In W. Stroup & U. Wilensky (Chairs) and C. D. Lee (Discussant), *Patterns in group learning with next-generation network technology*. The annual meeting of the American Educational Research Association, Montreal, Canada, April 11 – 15, 2005.

Collaboration and Equity in Classroom Activities Using Statistics as Multi-Participant Learning Environment Resource (S.A.M.P.L.E.R)

Dor Abrahamson & Uri Wilensky
The Center for Connected Learning and Computer-Based Modeling
Northwestern University
abrador@northwestern.edu uri@northwestern.edu

We examine an implementation of a probability-and-statistics participatory-simulation activity in a networked classroom to investigate the equity it affords in terms of student learning opportunities. The simulation activity, S.A.M.P.L.E.R, was designed for middle schoolers to learn basic statistical sampling theory. Students each drew their own samples from a shared population, then each inputted a value reflecting their quantification of their samples. Classroom input was plotted and compared to the true population value. Students devised strategies to coordinate classroom sampling and mathematize perceptual judgment to achieve accurate prediction. We compare the design to a non-networked collaborative construction project enacted in the same classroom and demonstrate and analyze advantages of the networked design, which was more demanding of student participation, more supportive, more student-centered, more inclusive, more suited to capitalize on classroom social dynamics, and more equitable in terms of emergent distribution of student roles and skill development.

This paper discusses a case study of collaboration-and-equity affordances of *networked classrooms* (e.g., Roschelle, Penuel, & L. Abrahamson, 2004; Ares et al., 2004). These technologies and the classroom activities they enable, such as *HubNet participatory simulations* (Wilensky & Stroup, 1999a, 1999b, 2002), are showing promise in supporting student engagement, student-initiated inquiry/exploration of scientific and mathematical phenomena, high-level discussion of challenging content, and real-time assessment (Abrahamson & Wilensky, 2004, 2005a; Berland & Wilensky, 2005; Ares, Stroup, & Schademan, 2004; Stroup, Ares, & Hurford, 2004). Examining the potential role technology can play in promoting equity in education, Lee (2003) concludes that, "computer-based technologies offer unique opportunities. Computer-based tools can provide underlying architectures that allow for multiple forms of modeling, of ways that learners can represent their understanding, and multiple routes for interactivity and appropriation" (p. 58; see also Hooper, 1996; for issues of equity in non-technological design for mathematics education, see, e.g., De La Cruz, 1999; Fuson & Lo Cicero, 2000). This paper focuses on one such underlying architecture designed to enhance participation by enabling multiple forms of modeling, representation, interactivity, and appropriation. The particular interest of this paper is that we compare the participation affordances of two designs, one using traditional media and another using networked technology. Both designs were implemented in the same classroom as part of a single unit. Specifically, the data set for this study is from an implementation of *ProbLab* (Abrahamson & Wilensky, 2002, 2005b, 2005c), a probability-and-statistics unit under the umbrella of the *Connected Probability* project (Wilensky, 1993, 1997), in middle-school classrooms (see also Abrahamson & Wilensky, 2005d).

In ProbLab, students work both in traditional and computer-based media (see also Abrahamson, Blikstein, Lamberty, & Wilensky, 2005, on our choice to mix media). Students analyze combinatorial spaces of stochastic objects and literally construct these spaces in the form of "picture bar charts." Students then work in the *NetLogo* modeling-and-simulation environment (Wilensky, 1999) to conduct simulations of empirical-probability experiments. Students compare the combinatorial spaces they have built to the distributions of random outcomes they receive. These comparisons support discussions of computer-based mathematical modeling, randomness vs. determinism, sampling, distributions, and the Law of Large Numbers (the Central Limit Theorem; see below for an overview of the Problab design). Finally, students participate in *S.A.M.P.L.E.R.* (Abrahamson & Wilensky, 2002), the statistics component of ProbLab that is implemented in the HubNet (Wilensky & Stroup, 1999a) networked technology.

In Abrahamson and Wilensky (2005c), a complementary paper, we examine issues of collaboration and equity in implementations of the *combinations tower*, the traditional-media component of ProbLab, in two 6th-grade classrooms. Here we investigate these same issues in implementations of S.A.M.P.L.E.R. We begin by overviewing the design of ProbLab. Next we focus on S.A.M.P.L.E.R. so as to contextualize the data we subsequently analyze. In particular, we will look at forms of participation in S.A.M.P.L.E.R. and the learning affordances of these forms. Following, we will describe the implementation of S.A.M.P.L.E.R. in our focus Grade 6 classrooms, including examples of student insight and inventions. Finally, we will compare the implementation of the combinations tower and S.A.M.P.L.E.R. in these classrooms. In light of the classroom excerpts we furnish and findings from post-test data, we will examine whether the participatory simulation afforded more students more mathematically meaningful participation. Conclusions from this comparison will then be analyzed in terms of the unique features of networked technology for collaborative-learning classrooms.

Overview of the ProbLab Design

ProbLab is designed to support student inquiry into connections between theoretical probability, empirical probability, and statistics.[1] ProbLab consists of a set of interrelated NetLogo (Wilensky, 1999) models and associated activities. The learning environment, including the tools and activities, is based on a design rationale that these three constructs are related, that students will understand these constructs better through coordinating them, and that therefore students need tools and activities that allow for such coordination of the perspectives (see Abrahamson & Wilensky, 2005e, for a cognitive perspective on ProbLab's design rationale). For example, one theme of ProbLab is for students to explore relations between the anticipated frequency distribution, which we determine through combinatorial analyses, and the outcome distribution we receive in computer-based simulations of probability experiments. To facilitate the exploration of the relationship between such theoretical and empirical work, we build tools that bridge between them. These *bridging tools* (Abrahamson, 2004) have characteristics of both the theoretical and empirical work. Specifically, the combinatorial spaces are designed in formats that resemble outcome distributions, and experiments are structured so as to sustain the raw data.

[1] To interact with ProbLab's NetLogo models, including simulations that relate probability to geometry and to basic genetics, see http://ccl.northwestern.edu/curriculum/ProbLab/index.html .

a. *b.* *c.* *d.* *e.*

Figure 1. Artifacts and activities in the ProbLab probability-and-statistics experimental unit.

