

## BOUDON'S MODEL OF RELATIVE DEPRIVATION REVISITED

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*The less limited one feels, the more intolerable all limitation  
appears*

*Lack of power, compelling moderation, accustoms men to it,  
while nothing excites envy if no one has superfluity.*

Durkheim, *Suicide*

(trans. John A. Spaulding and George Simpson. New York,  
The Free Press, Simon and Schuster Inc., 1951, p. 254)

I would like to pay homage to Raymond Boudon by returning to his analysis of the phenomenon of *relative deprivation* (Boudon 1982 [1977]: chap. 5; 1979: 52–56).

This analysis is both a synthetic and paradigmatic application of two of Boudon's fundamental contributions to “*sociology that really matter*” (Boudon 2002): a) the idea that sociology's explanatory power increases when we develop and use formal models of the mechanisms that generate the link between individual behaviours and aggregate regularities; b) the idea of studying such models by simulating them.<sup>1</sup>

The recent movement to institutionalize two sociological approaches — analytical sociology (Cherkaoui 2005, Hedstrom 2005) and computational sociology (Hummon & Fararo 1995, also Fararo's article here) — attests to the importance of these two ideas. The computer modelling method called multi-agent systems (Gilbert 2007) has been favourably

noted recently in computational sociology, as well as in some other fields (Epstein 2006), for its conceptual flexibility and computational power (Macy & Willer 2002, Sawyer 2003). I have applied this method here to reformulate and re-examine the hypotheses formalized by Raymond Boudon to account for the genesis of the phenomenon of *relative deprivation*.

I first situate this contribution within the contexts of classic sociology on the subject and more recent psychology and economics literature (Section 1). I then present the basic multi-agent system configuration I constructed using Boudon's original model (Section 2). Lastly, I discuss the results obtained by simulating this artificial society on several parameter combinations (Sections 3–5).

My main purpose is to demonstrate that the relation between the structure of objective opportunities available to actors and the quota of “dissatisfied” actors may take several forms, at least according to the mechanisms designed in my reformulation of Boudon's model. Depending on the number of lots, how attractive they are, and population size, that relation can actually be of several types: “*more opportunities, fewer satisfied actors*”; “*more opportunities, more satisfied actors*”; a combination of the two.

As I indicate in the conclusion, one advantage of these computational results is that they open up a new perspective on the different forms that econometric studies have been imputing for nearly 50 years to the empirical relationship between a country's wealth and the proportion of satisfied individuals in it.

## 1. WHAT EXACTLY DOES BOUDON'S MODEL MODEL?

The empirical observations that gave rise to the sociological literature on *relative deprivation* (hereafter noted RD) all noted an inverse relation between actors' perceptions of the conditions they act in and the “objective” quality of those conditions (Tocqueville 1967 [1856]: bk. III, chap. 4, p. 276, Durkheim 2004 [1987]: bk. II, chap. 5, p. 267, Stouffer *et al.* 1949: vol. I, p. 250, Runciman 1966: 3).

Stouffer and his colleagues (1949: vol. I, pp. 52, 125) were the first to use the concept explicitly to explain this seemingly paradoxical correlation. The hypothesis implicit in this “interpretative intervening

variable,” as Merton described it (1957: 229), is that actors’ assessments of their objective opportunities actually depend on their standards of comparison (Stouffer *et al.* 1949: vol. I, p. 125).<sup>2</sup> While empirical observation of a linear inverse relationship between objective opportunity structure and people’s perceptions of those opportunities was what first motivated the use of the RD concept, the problem of how general that relation is has not yet been resolved.

In sociology, this point was mentioned in passing by Merton (1957: 237, n. 7): “presumably, the relationship is curvilinear, and this requires the sociologists to work out toward the conditions under which the observed linear relation fails to obtain.” Runciman took up the point nearly ten years later (1966: 19–20): “this relation is both complicated and variable. ... it can as well take the form of an inverse correlation as a direct one” (*ibid.*, p. 247). In economics, the Easterlin paradox holding that “raising the incomes of all does not increase the happiness of all” (Easterlin 1973: 4) has been repeatedly re-analyzed (cf. Clark *et al.* 2008) to demonstrate that a positive relation between income and life satisfaction/happiness does exist, not only at the individual level but also at the aggregate level and not only within a given country but also among countries (Wolfers and Stevenson 2008).

Constructing formal models of the mechanisms that generate RD and analyzing the results deductively is unquestionably an attractive way of trying to resolve the difficulties raised by the “empirical solution” (Clark *et al.* 2008: 111–115), because it enables us to establish all the outcomes logically associated with a given mechanism (or several) and then compare this range of possibilities with the specific zone of the real covered by the empirical data under study.

However, the “theoretical-deductive” solution requires at least two preliminary specifications. First, it is essential to clarify the point of view from which the RD phenomenon is being studied; second, it is essential to have a map of the basic components used to construct the RD generating mechanisms (on this point see Gambetta 1998: 114–119).

On the first point, it is useful, following Runciman (1966: 10), to distinguish between RD frequency — i.e., the quota of actors who do not have what they want — and RD degree: the intensity of the feeling actors associate with this discrepancy (see also Elster 2007: 58). This suggests that the mechanisms that move a certain number of actors to perceive a discrepancy between reality and their desires may be different

from those that engender their specific reactions to this assessment. From this in turn it follows that the relations between objective conditions of well-being and subjective perceptions of those conditions can take different forms depending on which aspect of RD is under study and the type of mechanisms mobilized.<sup>3</sup>

Moreover, these mechanisms can be inscribed in a basic analytic space using axes that correspond to the comparison reference points that actors choose. Two main types have been identified (Tyler *et al.* 1997: ch. 2): a) actor-specific reference points, namely one's past condition or expectations (intrapersonal comparisons) and b) reference points external to the actor, namely other individuals or groups (interindividual and intergroup comparisons).<sup>4</sup>

Type a are present in the writings of Tocqueville (1967 [1857]: 278) and Durkheim (2004 [1897]: 274, 282); Durkheim also partially acknowledged type b.<sup>5</sup> "Social" comparisons are at the heart of *The American Soldier*, and this was the source for Merton's analysis (1957, chs. 7 and 8) of the "reference group" concept. Runciman (1966: 24–25) combined the two, positing a loop between a rise in individual expectations and a rise in reference group level.

What distinguishes Raymond Boudon's contribution to this sociological literature is formalization. Boudon put forward hypotheses about a combined set of rules, individual reasoning and interdependence structure that would lead a certain number of actors to rationally hope to obtain more than they could objectively obtain. This mechanism generates a situation of "aspirational deprivation" (Gurr 1970: 51): actors' expectations rise but their ability to satisfy them remains the same.

Boudon's RD model, then, in terms of the above distinction between RD-frequency and RD-degree, made it possible to analyze the genesis of the aggregate relationship between the objective opportunities structure that actors have access to and RD-frequency. However, as he formulated it, it does not allow for formally studying the relation between these objective conditions and individual feelings of satisfaction/dissatisfaction; i.e., RD-degree. In fact, Boudon implicitly admitted the existence of intrapersonal and interindividual comparisons that produce feelings of disappointment and envy, but these elements were not quantified and the comparisons underlying them were not formally represented.<sup>6</sup>

## 2. AN AGENT-BASED IMPLEMENTATION OF BOUDON'S RD-FREQUENCY MODEL

Kosaka (1986: 36, 37) harshly criticized Boudon's model in his analytic study of it for not going "beyond a mere illustration of the phenomena with some numerical examples." I suggest that we are now in a position to analyze the genesis of the RD phenomenon as Boudon hypothesized it, this time using a computer method that, while fully consistent with a "numerical examples" approach, also enables us to move forward in two directions. In conceptual terms, a multi-agent system allows for creating stylized actors who reason and choose and may interact in a given context according to specific rules; in computational terms, it enables us to study the outcomes produced by these sorts of artificial microcosms for an extremely high number of parameter combinations.<sup>7</sup>

To study Boudon's model, I wrote a computer program in NetLogo that uses all his basic analytic components: 1) an "institutional" structure in which actors act; 2) an individual reasoning; 3) an individual choice; 4) a procedure for allocating available resources.<sup>8</sup>

- 1) *Agents' action context* is specified by the following elements: a) a population of  $N$  agents; b) these agents compete to obtain limited numbers of two types of lots,  $L1$  and  $L2$ ; c) the sum of  $L1$  plus  $L2$  is always equal to  $N$ ; d)  $L1$  and  $L2$  differ in attractiveness in the sense that the benefit  $B1$  ( $> 0$ ) associated with  $L1$  is higher than the benefit  $B2$  ( $\geq 0$ ) associated with  $L2$ ; e)  $L1$  and  $L2$  also differ in accessibility in the sense that  $L1$  can only be obtained if agent spends  $C1$  ( $> 0$  and  $< B1$ ) whereas  $L2$  can only be obtained if agent spends  $C2$  ( $\geq 0$ ,  $\leq B2$  and  $< C1$ ); f) all agents have enough resources to be able to spend  $C1$  or  $C2$ .
- 2) *Agent reasoning* is constructed around the following elements: a) each agent knows the number of  $L1$  and  $L2$  lots available in society but does not know the number of agents  $A(S1)$  and  $A(S2)$  who will respectively adopt strategy  $S1$  (spending  $C1$  to obtain  $B1$ ) or strategy  $S2$  (spending  $C2$  to obtain  $B2$ ); b) each agent must therefore estimate the gain expected from  $S1$  ( $G[S1]$ ) compared to the gain expected from  $S2$  ( $G[S2]$ ) as a function of the number of agents likely to opt for  $S1$  (and therefore the number likely to opt for  $S2$ ); d) each agent therefore has to deal with two situations:

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d.1) For each  $A(S1) < \text{number of L1 lots}$ ,  $G[S1]$  remains intact whereas  $G[S2]$  will be cut proportionally to the surplus of agents assumed to have opted for  $S2$ :

$$G[S^1] = B^1 - C^1$$

$$G[S^2] = (B^2 - C^2) \times \frac{\text{number of L2 lots}}{A(S^2)}$$

d.2) For each  $A(S1) > \text{number of L1 lots}$ ,  $G(S2)$  remains intact whereas  $G[S1]$  will be cut proportionally to the surplus of agents assumed to have chosen  $S1$ , a loss partially compensated for by the gain from benefit  $B2$  associated with the less attractive  $L2$  lots, the probability of this compensation increasing with surplus of agents assumed to have opted for  $S1$ :<sup>9</sup>

$$G[S^1] = (B^1 - C^1) \times \frac{\text{number of L1 lots}}{A(S^1)} + (B^2 - C^1) \times \frac{A(S^1) - \text{number of L1 lots}}{A(S^1)}$$

$$G[S^2] = B^2 - C^2$$

e) for each  $A(S1)$ , each agent will choose  $S1$  if

$$G(S1) - G(S2) > r,$$

$r$  being the minimum gain demanded.

