88 \leq t < 4787.28, 8300.88 \leq t < 8784, \\ 605, & 395.28 \leq t < 3908.88 \\ 358, & 4787.28 \leq t < 8300.88 \end{array} \right.$ 

where *t* is the current simulation time in hours and is represented for one seasonal cycle of one year.

For product type 2 the optimal reorder points were 510, 473 and 440 for the regions of high, transient and low demand respectively.

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OP_2 = \left\{ \begin{array}{ll} 473, & 0 \leq t < 395.28, 3908.88 \leq t < 4787.28, 8300.88 \leq t < 8784, \\ 440, & 395.28 \leq t < 3908.88 \\ 510, & 4787.28 \leq t < 8300.88 \end{array} \right.
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The optimal lot sizes for product type 1 were 55, 44 and 34 for the regions of high, transient and low demand respectively.

 $LS_1 = \begin{cases} 44, & 0 \le t < 395.28, 3908.88 \le t < 4787.28, 8300.88 \le t < 8784, \\ 55, & 395.28 \le t < 3908.88 \\ 34, & 4787.28 \le t < 8300.88 \end{cases}$ 

The optimal lot sizes for product type 2 were 50, 34 and 36 for the regions of high, transient and low demand respectively.

 $LS_2 = \begin{cases} 34, & 0 \le t < 395.28, 3908.88 \le t < 4787.28, 8300.88 \le t < 8784, \\ 36, & 395.28 \le t < 3908.88 \\ 50, & 4787.28 \le t < 8300.88 \end{cases}$ 

The high, transient and low regions for product type 1 correspond to regions X, Y and Z respectively in Figure 1. Similarly, the high, transient and low regions for product type 2 correspond to regions Z, Y and X respectively. The optimal switching parameters for the dynamic adjustment,  $TP_1^{offset}$  and  $TP_2^{offset}$ , were both found to be 0.09 proportion of a half cycle. Similarly,  $TP_1^{prtr}$  and  $TP_2^{prtr}$ , were both found to be 0.80 proportion of a half cycle. This means the proportion of the full demand cycle covered by regions X and Z were each 0.40, while the two transient regions, Y, were each 0.10 of a full cycle.

### 5.2 Analysis of Inventory with and without dynamic adjustment

This section presents the analysis of total inventory, finished goods and work-in-process inventory with and without the dynamic adjustment of the replenishment parameters. The values of work-in-process inventory, finished goods inventory and total inventory were collected for two complete seasonal cycles at intervals of 10 hours. This resulted in 1761observations for each measure. These observations were then plotted as a 10-period moving average to smooth out fluctuations and more clearly present the performance measure trends. The left hand axis in the following figures provides reference for the inventory counts of each product type. These figures also show the mean demand rate, referenced on the right-hand axis, as a smooth solid line.

Figures 5 (a) and (b) show the behavior of finished goods inventory without and with dynamic adjustment of replenishment parameters. The finished goods inventory fluctuates with a pattern opposite to demand since inventories will be drawn down in periods of high demand. It is observed that the finished goods inventory becomes more stable through time with dynamic adjustment. The average finished goods inventory also decreased drastically. As well, it is observed that the reduction in finished goods inventory is greater for the highly seasonal product.



Figure 5: Behavior of finished goods inventory

Figures 6 (a) and (b) show the behavior of work-in-process inventory without and with dynamic adjustment of replenishment parameters. Work-in-process inventory was defined to include orders in transit or at the capacity-constrained resource. These graphs indicate that the work-in-process inventory fluctuated quite consistently with the demand. In other words, the patterns are similar and there was little effect of dynamic adjustment on work-in-process inventory.





(b) With dynamic adjustment

Figure 6: Behavior of work-in-process inventory

The behavior of the total inventory levels is shown Figures 7 (a) and (b). The net result of summing the finished goods and work-in-process inventory is that total inventory fluctuates in a pattern opposite to the demand. In other words, the total inventory for each product type tends to be lowest at high demand.

The optimal decision variables were also run for 5 independent replications in a separate simulation experiment to validate performance. Table 1 summarizes the finished goods inventory (*FGI*) performance, including the standard deviations (*SD*) and confidence interval half-widths (*HW*). Table 2 summarizes the total inventory (*TI*) performance. As well, the actual service levels were checked to confirm they

matched the target levels used during simulation-optimization. The observed 95% confidence intervals across the five replication were found to  $85.18\pm0.14$  and  $85.06\pm0.15$  with and without dynamic adjustment, respectively. These values are very close to the 85% target level.



#### (a) Without dynamic adjustment



Figure 7: Behavio	of total inventory
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	Without Dynamic Adjustment			With Dynamic Adjustment			
	FGI product 1	FGI Product 2	Total FGI	FGI Product 1	FGI Product 2	Total FGI	
Average	183.52	99.38	282.91	78.87	69.78	148.65	
SD	0.20	0.14	0.29	0.29	0.16	0.40	
HW	$\pm 0.50$	$\pm 0.35$	$\pm 0.71$	$\pm 0.71$	$\pm 0.39$	$\pm 1.00$	
Minimum	183.24	99.26	282.55	78.57	69.64	148.35	
Maximum	183.74	99.57	283.16	79.31	70.03	149.34	

#### Table 1: Finished goods inventory statistics

Table 2: Total inventory statistics

	Without Dynamic Adjustment			With Dynamic Adjustment			
	Total Inventory	Total Inventory	Total	Total Inventory	Total Inventory	Total	
	Product 1	Product 2	Inventory	Product 1	Product 2	Inventory	
Average	414.66	332.40	747.06	310.83	299.87	610.69	
SD	0.15	0.16	0.29	0.08	0.09	0.06	
HW	$\pm 0.37$	$\pm 0.40$	$\pm 0.73$	$\pm 0.20$	$\pm 0.24$	$\pm 0.16$	
Minimum	414.44	332.21	746.65	310.72	299.75	610.60	
Maximum	414.81	332.64	747.42	310.92	299.97	610.76	

### **6 CONCLUSIONS**

This paper has demonstrated the application of discrete-event simulation for the dynamic adjustment of replenishment parameters under a seasonal demand pattern. Re-optimization in different seasonal demand regions with the use of a heuristic-based optimization engine.

The results show it is advantageous to adjust and re-optimize replenishment parameters, such as reorder points and lot sizes, for different regions in a seasonal demand cycle. It was found that the dynamic adjustment resulted in reductions in total inventory and finished goods inventory of 18.3% and 47.5% respectively. The dynamic adjustment also leads to more stable inventory levels through time.

This approach could also be applied for other replenishment strategies, such as a Kanban strategy. As well, it could be generalized to handle different kinds of seasonal patterns.

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