The core activities in ProbLab are based on the *9-block*, a 3-by-3 grid in which each "square variable" receives one of the designated "color values." Figure 1 (see above) features several key design elements of the ProbLab unit that are related through the 9-block. Figure 1a shows one of 512 unique permutations of the green/blue 9-block. Students collaborate in finding and creating the 'combinatorial sample space' of the 9-block. They use paper, crayons, scissors, and glue, to create and assemble the *combinations tower* (Figure 1b, 1c) that rises from the classroom floor up to the ceiling, with columns of height (in 9-blocks) 1, 9, 36, 84, 126, 126, 84, 36, 9, and 1 (the coefficients of the binomial function *(a+b)*^9). Figure 1d features an empirical experiment in NetLogo, "9-Blocks," that dynamically builds outcome distributions of randomly generated 9-blocks (it is projected on the classroom overhead screen, and a student is presenting her understanding of why the outcome distribution in this probability experiment resembles in *shape* the combinations tower, even though the outcome distribution represents thousands of 9-blocks and not just the 512 different possible 9-blocks in the combinatorial space). Figure 1e shows a student participating in the S.A.M.P.L.E.R. participatory simulation activity. The student is sampling from a hidden population of thousands of green/blue squares; he is taking samples each of 9 squares (a 9-block) and 1 square (a 1-block).

Having overviewed the ProbLab unit, we will now elaborate on S.A.M.P.L.E.R.

The Design of S.A.M.P.L.E.R. and its Inherent Learning Opportunities

S.A.M.P.L.E.R., Statistics As Multi-Participant Learning-Environment Resource, is an activity for a networked classroom studying basic statistics concepts. We will now further explain the design and then present learning opportunities afforded by the design.

The S.A.M.P.L.E.R. activity. S.A.M.P.L.E.R. is a participatory simulation activity implemented in the HubNet architecture (Wilensky & Stroup, 1999a, 1999b). Students participate through *clients* (in the current version of S.A.M.P.L.E.R., these clients are personal computers[2]). These clients are hooked up to the facilitator's *server* that communicates with the clients and processes student input. In S.A.M.P.L.E.R. (see Figure 2, below), students take individual samples from a population so as to determine a target property of this population. The "population" is a matrix of thousands of green or blue squares (Figure 2a) and the target property being measured is the population's greenness, i.e., the proportion of green in the population. A feature of the activity is that population squares can be "organized"—all the green squares to the left, all the blue squares to the right (Figure 2b). This organization indexes the proportion of greenness as a part-to-whole

[2] Using the Computer–HubNet package (Wilensky & Stroup, 2002).

spatial extension that maps onto scales both in a slider (above it) and in a histogram of students' collective guesses (below it). When a round of S.A.M.P.L.E.R. begins, the population is hidden. By clicking on the interface of their computer screens, students each take from the population a set of different individual guesses (Figure 2c) and analyze these samples so as to establish their best guess for the population's target property. Students input their individual guesses and these guesses are processed through the central server and displayed as a histogram on the server's interface that is projected onto the classroom overhead screen (Figure 2d). Note that whereas all students sample from the same population, by default each student sees only their own samples, unless these are "pooled" on the server. Note also that students are each individually responsible for determining the value of greenness coming from their own samples—only once these values are input does the program display the distribution and calculate its central-tendency indices.

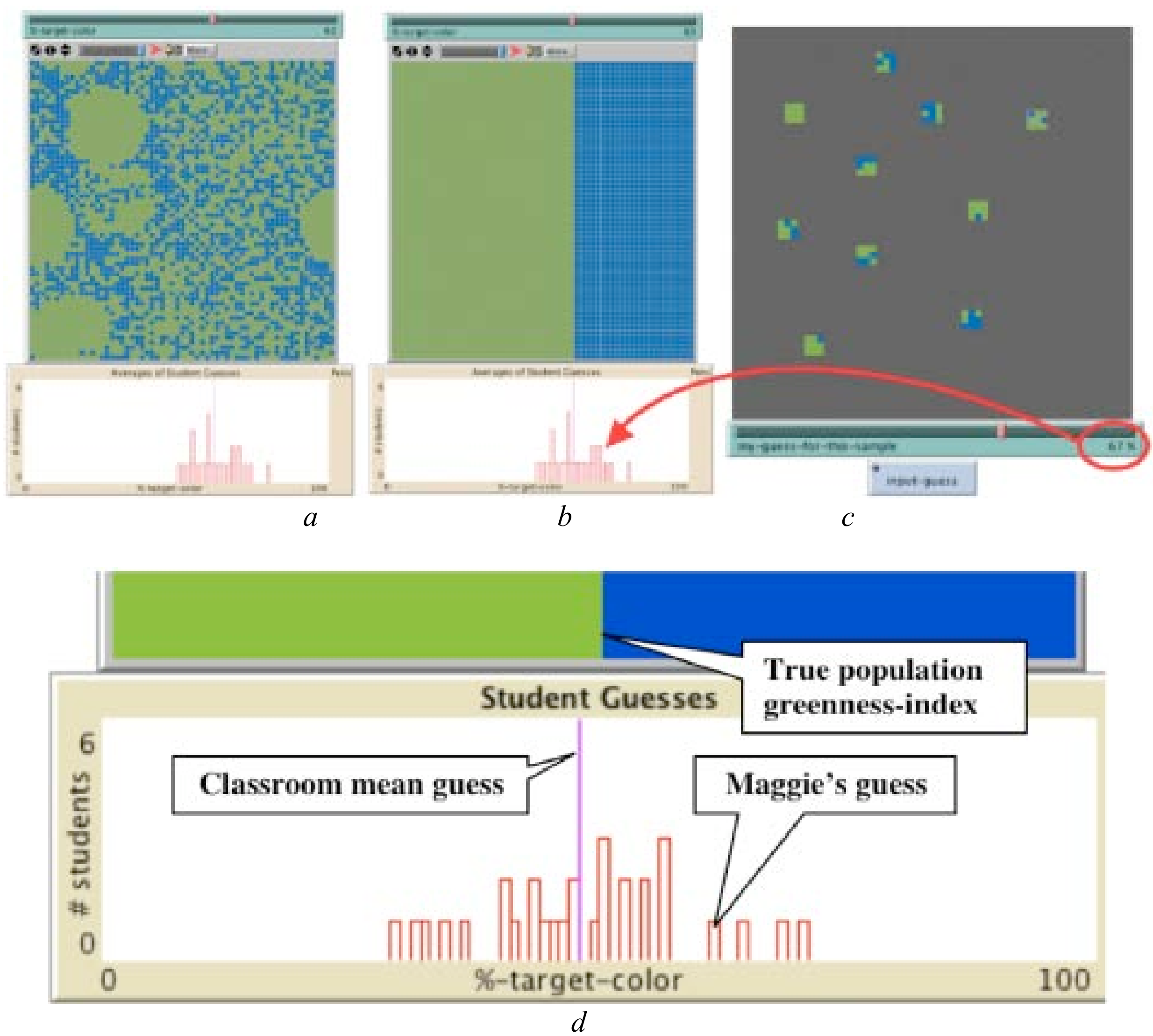


Figure 2. Selected features of the S.A.M.P.L.E.R. computer-based learning environment.