- 3) *Agent's choice.* Given the vector of choices for or against  $S1$  produced by the reasoning above, the probability of the agent ultimately deciding for or against  $S1$  increases non-linearly as a function of the proportion of cases in which agent chooses  $S1$ . Specifically, I chose a logistic function discretized by 10-unit intervals.<sup>10</sup>
- 4) *Allocation of available resources.* Once agents have shown their definitive preference for  $S1$  or  $S2$ ,  $L1$  and  $L2$  lots have to be allocated to the different players. There are three possible situations:
  - a) If the number of agents who definitively opted for  $S1$  is exactly equal to the number of  $L1$  lots, all agents are satisfied: those who wanted  $L1$  got  $L1$ ; the others, who wanted  $L2$ , got  $L2$ .

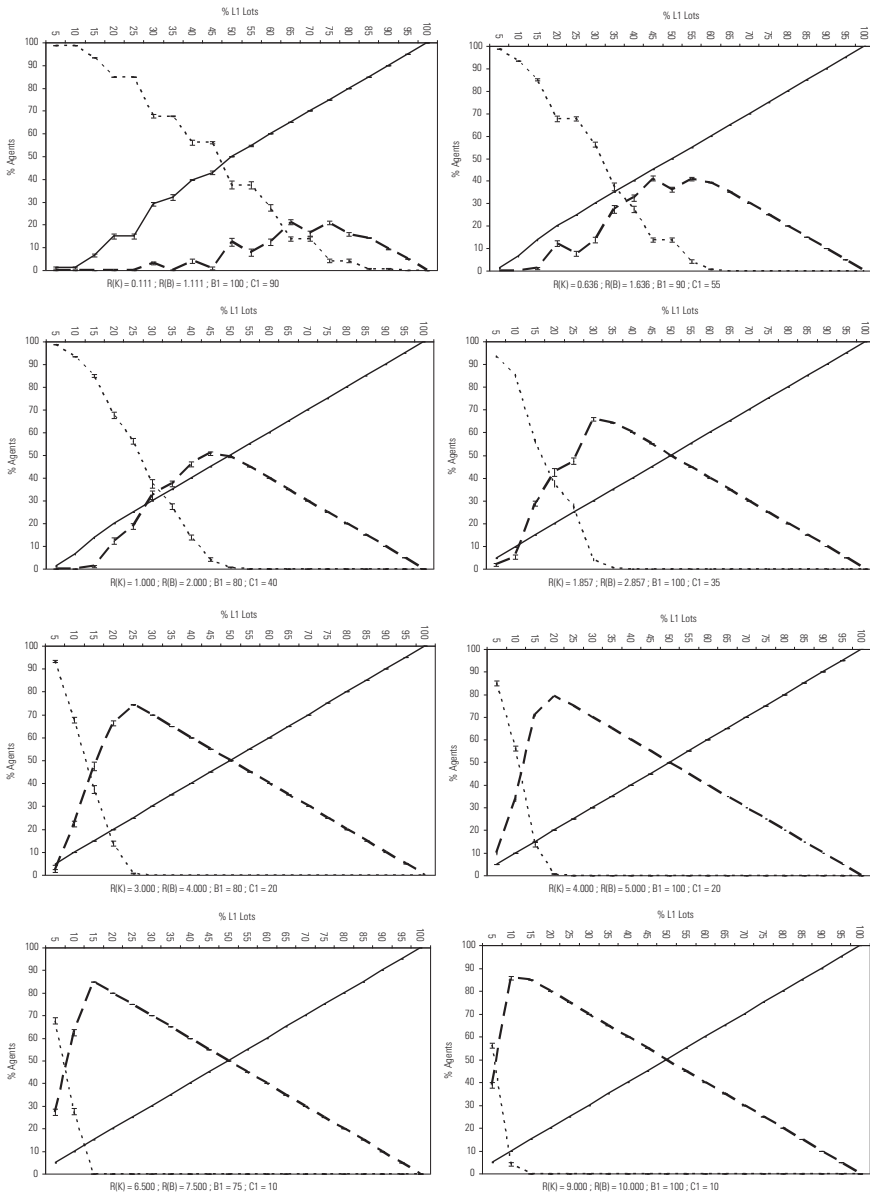
- b) If the number of agents who definitively opted for S1 is greater than the number of L1 lots, some of the agents who spent C1 to obtain B1 will actually only be able to obtain B2. As there are no individual or social screening traits, agents receiving the lesser benefit at the higher cost were determined by random selection.
- c) If the number of agents who ultimately opt for S1 is below the number of L1 lots, the number of agents opting for S2 will be above the number of available L2 lots.<sup>11</sup> Given that the game rule stipulating that B1 cannot be obtained by spending only C2 precludes allocating L1 lots to these agents, the simplest solution is to randomly allocate a zero-gain to the surplus of agents desiring L2.

Thus programmed, Boudon's model can now engender not one but two types of RD (hereafter indicated as RD1 and RD2) whose frequency can now be studied (respective frequencies hereafter indicated RD1-freq and RD2-freq). The first type affects agents who, having chosen L<sup>1</sup>, only got L2 because there were not enough L1 lots. Their reference group is of course made up of agents who spent C1 like them but did obtain B1. The second type of RD affects agents who wanted L2 but in fact got nothing given the rules of the game and because there were not enough L2 lots. Their reference group is clearly agents who spent C2 like them but did get B2.

In the following three sections, I systematically study variations in RD1-freq and RD2-freq as a function of two parameters: a) attractiveness of S1 compared to S2; b) "wealth" of objective opportunities available to agents. The first measure consists of two indices proposed respectively by Boudon (1982 [1977]: 118) and Kosaka (1986: 38) and here indicated respectively  $R(B) = (B^1 - C^1) / (B^2 - C^2)$  and  $R(K) = [(B^1 - C^1) - (B^2 - C^2)] / (C^1 - C^2)$ . The second measure is simply the percentage of L1 lots present in the artificial society.<sup>12</sup>

My point is to demonstrate that within the artificial microcosm driven by the mechanisms just described, the inverse relationship between objective conditions for well-being and subjective perceptions of those conditions is confined to a limited area of the model's parameter space.<sup>13</sup>

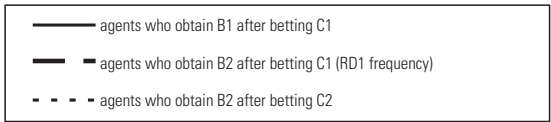
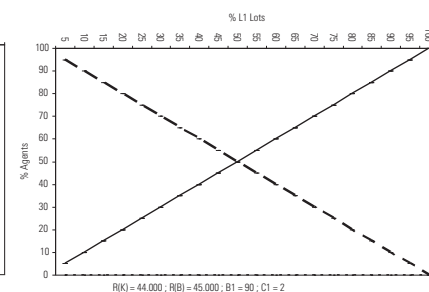
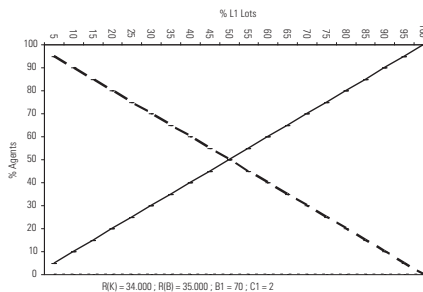
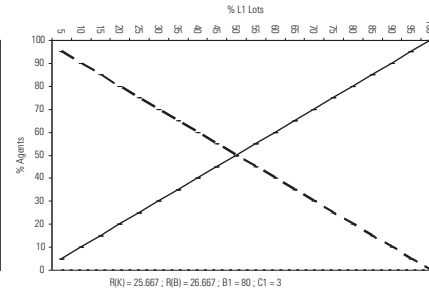
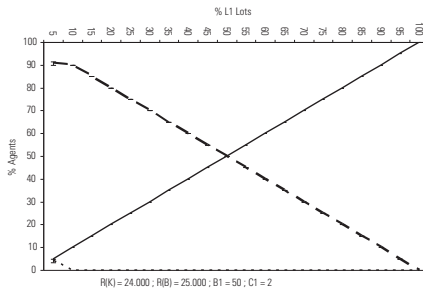
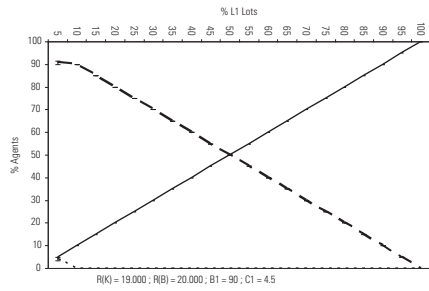
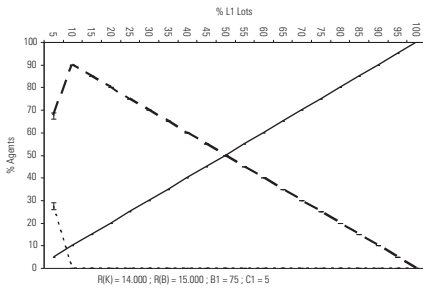
### PART THREE ON GENERATIVE MECHANISMS



**Figure 1:** For a typical series of  $R(K) - R(B)$  values (i.e. attractiveness of  $S1$  compared to  $S2$ ), percentages (95% confidence intervals) of agents who finally obtain  $B1$  or  $B2$  after betting  $C1$  or  $C2$  (y-axis) as a function of the percentage of available  $L1$  lots (x-axis) [median absolute difference  $B1 - C1$ : 66.5].



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### PART THREE ON GENERATIVE MECHANISMS

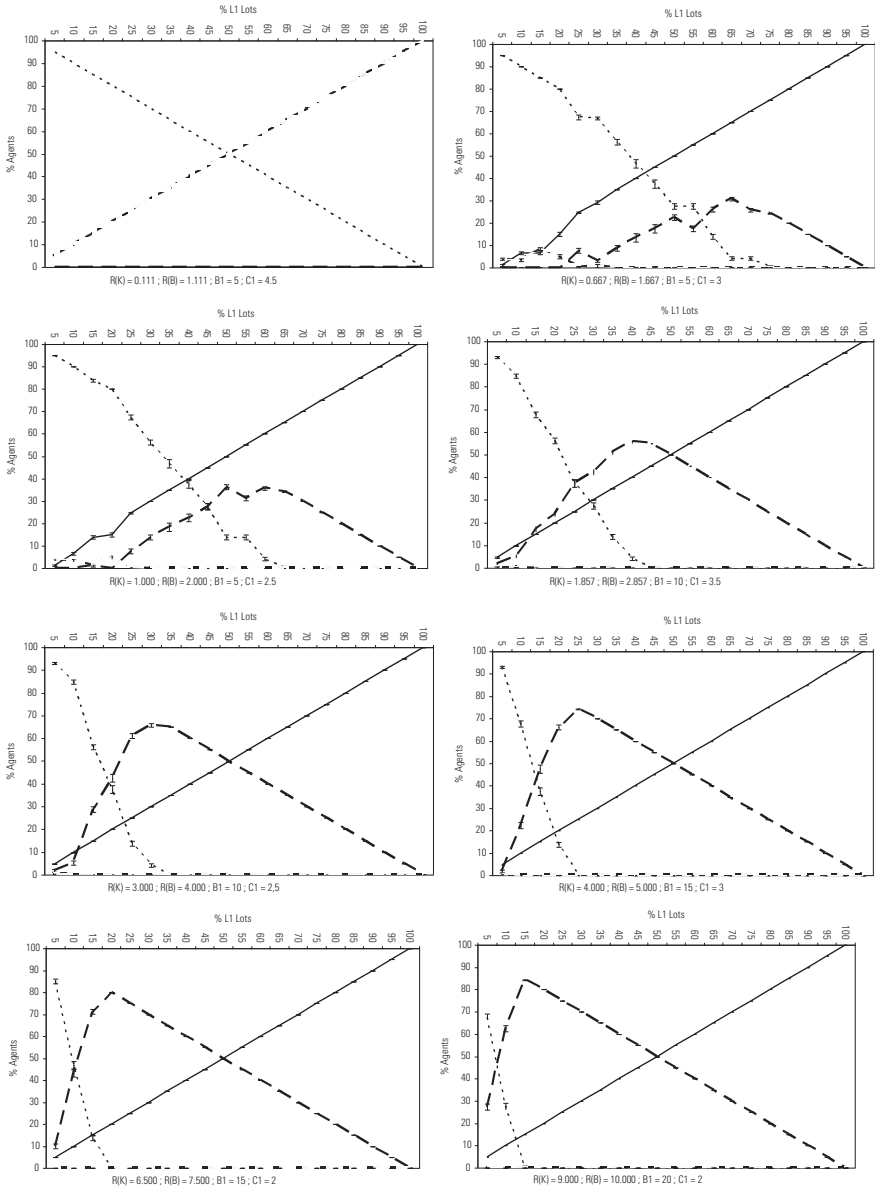
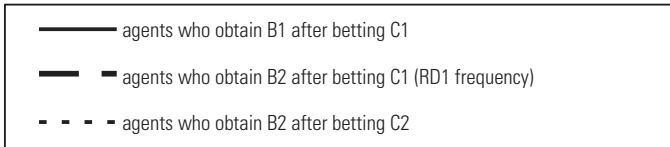
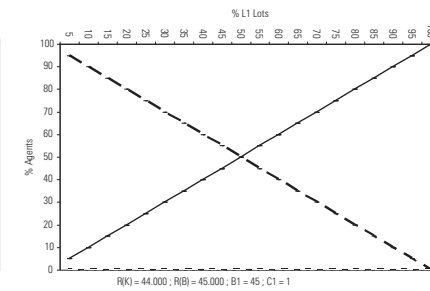
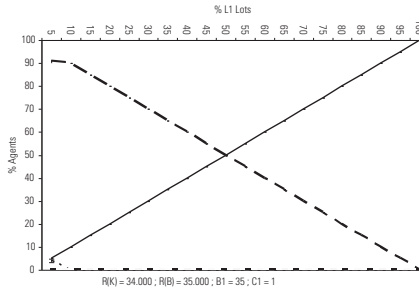
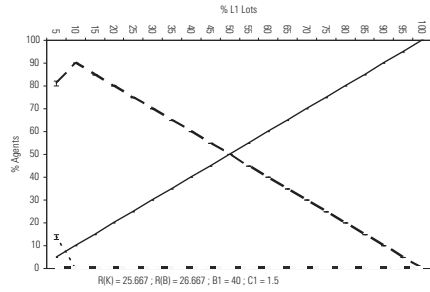
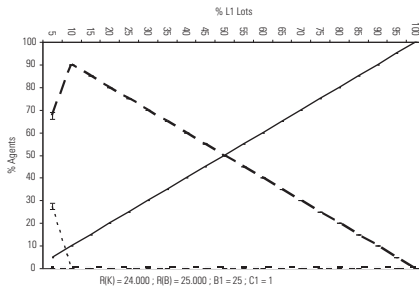
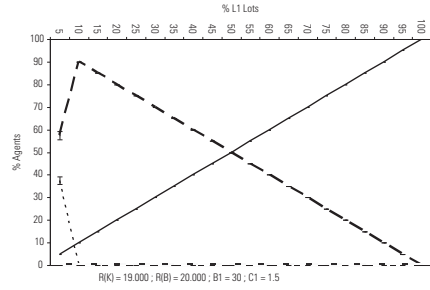
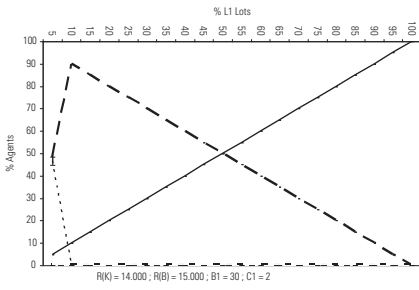


Figure 2: For a typical series of  $R(K)-R(B)$  values (i.e. attractiveness of  $S1$  compared to  $S2$ ), percentages (95% confidence intervals) of agents who finally obtain  $B1$  or  $B2$  after betting  $C1$  or  $C2$  (y-axis) as a function of the percentage of available  $L1$  lots (x-axis) [median absolute difference ( $B1 - C1$ ): 15.5]

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### 3. GENERATING RD-FREQUENCY PATTERNS: THE “ZERO-SECOND ALTERNATIVE” CASE

We can begin by considering the simplest case, where  $B2 = C2 = 0$ . Under this condition, not choosing S1 amounts to not playing the game: S2 is not a real alternative. For this case we can only study RD1-freq variations.<sup>14</sup>

I worked with populations of 100 agents demanding a minimum gain of  $r = 1$ . B1 and C1 values varied between 5 and 100 and between 0.5 and 95. 287 different combinations producing S1 attractiveness levels ranging from 0.53 to 199 (R(K)) and 1.053 to 200 (R(B)) were explored. For each of these values I took into account L1 lots percentages ranging from 5 to 100 (increments of 5). I simulated the model for 5740 parameter combinations, for a total of 57400 simulations, since each combination was simulated 10 times to assess the model behaviour variability linked to its random elements.

Figure 1 presents a set of typical patterns generated by the model for a specific series of R(B) and R(K) values.

What these graphs show first and foremost is that the relation between percentage of L1 lots and RD1-freq may be linear or non-linear depending on attractiveness of S1.

For R(B) and R(K) values under or equal to 25, the gradual expansion of L1 lots goes together first with an increase, then a fall, in percentage of agents who do not obtain the B1 benefit associated with L1 lots despite the fact that they spent C1. However, the greater the attractiveness of S1, the more quickly we arrive at a percentage of L1 lots at which a negative relation between L1 and RD1 freq appears and the higher the percentage of deprived agents.

For R(B) and R(K) values over 25, this critical point arrives so early that the positive section of the relation between percentage of L1 lots and RD1-freq disappears. Here we find the sign of the “spontaneously” expected relation: an improvement in agent living conditions (represented here by the increased number of L1 lots) is accompanied by a gradual decrease in the size of the dissatisfied population.

The interdependence structure within which agents reason and make their choices explains the appearance of these aggregate patterns. When S1 attractiveness is low, the probability of agent ultimately choosing S1 is also low because he is likely to want to spend C1 only if the gain

expected from S1 is “full,” a condition that will only be validated if he deems the number of agents choosing S1 lower than the number of L1 lots. This means that only as the number of L1 lots rises will an increasing percentage of agents definitively choose S1: clearly the conditions are being created for an increasing segment of the population to fail to obtain what they want. As S1 attractiveness increases, however, we move closer to the exact opposite situation: the gain expected from S1 falls less and less with the presence of other competitors, and this will move the agent to choose S1 regardless of how many other competitors he thinks there are. Here the conditions have been created for the entire population to choose S1: RD1-freq reaches its peak when the number of L1 lots available is very low; it can only fall as that availability increases.

Clearly, then, with B2 and C2 equal to 0 here, what ultimately generates these aggregate patterns are the absolute values of B1 and C1. Specifically, the difference between them determines how sensitive agents' reasoning and final choice will be to other agents' choices. The greater that difference is, the greater the probability that agents will choose S1 regardless of the number of agents they think are competing to obtain L1 lots.