The histogram (see Figure 2d) shows all student guesses and the classroom mean guess and interfaces with the color-separated green–blue population. Note the small horizontal gap (Figure

2d, middle) between the classroom mean guess and the true population index. This gap represents the classroom mean error—it is the difference between the true population value of greenness and students' collective guess of that value. Because a classroom-full of students takes different samples from the same population, the histogram of collective student input typically approximates a normal distribution and the mean approximates the true value of the target property being measured. Individual students are identified with data points on the plot and "embody" these data ("I am the 37"… "So am I!"… "Oh no… who is the 81?!"). That is, the classroom is plotted on the overhead screen as a distribution of data points, and individual students' location in this distribution is *mathematically* meaningful, largely because it is *personally* meaningful. Thus, students can reflect both on their individual guesses as compared to their classmates' guesses and compared to the true population value of greenness, and they can reflect on the classroom guess as compared to the population value. Such reflection and the discussion it stimulates is conducive to understanding sampling and distribution (Abrahamson & Wilensky, 2004).

The S.A.M.P.L.E.R. activities include a group game. Students each begin with 100 points. At each round, points are deducted from each student according to the distance of their guess from the true value. But students can choose whether to bet on their personal guess or on the classroom average guess. Over many rounds, it is more advantageous for students to go with the classroom average, because some populations are not uniformly distributed and so students are dependent on each other for achieving a collective guess that is usually more accurate than their own guess. Abrahamson and Wilensky (2004) found that student reasoning about sample means as associated with their classmates helped students ground essentials of the Central Limit Theorem in terms of the "Law of Large *Social* Numbers." In choosing to go with the group guess, students take a certain socio–mathematical leap of faith—they trust, adopt, and harness the power of large numbers (see also Surowiecki, 2004, on the "wisdom of crowds"). This mental leap to the *group* guess is difficult when students have confidence in their *own* input that reflects their own samples and calculation/estimation; students need to understand that their personal input, reliable as it may be, is just one data point in a distribution of sample means—their input may be completely correct per se yet nevertheless off the mark of the true population value.

S.A.M.P.L.E.R. activities are, therefore, designed to motivate both individual and collaborative work by creating feedback loops between students and the classroom forum. Specifically, whether students are competing against their classmates or collaborating with them, students *each* try to achieve as accurate a guess as possible. Moreover, students who have determined that the group guess is generally more accurate than most individual guesses and have decided to bet on the group-guess are motivated to help their classmates analyze their respective sampling data, because each and every student input will impact the collective guess and, therefore, their own personal success. Thus, the design encourages student-to-student mentoring.

Learning opportunities in S.A.M.P.L.E.R. Abrahamson and Wilensky (2004) found that 6th-grade students working with S.A.M.P.L.E.R. invented a variety of mathematically sound sampling strategies. Also, students leveraged classroom socio-mathematical dynamics. Abrahamson and Wilensky (2004) concluded that:

S.A.M.P.L.E.R. engages students in activities wherein a shared object serves as a platform for articulating intuitions, learning professional vocabulary, testing hypotheses, and debating strategies of statistical inquiry. The inherently collaborative activities in S.A.M.P.L.E.R., embodied primarily in students' interdependence for data and for estimates from these data—impelled students to scholastic argumentation that: (a) teased out individuals' intuitions; (b) afforded opportunities to engage in and refine mathematical terminology, representational forms, and conceptual tools; and (c) introduced and positioned 'distribution,' 'variability,' and complementary micro and macro perspectives in probability and statistics as social–mathematical constructs.

Methodology

Participants

A total of 40 students in two 6th-grade classrooms (the "AM" or morning classroom and the "PM" or afternoon classroom) participated in a three-day (80 minutes per day) implementation of S.A.M.P.L.E.R. in a middle school in a very heterogeneous urban/suburban district (school demographics: 43% White; 37% African–American; 17% Hispanic; 2% Asian; 36% free/reduced lunch; 5% ESL). From Abrahamson and Wilensky (2005c) we have these students' mathematical-achievement group (top, middle, and bottom thirds) and student SES (top, middle, and bottom thirds). We also know that, in this particular student body, student mathematical-achievement group, SES, and ethnicity are related. The teacher was a White female mathematics-and-science teacher in her second year as a teacher. The first author and the classroom teacher shared the facilitation of the activities. Another two researchers assisted with occasional facilitation, technological support, and with videotaping and interviewing the students. Students were seated in a large horseshoe shape. So each student could see most of the other students as well as the overhead screen

Collected Data

Our data include a total of about 8 hours of video footage from the implementation of the design. There are extensive field notes from each day as well as a volume of correspondence between the researchers and the teacher and within the design-research team. Two video cameras were employed to elicit student descriptions of their activities and thoughts. Also, we have student response to a post-test questionnaires designed to elicit student strategies and the reasoning behind these strategies.

Data Analysis

In order to investigate student forms of participation, we studied the video data, observing student sampling-and-calculation strategies, as seen on their individual computer screen and on scrap paper they used to perform calculations. Also, we paid close attention to individual student utterance both in on-the-fly interviews with the researchers and in classroom discussion. To study better these episodes, we transcribed student utterance. These transcriptions supported microgenetic analyses of these episodes as well as discussion within the design-research team over these analyses and over the degree to which the episodes characterize the entire unit (ranging from "typical" to "very rare"). Student post-intervention response on two items relevant to this study was tabulated according to whether students answered affirmatively or negatively on "yes/no" questions that were followed by requests for elaboration.

Results and Discussion

The collaborative construction project (the combinations tower) and the networked-classroom participatory simulation activity (S.A.M.P.L.E.R.) were each implemented over 3 days. Both designs broke away from traditional classroom dynamics in that students were given space and time to pursue their personal ideas towards problem solving a mathematical challenge. The combinations-tower design demanded a concerted effort in analyzing a complicated combinatorial space, and S.A.M.P.L.E.R., too, demanded a pooling of classroom recourses in analyzing statistically properties of a population. Yet, whereas in the combinations-tower design students self organized into skill-specific roles, thus denying many students opportunities for practicing the range of necessary problem-solving skills, in S.A.M.P.L.E.R. students all had ample opportunity to reason through the core problem. Still, high mathematical achieving students had a greater voice in the activity, but this advantage was constantly reestablished "democratically" through evaluation of the efficacy of these students' suggestions and not through teacher-mandated division of labor or institution of a stable hierarchical division of roles (see Abrahamson & Wilensky, 2005c).

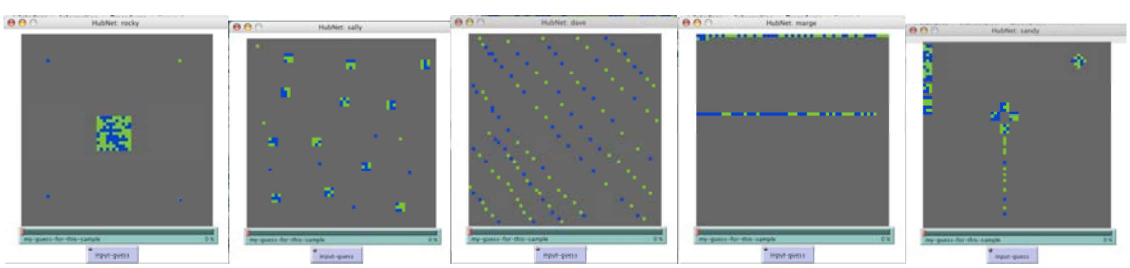
This final section of the paper begins with a description of the implementation. We will go into detail in narrating classroom episodes for two reasons. Firstly, networked classroom design is still relatively new, and so it may be useful for some readers to have a better picture of "what it looks like," in order better to evaluate such design in terms of the classroom dynamics and learning opportunities it creates. Mainly, though, we wish to demonstrate broad and inclusive forums of mathematical reasoning that, at least in our interpretation, are enabled by participatory simulation activity. In particular, we wish to describe how students of all mathematical achievement levels and/or proneness to leadership both contribute equitably to classroom discussion and receive feedback from the forum. This feedback comes from the entire range of classroom mathematical reasoning and not only from a narrow cohort of similar mathematical level, as might occur in designs where student cohorts are either created by teacher grouping or emerge through student-to-student interaction (see Abrahamson & Wilensky, 2005c).

Description of Implementation

In describing the implementation, we will treat the two classrooms as though they are one, unless distinctive classroom-specific behavior emerge that can be tied to classroom-specific student make-up or facilitation emphases. Each implementation day will be overviewed, followed by selected transcriptions that highlight student reasoning in working individually or collaboratively and connections between these types of student work. Specifically, we will demonstrate student: (a) personal sampling strategies; (b) understanding of distribution, mean, and randomness; (c) ideas for classroom collaboration, including coordinated distribution of population segments, sharing sample data and methods for quantifying these data, and optimizing the inputting of guesses once the sample-mean values have been determined. Also, we will demonstrate cases in which the content itself, as shaped by the technology, supported student insight and collaboration.

Overview of Day 1. Lessons began with an introduction to the S.A.M.P.L.E.R. population that was projected on the overhead screen. Students' personal computers were not yet logged in. Students interpreted the population in terms of a collection of 9-blocks and explored quantitative implications of these relations. Students logged in, so each could watch the population from closer, on their personal screens. Students each described what they were seeing (e.g., an elephant with a nose, a diamond, superman, a teddy bear, the collection of all possible 9-blocks like in the combinations tower, one possible combination of a giant "1000-block"). Building on student input, the facilitators gradually steered the conversation to quantitative aspects of the population, and asked how green the population is. Students suggested sampling as a means to focus and calculate the greenness, and the classroom debated the size, number, and location of the samples, and, once samples were taken, how to determine the greenness value from them. We enabled students' logging in, they inputted their individual guesses, and the classroom analyzed the collective histogram. Students interpreted the classroom mean guess and each compared it to their own guess. Once we exposed the population's true value of greenness, students compared the classroom mean guess to their individual guesses for this value, evaluating these in terms of accuracy and how this accuracy reflected the specific samples taken.

We created a new hidden population and enabled student sampling. Each student could now manage their own sampling, and, indeed, each student chose their own sampling attributes (size, number, and location). The teacher moved between students, speaking to them individually, with each student explaining their strategy and the teacher supporting them in moving from qualitative reasoning, e.g., "there is much more blue than green," to more quantitative reasoning, e.g., "I think it's 11% green, because I sampled 100 little squares and 11 of them are green." Each student had different samples, and students had a variety of strategies for determining the population greenness as based on those samples, and so each student had different guesses for green that were based on those samples and strategies. Often, some students collated their samples in one big square, so as to view a "miniature replica" of the large population, whereas other students distributed their samples so as to cover as much ground as possible (see Figure 3, below, for typical sample distribution strategies). Again, students inputted their guesses, these were plotted as a histogram, and students discussed their performance.



a. *b.* *c.* *d.* *e.*

Figure 3. Typical examples of student sample distribution in the same population, each with 125 squares: (a) concentrating in the middle to view a "mini-population"; (b) covering larger grounds with 9-blocks; (c) maximizing coverage with 1-blocks; (d) sample rows; and (e) creative picture.

Sample transcription from Day 1. The following transcription is from the first day of the implementation, when students were sampling from the population on their personal computers. The data demonstrate that whereas students were engaged in solving the same problem, each student, working individually, brought to bear their own mathematical ideas. This transcription is from student utterance over a period of seven minutes—student ideas were elicited by a researcher who interviewed a total of seven students on the fly, moving from one student to the next and asking each one of them for their sampling strategies. The researcher focused on understanding student reasoning and not so much on supporting student progress. Yet, this transcription also demonstrates the teaching *opportunities* that the *teacher* had during this same implementation of S.A.M.P.L.E.R. Preceding each conversation excerpt, we introduce the student. In addition to the student's mathematical-achievement group, which is the focal factor for this paper on equity in learning opportunities, we have included the student's ethnicity, gender, and SES group. We include students' ethnicity and SES factors to demonstrate the demographical diversity of our study classrooms as well as to suggest that this study, which looks at issues of equity among students of varied mathematical achievement, may carry larger social issues, at least in the school district of our study classrooms.