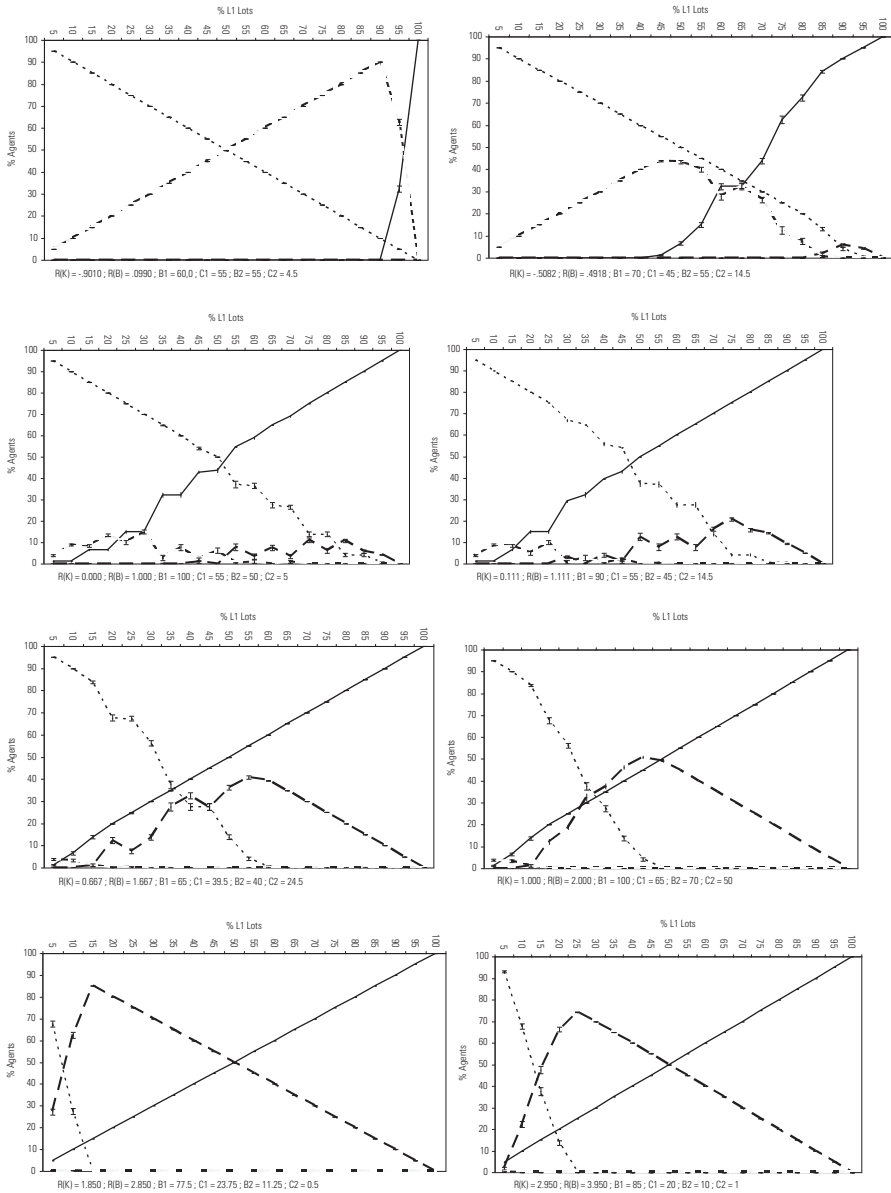
The results reported in Figure 2 demonstrate the soundness of this hypothesis. In relation to the simulations I have just commented on, only one element changes: the absolute values of B1 and C1 are lower. If we compare the graphs in Figures 1 and 2 for the same R(B) or R(K) value, we see that under this new experimental condition a) the percentage of L1 lots at which the relation between L1 and RD1-freq becomes negative is higher; b) RD1-freq levels are lower; c) the S1 attractiveness value above which the relation becomes entirely negative (*more opportunities, more deprived agents*) is higher.

What happens is that the lower B1 and C1 values delay the moment at which agents choose S1, regardless of the number of agents they think are making the same choice they are.

#### **4. GENERATING RD FREQUENCY PATTERNS: THE “NON-ZERO-SECOND ALTERNATIVE” CASE**

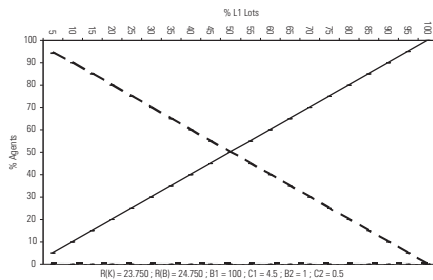
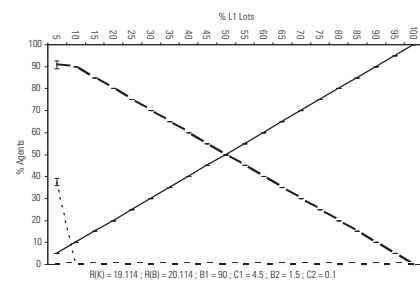
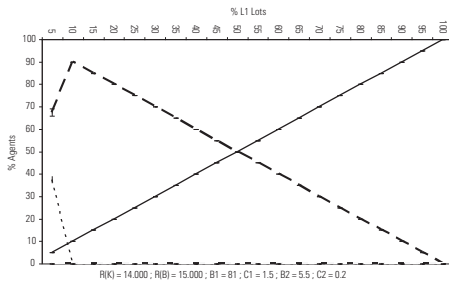
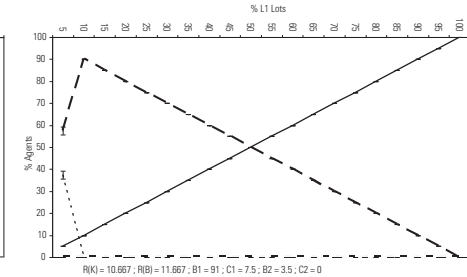
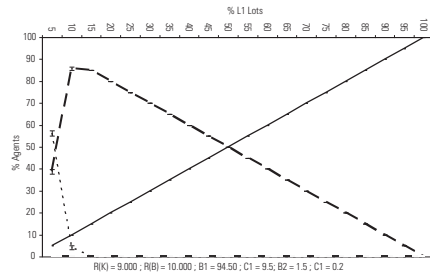
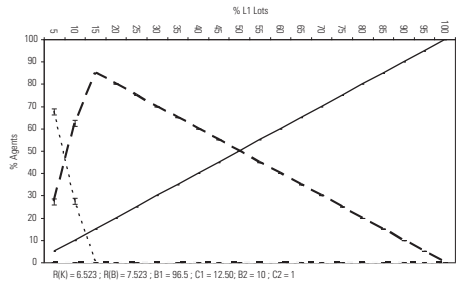
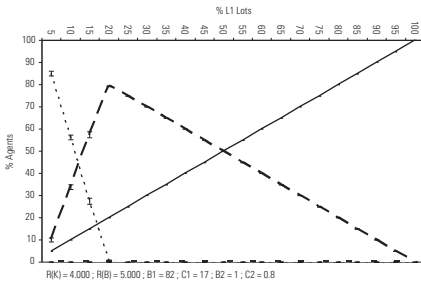
We can now examine a somewhat more complex artificial society, where S2 represents a real alternative to S1 ( $B2 \neq 0$  and  $C2 \geq 0$ ). In this case we can study both RD1-freq and RD2-freq variations.

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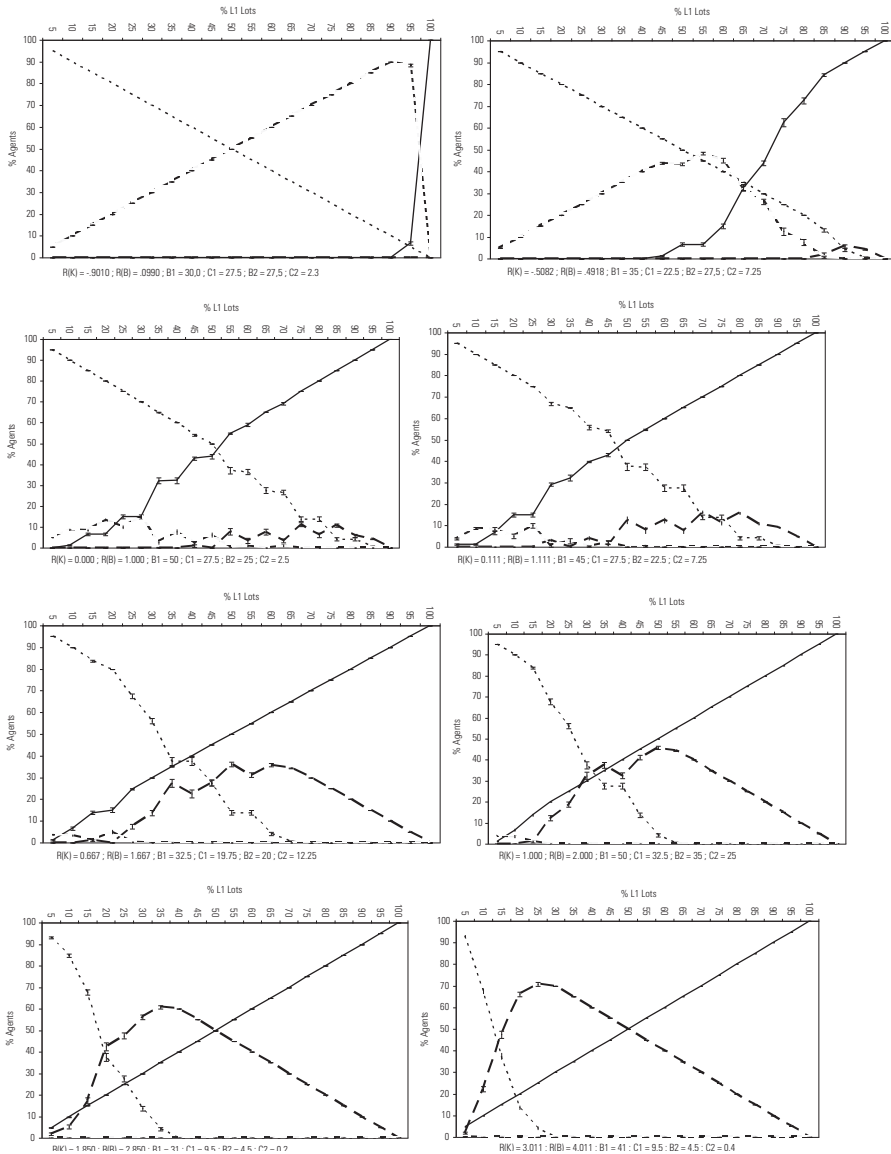
**Figure 3:** For a typical series of  $R(K)$ – $R(B)$  values (i.e. attractiveness of  $S1$  compared to  $S2$ ), percentages (95% confidence intervals) of agents who finally obtain  $B1$ ,  $B2$  or nothing after betting  $C1$  or  $C2$  (y-axis) as a function of the percentage of available  $L1$  lots (x-axis) [median absolute differences  $B1 - C1$ : 75;  $B2 - C2$ : 15;  $(B1 - C1) - (B2 - C2)$ : 56].

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- agents who obtain B1 after betting C1
- - - agents who obtain B2 after betting C1 (RD1 frequency)
- · · agents who obtain B2 after betting C2
- · - agents who obtain B1 after betting C2 (RD2 frequency)

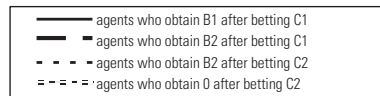
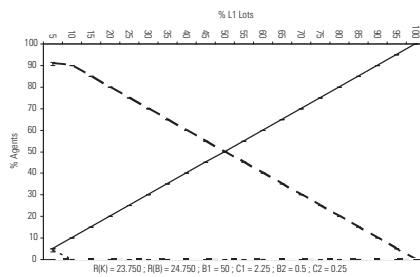
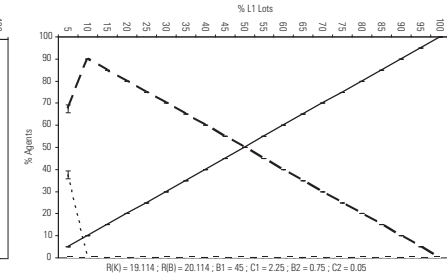
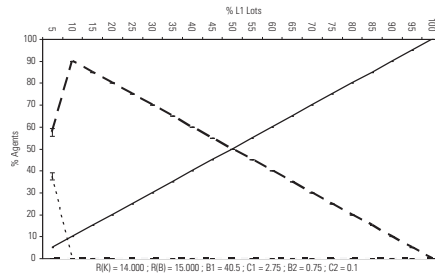
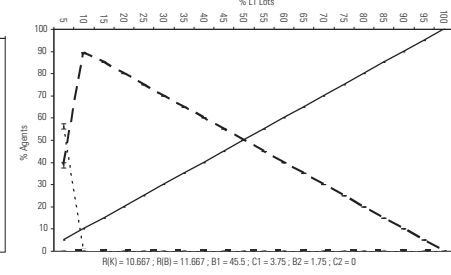
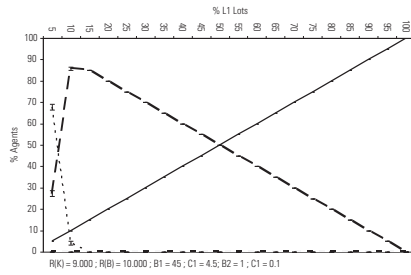
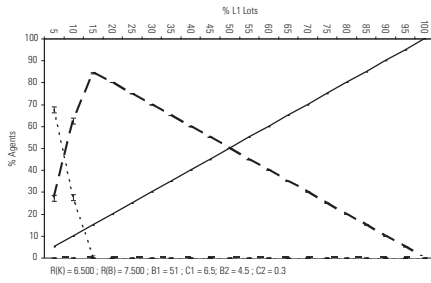
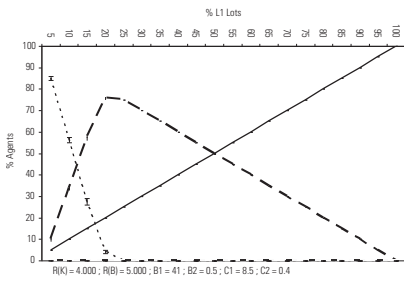
### PART THREE ON GENERATIVE MECHANISMS



**Figure 4:** For a typical series of  $R(K)$ – $R(B)$  values (i.e. attractiveness of  $S1$  compared to  $S2$ ), percentages (95% confidence intervals) of agents who finally obtain  $B1$ ,  $B2$  or nothing after betting  $C1$  or  $C2$  (y-axis) as a function of the percentage of available  $L1$  lots (x-axis) [median absolute differences  $B1 - C1$ : 36.5;  $B2 - C2$ : 7.5;  $(B1 - C1) - (B2 - C2)$ : 27.4].



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### PART THREE ON GENERATIVE MECHANISMS

I once again examined populations of 100 agents demanding a minimal gain of  $r = 1$ . B1, C1, B2 and C2 values varied respectively between 10 and 100; 1.5 and 95; 5 and 90 and 0 and 50. 1976 different combinations producing S1 attractiveness levels ranging from  $-0.90$  to  $98.5$  (R(K)) and  $0.09$  to  $99.5$  (R(B)) were studied. To limit calculation time, the percentage of L2 lots was increased from 5% to 100% by increments of 10 in 1630 cases and by increments of 5 in the 346 other cases. I simulated the model for 20700 parameter combinations, for a total of 207,000 simulations, since each combination was simulated 10 times to assess the model behaviour variability linked to its random elements.

Figure 3 presents a set of typical patterns generated by the model for a specific series of R(B) and R(K) values.

The first two graphs show that if S1 is infinitely less attractive than S2, RD1-freq (percentage of agents spending C1 and obtaining only B2) is low, even nil, whereas RD2-freq (percentage of agents spending C2 and obtaining 0) soars.

The dynamic that produces this aggregate result is simple. When the B1–C1 gain expected from S1 is below the B2–C2 gain associated with S2, the probability of agent ultimately choosing S1 is zero. Since all agents, then, are opting for S2, the proportion of the population obtaining 0 after attempting to obtain the B2 benefit associated with S2 lots can only increase as the number of L2 lots falls (i.e., as the number of L1 lots rises). Meanwhile, the probability of each agent ultimately choosing S1 starts to move away from zero when the number of L2 lots is so small and the number of agents assumed to want to obtain those lots so high, that the B2–C2 gain associated with S2 is undercut by a value so high that it necessarily falls below the B1–C1 gain expected from S1. In this case, because the number of L<sup>1</sup> lots is high, the increase in RD2-freq is more or less quick to disappear.

The following graphs show that RD1-freq gradually takes the place of RD2-freq as the difference between the B1–C1 gain expected from S1 positively overtakes the B2–C2 gain associated with S2.

In this area of parameter space we find familiar aggregate patterns. The more attractive S1 is, the higher the probability of observing a situation where a very high quota of the population is moved to choose S1, even if the number of L1 lots is limited. For R(B) or R(K) values above 20, this quota is so high that the positive part of the relation between the

percentage of L1 lots and RD1-freq disappears. Under this condition, a negative linear relationship appears: the higher the percentage of L1 lots, the fewer the number of agents spending C1 and obtaining only B2.

The dynamic behind this aggregate pattern is the same as in the  $B2 = C2 = 0$  case. However, from a formal viewpoint, what counts here is the absolute value of the difference between the  $B1-C1$  and  $B2-C2$  differences. The greater that difference, the less sensitive agent reasoning and final choice will be to other agents' final choices: S1 will be chosen whatever happens.

The experimental data in Figure 4 show the soundness of this hypothesis. In relation to the simulations I have just commented on, only one element changes: generally lower B1, C1, B2 and C2 values produced virtually the same R(B) or R(K) values. Under this new condition we observe once again a change from a non-linear relationship between L1 and RD1-freq to a negative linear relation (*more opportunities, fewer deprived agents*). However, this shift occurs more slowly.

What happens is that the lower B1, C1, B2 and C2 values delay the moment at which agents choose S1 rather than S2, regardless of the number of agents they think are making the same choice they are.

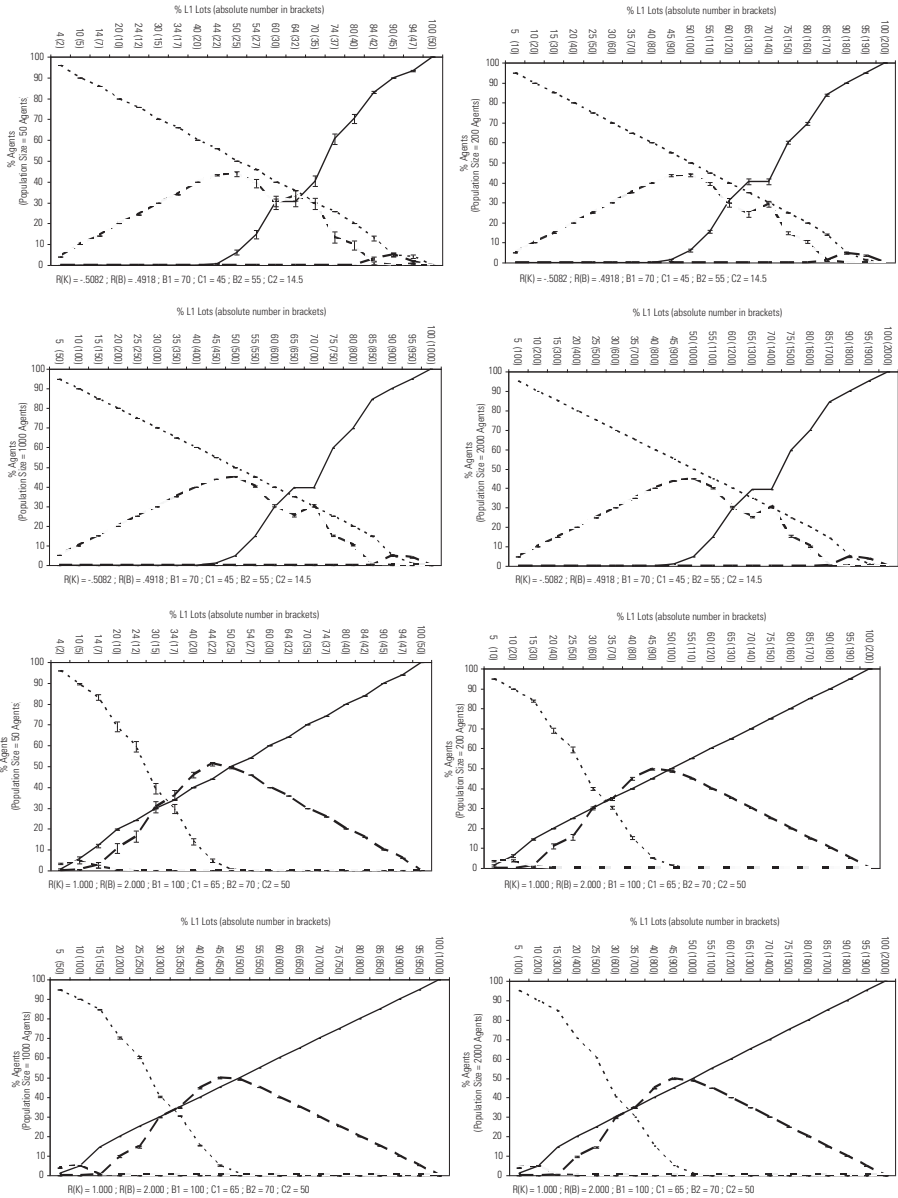
## 5. GENERATING RD FREQUENCY PATTERNS IN AGENT POPULATIONS OF VARIABLE SIZE (IN THE "NON-ZERO-SECOND ALTERNATIVE" CASE)

Does the size of the population in which agents act play a role in the appearance of this phenomenon, i.e., the gradual erosion of agent sensitivity to others' choices?<sup>15</sup>

Figure 5 reports the results of a set of model simulations in cases where N is lower (50) or higher (200, 1000, and 2000) than the number used thus far (100). The distinctive feature of these experiments is that the absolute number of L1 lots in the society was changed proportionally with change in population size.