> [El is an African–American female student, bottom-third SES, and middle-third mathematical-achievement group.]
> El: I'm going to start… I want to keep going down and down [parallel columns, beginning on the top-left corner and moving to the right], like start on the line right here [left column] and then I'm, like, going to go around, and then I want to, like, I want to see, like, how much blue there is and how much green there is, <u>then I'm going to try and make a percent</u>.
> [El has a systematic method for sampling. She articulates an intent to use multiplicative reasoning]
>
> [Ta is an African–American female student, bottom-third SES, and middle-third mathematical-achievement group. She is preparing a pencil and a sheet of paper.]
> Ta: The last time, I got pretty close—I was just two [%] off. And what I did was, uhhm, I counted the nine squares, and then took how many [blue squares] is in there, and did however-many was there…. Oh, wait… Ok, if there were 2 I did "9 – 2" [to find how many green], and then, so that was 7, and then I just did that to all of them [all of the samples, each of size 3-by-3]. And then I got … and <u>I added everything once I got that, and then I got to my answer</u>.
> [Ta's reasoning is apparently not multiplicative, unless she is careful to take a total of exactly 100 sample squares, in which case the total number of green squares is also the percentage green of the samples.]
>
> [To is an African–American male student, bottom-third SES, and bottom-third mathematical-achievement group. He has taken eight 3-by-3 samples in the center of the screen and one large rectangular collection of samples on the bottom-left corner of the screen]
> To: Well, first I put it up to 11 squares [a sample of size 11-by-11]—I thought it was going to help better. But then it took too long so I brought it down to 3 [3-by-3]
> Researcher: How did you decide where to put the squares?
> To; <u>I did it randomly</u>
> [To says "randomly," yet his sample distribution is nearly uniform. He has not yet learned to articulate his intuitive method.]

[Ash is an African–American female student, middle-third SES, and middle-third mathematical-achievement group. Ash's samples, of size 3-by3- and 1-by-1, are very uniformly distributed across the screen]

Ash: I used 13 of the bigger blocks [each 3-by-3 squares], and then I figured to use more of the smaller boxes, mostly because my sampling allowance was running out and because <u>I'll probably get a closer guess by looking at it</u>, and then by looking at the big ones I probably wouldn't.

[Ash articulates well her sampling method, but it is not clear how she intends to mathematize her samples or why she considers the smaller samples as particularly useful.]

[Rac is a White female student, middle-third SES, and top-third mathematical-achievement group. Rac's screen is uniformly strewn with samples of size either 3-by-3 or 1-by-1]

Rac: I did 3-by-3 boxes, and I did, like, a kind of pattern, and then, with my [sampling allowance] left, I did just 1-by-1 boxes, to get an idea of what was where. Then I thought there was a little more green than blue, so I guess "52" [% greenness in the population]

Researcher: [] Why not "55" or "57?" Why "52?"

Rac: Because, from my thing [samples] it looked pretty much even, but there are more green; but it looked really close.

[Rac articulates her sample distribution method as enabling her to see "what's there" in the entire population. She the uses estimation to mathematize the samples.]

[Jo is an Hispanic male student, bottom-third SES, bottom-third mathematical-achievement group. Jo has taken two larger samples, one beside the other, all on the top of the screen.]

Jo: I don't know

Researcher: Like, how did you decide to take those samples?

Jo: I don't know—I just chose them. [grins]

Researcher: Kind of "whatever?"

Jo: Yeah.

[Jo has difficulty articulating his mathematical reasoning.]

[Ja is a White male student, upper-third SES, upper-third mathematical-achievement group. Ja has a single very large sample in the top-right corner of the population window.]

Ja: I just tried to make as big a box as I could….so that I could see what's behind; so I could see more, so it would be easier to guess.

Researcher: Why is that easier to guess? Some kids here say they want smaller ones, some say they want bigger ones.

Ja: So you can guess—so it shows you, if… the bigger you get the more you can slide to the right and slide to the left, so you can just guess. You can see how many greens there are and then like try and move them to the side.

Researcher: Oh, I see, just like in the big window when we… ["organize"]

Ja: Yes.

> [Ja appropriated the “organize” technique. By “slide” he means imagining as though all the green squares move to the left, and all the blue squares move to the right. His strategy of concentrating all the samples as one large square is conducive to this imaging estimation technique.]

In sum, each student had an opportunity to problem solve individually. This occurred in the combinations-tower implementation, too, on the first day. However, in that implementation, middle- and lower-achieving students’ problem-solving opportunities decreased drastically after the first day, once students began to actively collaborate in constructing.

Note that this slice of data focused on student individual work within the S.A.M.P.L.E.R. design. It is typical in S.A.M.P.L.E.R. that students only gradually learn that they should collaborate and how they should do so. Our next excerpt will demonstrate group work.

Overview of Day 2. On the second day of the implementation, student sampling strategies became more sophisticated. Partnerships first emerged in the form of students joining up in pairs, and later in groups of three students. Students showed each other their samples, and some student pairs even planned their sampling so as to maximize the coverage of the population. For instance, one student might sample on the left side of the screen and the other on the right. *Thus, while each student worked at their own station, the* <u>*content*</u>*, as it was facilitated in this design, engendered spontaneous collaboration*.

Students still had difficulty in mathematizing and coordinating these pooled samples. For example, one student estimated, based on her samples, that the population was 47% green, while her partner estimated the same population at 33% green. These two particular students argued over the value of greenness, trying to choose the better value of the two, but did not attempt to negotiate a value, such as 40%. In other partnerships, one student input an average of the two values, while the other student did not. In yet other groups, both students inputted their average guess. One student explained that pairs should decide either each to input a guess based on their own samples or both to input a guess based an average of their pooled guesses. Later in this lesson period, group members cooperated by counting up their pooled green and blue squares and calculating an overall ratio and all inputting the same value. Because progressively more groups were acting this way, the histograms, from run to run, had smaller ranges and taller bars (see, for example, in Figure 4, below, the distribution in the histogram that has a minimum of two students per guess).

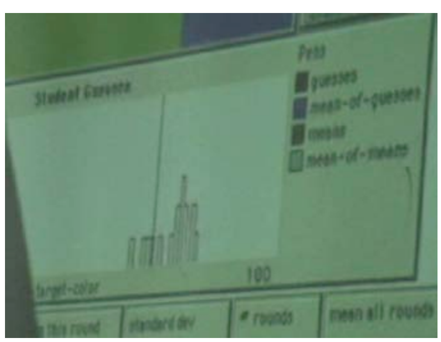


Figure 4. Distribution of student input. Modes of two or more and low distribution variance reflect student data pooling and coordination of input values.

In the afternoon class, a unique mathematical situation pushed up student spontaneous collaboration from pairs and triplets to the classroom level. In a sampling round about half way through the lesson, it so occurred that students were each only receiving completely blue samples. Students were concerned whether they should input "0%," in case somewhere on the screen there were some green squares. So students asked each other whether they had received any green squares, and soon students were asking the question out loud. A precedent was thus created by which the entire class pools individual information. As it turned out, the population was indeed 0% green. *Thus, a unique mathematical case—an extreme value along the continuum of the sample space, engendered spontaneous classroom-level collaboration*.

Sample transcriptions from Day 2. During the first round, after students had input their guesses, we pooled all the student samples on the main screen and displayed the histogram of student guesses. The classroom mean was 27.5% green. Then, students were first given an opportunity either to bet on their own guess or on the group guess. Not a single student chose to bet on the group guess. We revealed the population, and the true value was 21%. Many students lost points. The researcher, addressing the whole classroom, asked why nobody had chosen to go with the group guess.

> [Ky is an African–American female student, bottom-third SES, bottom-third mathematical-achievement group]
> Ky: Maybe they're not right, so you should go with your own [guess].
> [Ky trusts her own guess more than other student guesses. She does not articulate her idea in terms of the samples, the population, or the classroom mean]
>
> [Ka is a White female student, upper-third SES, middle-third mathematical-achievement group]
> Ka: 27.5%, or whatever it was [the classroom mean], uhhhm, it seemed a little bit high for what was there [all the pooled samples]
> Researcher: Oh, so you felt that it was too high. What was *your* guess?
> Ka: 20.
> [Ka, like Ky, trusts her own guess more than other student guesses. She articulates her idea in terms of mistrusting the classroom mean as compared to the data]
>
> [Je is a White female student, upper-third SES, upper-third mathematical-achievement group]
> Je: Well, I, uhhm didn't go [with the group], because some people guessed really high and some people guessed really low, and just because it's the average it's not necessarily going to be what, uhhhm… because, if somebody decides to get really high, because when they do their sampling they got a lot of greens, and then some people got almost all blues, it's just going to have to average those, it's not going to necessarily go with the most accurate one. So that's why I went with my own guess, which was 21%.
> [Je interprets the distribution of student guesses in terms both of the population's true properties—how the green and blue squares are distributed—and student "measurement error"—students' over- or under-guessing based on their samples. Later, Je explained her strategy. She had only taken samples of size 1-by-1. She took 4 sets of such samples, each with 10 squares, for a total of 40 exposed squares. She then calculated a proportion.

> As it turned out, however, she had calculated the proportion of green squares out of blue squares, 7/33, rather than green squares out of all squares, 7/40. Had we not discussed her strategy with her, she would not have realized her fortuitous error.]

Note that Ka and Je, whose mathematical reasoning was more advanced than other students, also achieved high accuracy. This is important for other students to witness, so that they are encouraged to be more mathematically sophisticated—to understand that rigorous methods are conducive to higher accuracy; that guessing is insufficient.

On a subsequent run, we intentionally created a population with a non-uniform distribution of the green squares. Consequently, student individual samples varied more than on the previous run. This apparently caused some concern, since more students shared resources. For example, Lot and Devvy peered at each others' samples. Lot said to the researcher, "I'm going with the group guess this time. Because I got like… He [Devvy] says he got all that green, [yet] I have [only] 5 green. I don't think mine's accurate." Other students said they would decide whether or not to go with the group guess only once the group guess is revealed. Students were reluctant in committing themselves to the group guess before having seen all the guesses, but over three additional round, more and more students chose to go with the group guess. Also, from round to round, the classroom guess was increasingly accurate.

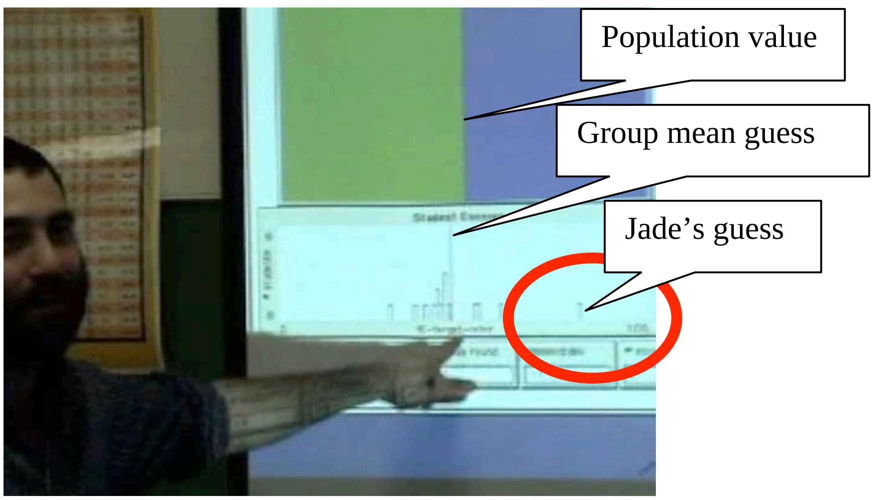


Figure 5. A researcher–facilitator pointing to a distribution with an outlier.