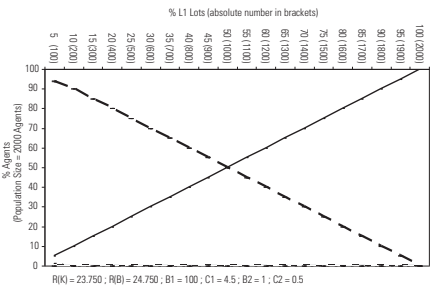
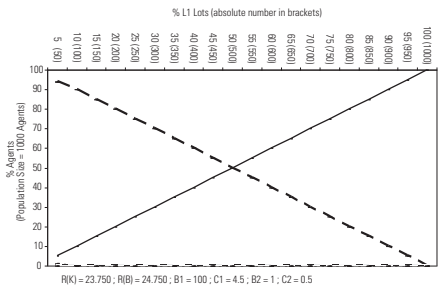
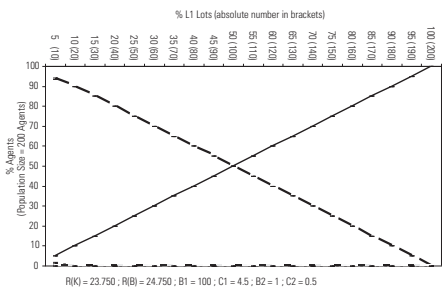
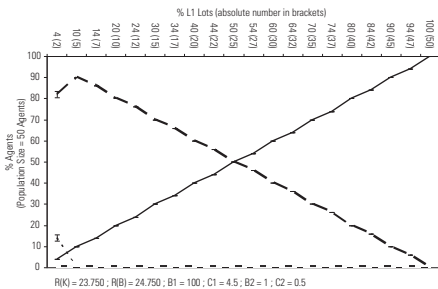
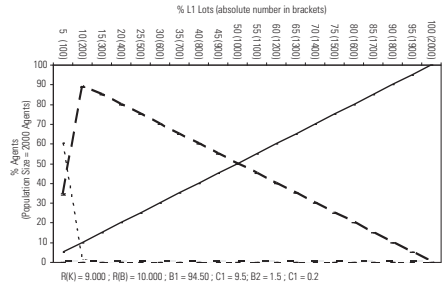
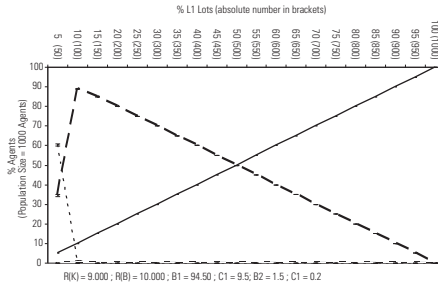
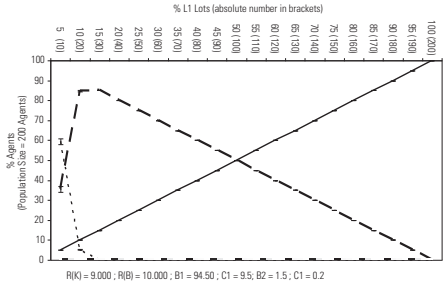
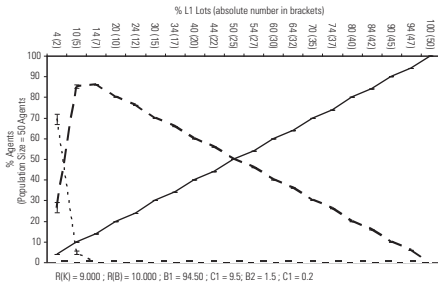
Under this condition — i.e., a constant level of opportunity — the effect of population size on the RD1-freq and RD2-freq levels generated by the model is virtually nil regardless of S1 attractiveness. The form of the relation between L1 and RD1-freq (and RD2-freq) is not affected either; it remains non-linear or negative linear (*more opportunities, fewer deprived agents*), depending on R(B) or R(K) values.

## PART THREE ON GENERATIVE MECHANISMS



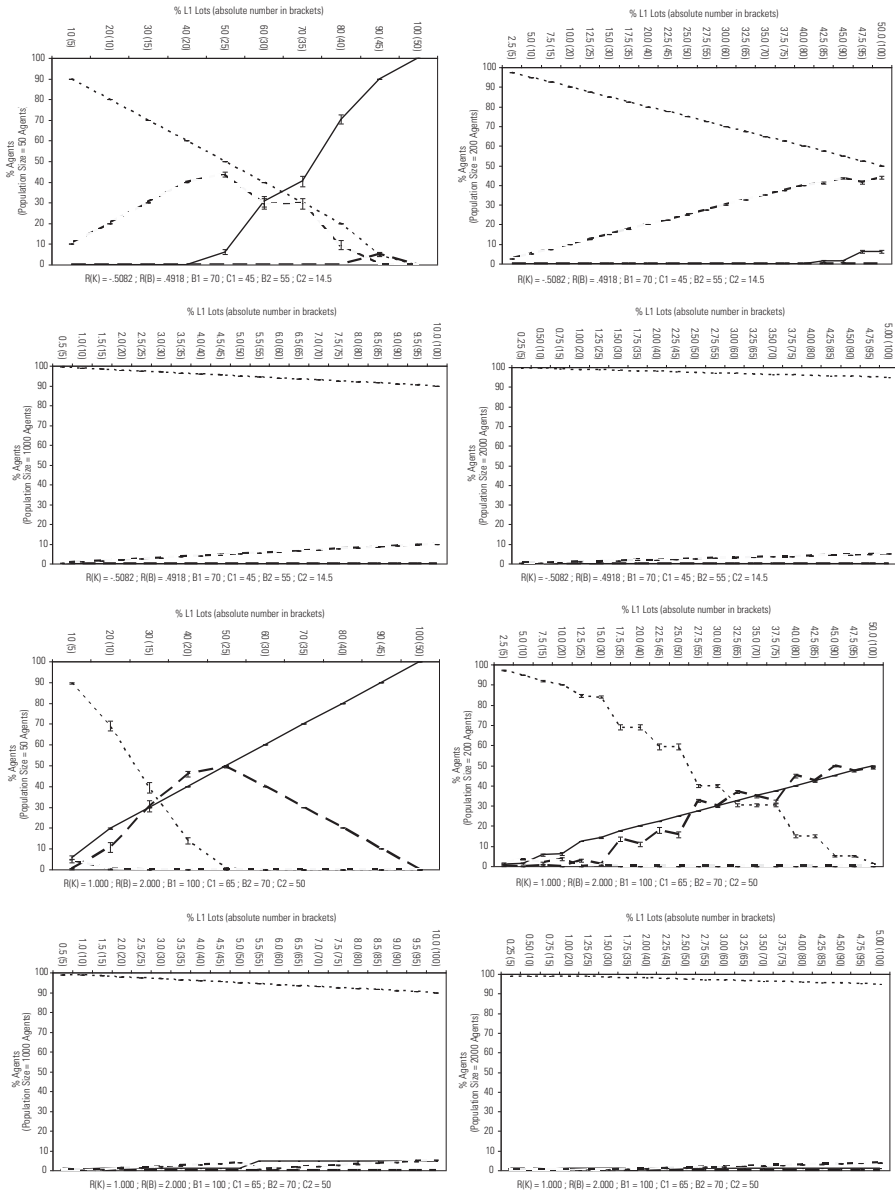
**Figure 5:** For four  $R(B)-R(K)$  values (i.e. attractiveness of  $S1$  compared to  $S2$ ), percentages (95% confidence intervals) of agents who finally obtain  $B1$ ,  $B2$  or nothing after betting  $C1$  or  $C2$  (y-axis) as a function of the percentage of available  $L1$  lots (x-axis) changing proportionally to changes in agent population size.

# GIANLUCA MANZO BODON'S MODEL OF RELATIVE DEPRIVATION REVISITED



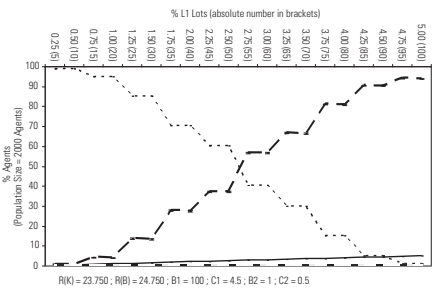
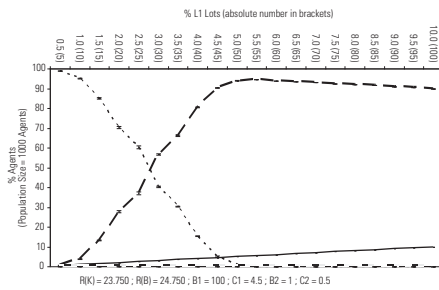
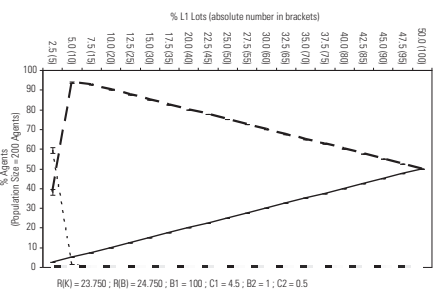
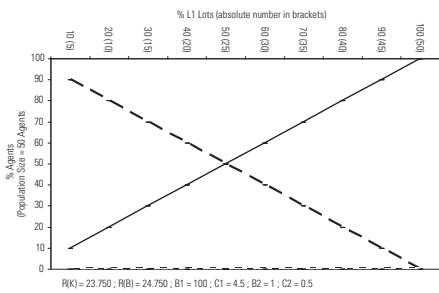
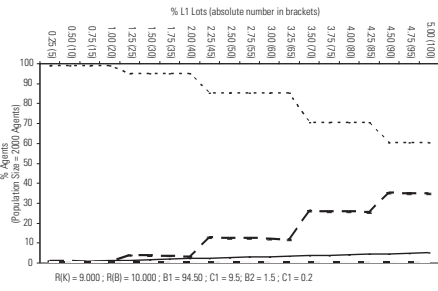
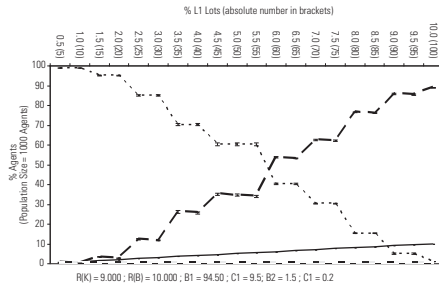
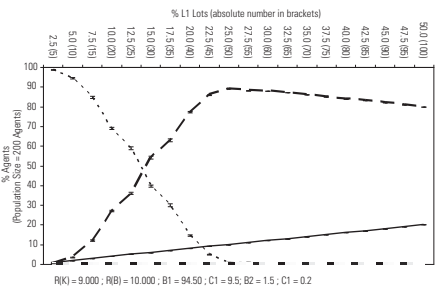
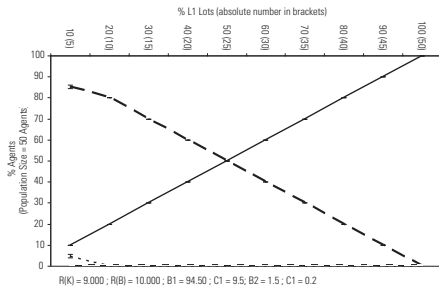
- agents who obtain B1 after betting C1
- - - agents who obtain B2 after betting C1 (RD1 frequency)
- · · agents who obtain B2 after betting C2
- · - agents who obtain 0 after betting C2 (RD2 frequency)

### PART THREE ON GENERATIVE MECHANISMS



**Figure 6:** For four  $R(B)$ - $R(K)$  values (i.e. attractiveness of  $S1$  compared to  $S2$ ), percentages (95% confidence intervals) of agents who finally obtain  $B1$ ,  $B2$  or nothing after betting  $C1$  or  $C2$  (y-axis) as a function of the percentage of available  $L1$  lots (x-axis), which is constant relative to changes in agent population size.

# GIANLUCA MANZO BOUDON'S MODEL OF RELATIVE DEPRIVATION REVISITED



- agents who obtain B1 after betting C1
- - agents who obtain B2 after betting C1 (RD1 frequency)
- ... agents who obtain B2 after betting C2
- . agents who obtain 0 after betting C2 (RD2 frequency)

This behavioural invariance is easy to understand. Changing the absolute number of L1 lots proportionally to the change in population size means leaving intact the ratio between L1 (and therefore number of L2 lots) and numbers of agents assumed to choose S1 (or S2). Consequently agents' assessments of the gains associated with S1 and S2 remain unchanged. The dynamic of the model is therefore the same, as are the aggregate results.

Figure 6 suggests that this is not at all the case when the change in absolute number of L<sup>1</sup> lots does not proportionally follow the change in population size. In this set of simulations, the absolute number of L1 lots varies only between 5 and 100 by increments of 5 while the population was decreased (to 50) or increased (to 200, 1000 or 2000).

Under this condition — variable opportunities — we observe that for low S1 attractiveness values ( $R(K) = -.508$  or  $R(K) = 1.000$ ), the greater the population size increase, the lower the maximum RD2-freq and RD1-freq levels. The form of the relation between number of L1 lots and RD1-freq (and RD2-freq) also changes: a positive linear relation (*more opportunities, more deprived agents*) takes over. For higher S1 attractiveness values (here  $R(K) = 9.00$  or  $R(K) = 23.75$ ), the decrease in maximum RD1-freq levels that follows on an increase in population size is gradually effaced; instead, those levels soar. The negative linear part of the relation between number of L1 lots and RD1-freq (*more opportunities, fewer deprived agents*) also tends to disappear here.

The dynamic generating these aggregate patterns only seems enigmatic. For what does it mean to analyze agent populations that increase in size while varying the number of L1 lots within a fixed interval between 5 and 100? It amounts to studying the relation between percentage of L1 lots and RD1-freq (and RD2-freq) for increasingly poor opportunity structures. Each of the graph series in Figure 6 is actually an increasingly specific zoom on an increasingly limited section of the left side of the corresponding graphs in Figure 5.

This means that the gradual effacement of agent choice interdependence — a phenomenon responsible for the appearance of the expected “*more opportunities, fewer deprived agents*” — is not validated here, simply because B1, B2, C1 and C2 values are too small to move agents to disregard the fact that the number of L1 lots is extremely limited.



## CONCLUDING REMARKS

Raymond Boudon's contribution to understanding RD stands apart from sociological literature on the phenomenon in two ways. First, it includes a formalized model of a mechanism likely to generate quotas of dissatisfied actors. Second, it studies the aggregate effects of this mechanism by numerically simulating how it works.

My purpose here has been to suggest that we are now in a position to reap all the benefits of the research avenue opened by Raymond Boudon, if we adopt certain recently developed computer simulation methods. The agent-based simulation technique allows both for specifying the conceptual structure of the mechanisms postulated and exploring their effects under numerous and varied conditions.

With regard to model design, constructing an agent-based system as I have done here enables me to advance on three points. First, the agents studied here assess the expected gains of two strategies, both in the case where the number of agents choosing the more profitable prize is below the number of those lots and in the opposite case (i.e., the one Boudon studied). Second, in constructing agents' final choices, I can readily take into account the entire range of possible frequencies of choosing the more attractive strategy, also linking these frequencies to agent choice by a non-linear probability function. The cases studied by Boudon — dominant strategy and perfect split between choices for and choices against — thus become instances of a more general choice function. Third, the process of prize allocation programmed here also encompasses the case where the number of agents choosing the more attractive strategy is below the number of more attractive lots.

These developments endow Boudon's model with an extension that has gone unused until now. Thus modified, the model can engender two types of RD instead of one: first, the RD affecting agents who have obtained the less attractive lot at the price of the more attractive one; second, the RD affecting agents who have obtained nothing at the price of the less attractive prize.

Thanks to the flexibility of computer simulation, I was able to explore variations in the frequency of these two types of RD to an extent that could not be reached when Boudon conceived his model.

This sensitivity analysis brings out the plurality of forms that may be assumed by the relationship between objective opportunity structure

and proportion of dissatisfied actors in a given population. The positive linear form designated by “*more opportunities, more dissatisfied persons*” — i.e., the take-off point of sociological literature on RD — is only validated in a specific region of parameter space. The more general situation is in fact that of a non-linear relation made up of a positive part (“*more opportunities, more dissatisfied agents*”) followed by a negative part (“*more opportunities, fewer dissatisfied agents*”). In some cases, the negative linear relation actually takes over entirely.

For the specific mechanism posited by this model, a simple dynamic generates the gradual shift from one to another of these three situations. As the more costly lots become more attractive, the dependence of agent’s choice on the choices of others declines: agents will try to obtain those lots even if their availability is limited. This means that when choosing the more costly prize becomes the dominant strategy for each agent, opening up the objective opportunity structure can only reduce the quota of dissatisfied agents.

This dynamic does not vary with agent population size on condition that a change in number of agents goes together with a proportional change in number of more attractive lots. If this is not the case, then at a given level of attractiveness for the more costly lots, increasing population size amounts to reducing prize attractiveness. In this case, only the non-linear or positive linear form of the relation — i.e., “*more opportunities, more dissatisfied agents*” — will tend to emerge.

These computational regularities enable us to consider from a new perspective the contradictory results obtained in recent econometric studies on the relation between a country’s wealth and the proportion of dissatisfied individuals in it. If the mechanism operative in my artificial societies were at work in real ones, positive or negative linear relationships between these two variables as well as positive/negative non-linear ones could also be observed because the empirical data studied underlie different three-way “attractiveness of lots at stake-number of more attractive lots-population size” combinations. Depending on the value assumed by the components of these three-way combinations or, in the case of diachronic data, the particular trend in these values, empirical data would reflect a specific segment of the curves discussed here.

What are the limitations of such an interpretation? First, we do not know if the mechanism posited by the model is really operative behind the data used in the literature in question. Second, the mechanism pos-

ited by the model includes only two alternatives, whereas it is likely that empirically quantified levels of individual satisfaction derive from a plurality of goods, and that actors assess their achievements in terms of this plurality. Third, the model formalizes a mechanism that generates satisfied/dissatisfied actor quotas but does not contain a mechanism responsible for the levels of individual satisfaction/dissatisfaction specifically associated with those states. There is no reason to believe that RD frequency and RD degree mechanisms are the same. Fourth, the model posits that a single RD frequency mechanism is at work when in fact — and this would depend on the good to be gained — we have no reason to exclude the possibility of RD frequency and RD degree being determined by different mechanisms (and the mechanisms may also differ from one individual actor to another).

It therefore seems essential to continue the research on two parallel lines. First, to collect more precise information on the reasoning, comparisons, and feelings, as well as the specific social objects, that fuel individual actors' satisfaction/dissatisfaction judgments. Second, to strengthen construction and analysis by simulating models of RD-frequency and RD-degree generating mechanisms.

With regard to this second line, in the multi-agent implementation of Boudon's model studied here, we could, for instance, a) accentuate the randomness of agent reasoning and introduce forms of individual learning; b) multiply sources of interindividual variability; c) make the model dynamic more complex by transforming it into a "repeated game"; d) quantify the individual satisfaction that agents associate with their state of deprivation; e) construct a network of dyadic interactions between agents that would influence their assessments of lot attractiveness, their reference groups, and the intensity of their feelings of satisfaction/dissatisfaction.

It is up to empirical research to indicate where this gradual improvement of formal modelization should stop. But it is through formalization that we will urge the imagination forward and increase the conceptual precision of empirical research. Clearly no principle of opposition exists between sociological theory, empirical research and formal modelization. This is one of the most general lessons I come away with from the groundbreaking work of Raymond Boudon.