Of particular interest was a run in which the group guess was better than any individual guess (see Figure 5, above). Note the close clustering of most student guesses around the mean and the quasi-normal distribution of guesses. Also note that one student guess deviated significantly from the guess cluster (see, in Figure 5, above, the single bar on the far right, highlighted by a ellipse). In a classroom discussion around this guess, Jade identified herself as the student who had input this guess. A student interpreted Jade's guess as indicating that her samples must have been greener than other student guesses. If Jade were working on her own, she may not have received feedback on her performance (which had been flawed). Another student came up to the board and analyzed the histogram, explaining that had it not been for Jade's high guess, the classroom group guess would not have been so accurate—Jade's guess pulled the classroom mean up. Following this analysis, Jade was thanked, especially by students who had chosen to bet on the group guess. This episode is an example of how personal and interpersonal planes interface in the participatory simulation activity—Jade's bar connected between her own screen (private space) and the server screen (public space): Jade uploaded her guess to the classroom histogram, and the histogram, in turn, contextualized Jade's guess in light of other students' work. The episode is also an example of how a facilitator can help turn a social event into a mathematics learning opportunity, and vice versa.

In another notable episode, students, who had all input their guesses, requested that we pool all the samples so that they see all of the samples together. The facilitator pooled the samples and asked that any student who wishes to change their initial guess could do so as long as they explain the reason for their change. Many students did change their guess, explaining that once they saw a "bigger picture," they realized that their previous guess was inadequate. The facilitator asked how all these changes in input might affect the histogram. Students replied that the histogram would be more "bunched up." Indeed, once all the new guesses were input, there was a big clustering of guesses (lower variance than usual). *Thus, students grounded new mathematical constructs in their collaboration strategies.*

Overview of Day 3. On the third day of the implementation, we played a game in which the morning and afternoon classes competed. There were four rules to this game:

1. Everyone goes with the group guess
2. No "Pool Samples" [students do not see all the samples at once]
3. Five rounds of 10 minutes each
    a. 7 minutes to input
    b. 3 minutes discussion
4. A limited "sampling allowance" of 100 squares per student

The rationale of this competition was to enhance student-to-student sharing of strategies and support. A new monitor introduced on the interface showed students' mean points. Once these rules were stated, a classroom debate erupted over best strategies for optimizing the accuracy of the group guess. One possible strategy the facilitators were hoping to steer students towards was that students each look at a different part of the population. Also, we were wondered if students could reach an understanding that one and the same classroom mean guess would result from the following two group-level plans: (a) each student calculates their sample mean and inputs it; or (b) students first pool their samples and then calculate the mean of all of samples and all input that mean. The former course of action would result in a distributed histogram, whereas the latter would only give the mean. Also, the latter strategy would be redundant, in that the computer program calculates the mean. Finally, we were hoping to foster opportunities for students to reason proportionately. To scaffold student proportional reasoning, we began with each student having an allowance of 100 squares (see above, Rule #4). The following transcription demonstrates the richness of the designed learning tools. Also, note how each student contributes an element within this richness, introducing content from the previous week, when students studied the combinatorial space of the 9-block.

> Pri: We can work together. Three or four people can take part of the square [the population].

Teacher: Some of you were doing that on Tuesday already, right? You were dividing it up so you could sample more area.
Rac: What if half of the class does the top half of the square, and the other half of the class does the bottom half of the square?
Student: Yeah.
Teacher: And how are you going to look and decide what the numbers are?
Ja: Count how many greens you have!
Researcher: So we just count up all the greens? And then what do we do?
Ja: You subtract it from a hundred.
Researcher: Like… what?… Count up yours or everyone's or what?
Ja: Everyone's.
Researcher: Ok, so say we get… I don't know… If we get 212…?
Ja: Then…mm'mm? [Ja is nonplussed; some laughter in the classroom]
Researcher: I like where you're going. We just have to figure out what to do with it.
Pr: Subtract it by 512.
Researcher: Subtract it from 512. Oh, I think I know where you're getting that number from. That's the number of combinations in the combinations tower.
Pri: But isn't that the same amount as there is in the…[population?]
Researcher: [reminds students of earlier lesson, when they had determined that there are about 400 9-blocks in the population]
Mo: It wouldn't do any good to count all the blues and the greens, because you don't know how many squares there are in all, so, like, you could count 50 blues, but it still wouldn't be half and half, because you don't know how many…
Researcher: But we *do* know that each kid has 100 squares allowance.
Mo: Right, but… // But, I mean, we don't know how many squares there are in the whole thing, so the mathematical stuff wouldn't really work.
Researcher: [reminds students of earlier lesson, when a student counted 61 squares per row and per column in the population]
Jo: [says the population doesn't look like a square. The researcher responds that this is due to projection distortions.]
El: But when you click on the thing and it shows up, it doesn't have… How can you count the squares unless you make a grid, because they're all together.
Teacher: That's a good question. You can think about it as you're working.

The above transcription captures student collaborative statistical inquiry. Students' contributions to the conversation differ in the initiative and mathematical reasoning they reflect, for instance Mo was moving in the direction of proportional reasoning, whereas Ja did not know how to process the data at all. The content, the affordances of the learning tools, and methods for using these tools are interwoven, for instance, in talking about the squareness of the population or needing a grid to count the squares. That students have trouble understanding the content through sorting out the elements of the learning tools they are working with is testimony of a need for such classroom-level discussion of the activities as well as testimony of these Grade 6 students' poor fluency in proportional reasoning.[3] The collaborative-learning affordances of PSA together with student difficulty with the mathematics necessary for successful participation in these PSA suggest that these PSA be implemented *not* in later years but in earlier years. Through such engaging activities, students could potentially bootstrap the mathematical content.

In the other classroom, many students were enthusiastic to use a "telephone game" method, by which they all pass on their guesses. The debate then focused on whether or not it was a good idea for students to share their guesses even before they have input them. Most students thought this was a good idea, but then Sa, rated by the teacher as the top mathematical thinker in his classroom, said

> It's not a good way, because if one person says it, and they're way off, and everyone trusts them, then the group guess will be way off, and it will screw up the whole guessing thing, so it's better that everyone guesses their own guess. Or guesses, like,… not everybody follows the same one.

To this, another student responded that, "Actually, it's a *good* idea, Sa, because if it does get way off, you'll tell us, and then we can change it." Sa retorted, "but I'm not always right." Lot said that Ray should be the "marshal," because he had done the best on the previous day. Ka suggested that students each present their guess and then the classroom should debate which guess is the best.

Students shared their samples loudly. They divided the population among the classroom. A student called out, "Who got more green [than blue]?" and then, "Who got more blue [than green]?" All students responded by raising their hands at the appropriate moment. One could have used this poll as an index of the greenness. For instance, if three-quarters of the class got more green, then 75% would be a good guess. At that moment, however, students did not think of that strategy.