*Trans. from French by Amy Jacobs*

## NOTES

1. Raymond Boudon presented these two programmatic ideas most explicitly in his 1979 article than in his oft-cited answer (Boudon 1976) to Hauser's criticism of *L'Inégalité des chances*. At that time, Fararo (1969) and Sorensen (1976) too were suggesting that it would be worthwhile to proceed by formalizing "mechanisms." Boudon cited Fararo as the source of the term "generating model" (Boudon 1979: 63, n. 6). Sorensen, meanwhile, unlike Boudon, did not think there was much use in simulation.
2. Runciman (1966: 10) was the first to give a more developed definition: "We can roughly say that A is relatively deprived of X when (i) he does not have X, (ii) he sees some other person or persons, who may include himself at some previous or expected time, as having X (whether or not this is or will be in fact the case), (iii) he wants X, and (iv) he sees it as feasible that he should have X." A pioneering definition developed in social psychology adds a fifth component: "lack[s] a sense of responsibility for failure to possess X" (Crosby 1976, Table 1).
3. This analytic distinction is made clearly in contemporary social psychology definitions of RD; see for instance Pettigrew (2002: 253) and Tyler *et al.* (1997: 17).
4. Recent social psychology studies have attempted to show that these two types of comparisons actually proceed from a single more general type known as counterfactual comparisons: "comparisons of one's current outcomes with outcomes that one might have obtained but did not" (Olson & Roese 2002: 266).
5. See Cherkaoui 2005 (chap. 1) for a perceptive reading of these mechanisms in Tocqueville's thought. See Cherkaoui (2001), Coleman (1990: chap. 8), Tyler *et al.* (1997: chap. 7), and Dubé & Guimond (1986) for reviews of the literature relating RD to collective action.
6. Since Kosaka (1986) and Yamaguchi (1998) in their re-analyses of Boudon's model only studied RD frequency, their works attest implicitly that Boudon's model is one of RD-frequency.
7. We have now been told that the relations between "analytical solutions" and computational approaches are various and complex (Axtell 2000, Epstein 2006, chap. 2). Boudon (1965) was already perfectly aware of this.
8. Constructing and analyzing a multi-agent system is still a fairly costly operation (see Janssen *et al.* 2008). Here I used the agent-based simulation platform option (Railsback *et al.* 2006), specifically NetLogo 4.0.2 (Wilensky 1999, Tisue & Wilensky 2004a, b). The model can also be run (without changing the code) in NetLogo 4.0.3, a new version of the platform that came out as this article was going to press.
9. In the case of  $A(S1) < \text{number of } L1$ , quantification of  $G(S2)$  cannot include a compensation equal to the one in the  $G(S1)$  calculation for  $A(S1) > \text{number of } L1$  lots, because rule 1e precludes an agent who spends only  $C2$  from obtaining  $B1$ . Neither Boudon nor Kosaka considered the case where  $A(S1) < \text{the number of } L1$  lots. This omission is probably due to the fact that both authors were studying the model only for  $B2 = C2 = 0$  and  $r = 0$ , though Boudon (1979: 53) also considered the case of  $r = 1$ . Under the condition  $r = 0$ , the case of  $A(S1) < \text{number of } L1$  is not of much interest because  $S1$  will always be more advantageous than  $S2$ . But if the intention is to run the simulation on a vast range of parameter

combinations, this generalization of agent reasoning has to be included. And as I discovered, the effects of this change are considerable.

10. The two situations studied by Boudon — half of all actors choose S1, all actors choose S1 (dominant strategy) — thus become respectively the equilibrium point and the upper limit of a more general choice function.
11. Boudon does not take into account this third possibility. But in writing a computer program to study the model we are almost spontaneously led to introduce the modification just proposed. If we were to stick to Kosaka's axiom (1986: 36) holding that "every player is entitled to gain at least B2 as long as he participated in a game by betting either C1 or C2," the model would be computationally incomplete and insoluble.
12. The sensibility analysis discussed further on was performed using the NetLogo 4.0.2 BehaviorSpace module. To ensure that result reproducibility, I used a series of ten seeds (values omitted here for the sake of brevity).
13. In a forthcoming paper I present a more general analysis of the relations between S1 attractiveness compared to S2 attractiveness; objective opportunities structure, RD1-freq (and RD2-freq) and a possible quantification of feelings of agent dissatisfaction in RD1 and RD2 situations (that is, RD1 and RD2-degrees).
14. The two numerical examples that Boudon began with (1982 [1977]: 110–117) are based on the hypothesis that  $B2 = C2 = 0$ . In the more general analysis presented in Tables 5.1 and 5.2, it is not clear whether the author has kept or discarded this restrictive hypothesis. Kosaka's analysis raises the same question. Since both authors studied RD1-freq only, it seems likely that they were reasoning in terms of  $B2 = C2 = 0$ .
15. Boudon (1982 [1977]: 219, n. 10) raised the question of the behavioural stability of his model for large populations, intuiting that "the conclusions that may be drawn from the preceding examples may, in certain circumstances, be taken also to refer to cases in which N is large."

## REFERENCES

- Axtell, R. (2000) "Why agents? On the Varied motivations for agent computing in the social sciences". In: Robert Axtell (ed.), *Proceedings of the Workshop on Agent Simulation: Applications, Models and Tools*. Argonne, IL, Argonne National Laboratory: 3–24.
- Boudon, R. (1965) "Réflexion sur la logique des modèles simulés". *Archives Européennes de Sociologie*, 6(1): 3–20.
- (1976) "Comment on Hauser's review of *Education, Opportunity, and Social Inequality*". *American Journal of Sociology*, 81(5): 1175–1187.
- (1979a) "Generating models as a research strategy". In: Robert K. Merton, James S. Coleman, Peter H. Rossi (eds.), *Qualitative and Quantitative Social Research*. New York, The Free Press: 51–64.
- (1982 [1977]) *The Unintended Consequences of Social Action*. London, The Macmillan Press Ltd.

### PART THREE ON GENERATIVE MECHANISMS

- (2002) “Sociology that really matters”. *European Sociological Review*, 18(3): 1–8.
- Cherkaoui, M. (2001) “Relative deprivation”. In: *The International Encyclopaedia of Social and Behavioural Sciences*. London, Elsevier, 2002: 3522–3526.
- (2005) *Invisible Codes: Essays on Generative Mechanisms*. Oxford, Bardwell Press.
- Clark, A. Frijters, P. and Shields, M. (2008) “Relative income, happiness and utility: an explanation for the Easterlin paradox and other puzzles”. *Journal of Economic Literature*, 46(1): 95–144.
- Coleman J.S. (1990) *The Foundations of Social Theory*. Cambridge, Mass., Harvard University Press.
- Crosby, F. J. (1976) “A model of egoistical relative deprivation”. *Psychological Review*, 83: 85–113
- Dubé, L. Guimond, S. (1986) “Relative deprivation and social protest: the personal-group issue”. In: James Olson, Peter Herman, Mark Zanna (eds), *Relative Deprivation and Social Comparison*. Hillsdale, Lawrence Erlbaum Associates, chap. 11.
- Durkheim, É. (2004 [1897]) *Le suicide*, Paris, PUF “Quadrige”.
- Easterlin, R. (1973) “Does money buy happiness?” *The Public Interest*, 3(Winter): 3–10.
- Elster, J. (2007) *Explaining Social Behaviour: More Nuts and Bolts for the Social Sciences*. New York, Cambridge University Press.
- Epstein, J. (2006) *Generative Social Science: Studies in Agent-Based Computational Modeling*. Princeton, Princeton University Press.
- Fararo, T.J. (1969) “The nature of mathematical sociology”. *Social Research*, 36(1): 75–92.
- Gambetta, D. (1998) “Concatenations of mechanisms”. In: Peter Hedström, Richard Swedberg (eds), *Social Mechanisms. An Analytical Approach to Social Theory*. Cambridge, Cambridge University Press: 102–124.
- Gilbert, G.N. (2007) *Agent-Based Models*. London, Sage Publications.
- Gurr, T.R. (1970) *Why Men Rebel*. Princeton, Princeton University Press.
- Hedström, P. (2005) *Dissecting the Social: On the Principles of Analytical Sociology*. Cambridge, Cambridge University Press.
- Hummon N.P. and Fararo T.J. (1995) “The emergence of computational sociology”. *Journal of Mathematical Sociology*, 20(2–3), Special Issue “Sociological Algorithms”: 78–87.
- Janssen, M. Aless, L. N. Barton, M. Bergin, S. Lee, A. (2008) “Towards a community framework for agent-based modelling”. *Journal of Artificial Societies and Social Simulation*, 11(2).
- Kosaka, K.(1986) “A model of relative deprivation”. *Journal of Mathematical Sociology*, 12(1): 35–48.
- Macy, M.W. and Willer, R. (2002) “From factors to actors: computational sociology and agent-based modeling”. *Annual Review of Sociology*, 28: 143–166.
- Merton, R.K. (1957) *Social Structure and Social Theory*. London, The Free Press of Glencoe.

- Olson, J. Roese, N. (2002) "Relative deprivation and counterfactual thinking". In: Walker, I. and Smith, H.J. (eds), *Relative Deprivation: Specification, Development and Integration*, Cambridge, Cambridge University Press, chap. 12.
- Pettigrew, T. (2002) "Summing up: relative deprivation as a key social psychological concept". In: Walker, I. and Smith, H.J. (eds), *Relative Deprivation. Specification, Development and Integration*, Cambridge, Cambridge University Press, chap. 16.
- Railsback, S.F. Lytinen, S.L. and Jackson, S.K. (2006) "Agent-based simulation platforms: review and development recommendations". *Simulation*, 82: 609–623.
- Runciman, W.G. (1966) *Relative Deprivation and Social Justice: A Study of Attitudes to Social Inequality in Twentieth-century England*. London, Routledge and Kegan Paul.
- Sawyer, R. K. (2003) "Artificial societies: multiagent systems and the micro–macro link in sociological theory". *Sociological Methods and Research*, 31(3): 325–363.
- Sorensen, A.B. (1976) "Models and strategies in research on attainment and opportunity". *Social Science Information*, 15(1): 71–91.
- Wolfers, J. and Stevenson, B. (2008) "Economic growth and subjective well-being: reassessing the Easterlin paradox". *IZA Discussion Papers*, no. 3645, <http://ideas.repec.org/p/iza/izadps/dp3654.html>.
- Stouffer, S.A. Suchman, E.A. Devinney, L.C. Star, S. Williams, R.M. (1965 [1949]) *The American Soldier. Vol. 1, Adjustment During Army Life*. New York, John Wiley & Sons.
- Tisue, S. Wilensky, U. (2004a) "NetLogo: design and implementation of a multi-agent modeling environment". Evanston, IL, Center for Connected Learning and Computer-Based Modeling, Northwestern University, <http://ccl.northwestern.edu/papers/>.
- (2004b) "NetLogo: A simple environment for modeling complexity". Evanston, IL, Center for Connected Learning and Computer-Based Modeling, Northwestern University, <http://ccl.northwestern.edu/papers/>.
- Tocqueville, Alexis de, (1967 [1856]) *L'ancien régime et la révolution*. Paris, Galimard.
- Tyler, T.R. Boeckmann, R. J. Smith, H. J. and Huo, Y.J. (1997) *Social Justice in a Diverse Society*, Boulder, Westview Press
- Wilensky, U. (1999) *NetLogo*, Evanston, IL, Center for Connected Learning and Computer-Based Modeling, Northwestern University, <http://ccl.northwestern.edu/netlogo>.
- Yamaguchi, K. (1998) "Rational-choice theories of anticipatory socialization and anticipatory nonsocialization". *Rationality and Society*, 10: 163–99.