After all the students had input their guess, we displayed the histogram of all the guesses and gave students an opportunity to change their guess. Several students stated that their guess is higher than the classroom average and the classroom average is too low, and so they would increase their guess just so as to drag the classroom average up. They did so and the guesses were replotted. Once the true value of green was revealed, it turned out that the classroom average was too high and, so, students lost several points. This annoyed some students, who explained that participants should not change their guesses; that if they happen to have greener samples than other students, they need not assume that the entire population is as green as their samples. The classroom debated whether they should each input their personal guess or each input some agreed upon value.

[3] The classroom was challenged when asked to calculate the proportion of 10 green squares out of a total of 50 squares. Several students called out "5%." The researcher asked the teacher whether students had studied proportion, and she replied that indeed they had.

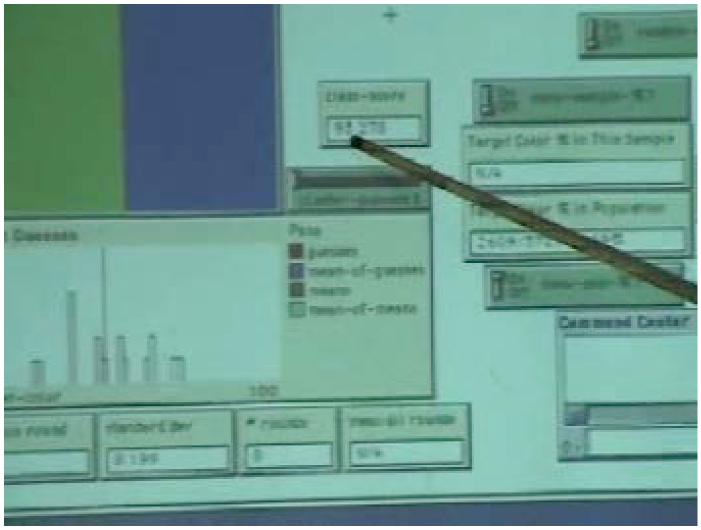


Figure 6. Histogram bars reflect student work in pairs and triplets.

Figure 6, above, demonstrates that on the subsequent round many students chose to “co-guess,” based on local averaging. Students had not changed their guesses as in the previous round. Note that the classroom guess is only 2.3% away from the true value. Sa commented:

> Maybe that says that if we don’t talk—don’t set it up with each other—the better we do. Maybe if we set it up in a different place, we all get the same *things*, but if we all do it randomly, we’ll all get different things, most likely.

The teacher asked whether students should be calculating their guesses rather than just basing these guesses on perceptual judgment, and Lot suggested counting up the green squares and determining a ratio.

Thus, the classroom struggled with several difficult mathematical problems simultaneously: (a) Should students “guess and then discuss” or “discuss and then guess?”; (b) Should students coordinate their samples or just sample randomly?; and (c) Should students bother to calculate sample means, or is this redundant, because errors from perceptual judgments cancel each other out?; In fact, might it not be the case that student guesses will be *less* accurate if they attempt to mathematize their perceptual judgments, due to the limitations of student mathematical skills?

These questions are fundamental to statistics. They make sense only as coordination issues *between* students. Thus, statistical content, in this design, is dependent on authentic collaboration in problem solving. Students must address these issues in order to perform well, and so the content is grounded in a meaningful and shared classroom experience, an experience that can be referred to in subsequent classroom conversations, after the intervention is over. For this sharing to be truly inclusive, all students must have a voice. The following incident, though singular, unplanned for, and brief in the overall picture of the implementation (the incident lasted 30 seconds), sheds light on the role of the networked technology in enabling equitable classroom discourse.

Two students, Je and Cha, had stepped out of the classroom, but their computers were still logged in to the system, and so we needed guesses inputted from these computers. The teacher, standing by these two computers, addressed the classroom, asking what values she should input for these absent students. The task is not spelt out, so some students may have understood that these guesses should be neutral in terms of the classroom overall guess, yet other students may have regarded this as an opportunity to trumpet their personal vote. Students began calling out their opinions, first in turn, with each student addressing the previous student, but then simultaneously. The teacher, facing this shouting-poll heterophony, must decide.

> Teacher: So what should I put in for Je, here, and Cha?
> Students: 15. 20. No, 24! 25. No, 30!!! 20.16. [voices rise]
> Teacher: 15, 20, 24, 16…
> Students: [overlapping voices] 23. 25. 20. 20. 20. 16.There's not a lot of green. 20. It's too high! 20, 20, 20. 20. There's not a lot of green. 20, 20. 30. There's not a lot of green. 20, 20, 20.
> Teacher: I'm going to put in 20, right down in the middle.

Of note is that only about half of the students participated in this Attic vote. It is not entirely clear whether the growing preponderance of "20" is due to students converging on this value, whether it reflects the number of students intending to input "20," or if it is just a strong lobby for this value. Yet, towards the end (see transcription, above), one student voted "20" often, using a different voice for each vote, as though he were spontaneously changing identities—impersonating and embodying a larger crowd—thus thickening the vocal mode of "20." The teacher's decision to input "20" is fairly reflective of the shouts, albeit "23" would have incorporated each voice only once (and would have been only 2% and not 5% away from the true population value). However, in comparing these votes to students' subsequent computer-mediated input in this round (see Figure 7, below), we note the absence of the voices "5," "17" (two students), "18," and "19" (two students). Thus, the technology supports a representation of student voice distribution that is both more reliable and more accurate.

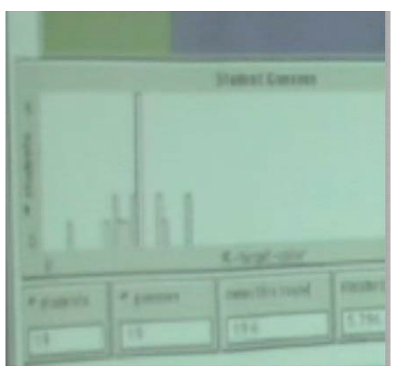


Figure 7. Student guess distribution with a mode of 20 and a mean of 19.6

From one round to the next, more and more students were counting their samples and not just eyeballing them. These students, upon receiving feedback as to the relatively high accuracy of their individual guesses, became a vocal lobby for counting up samples and not just using perceptual judgment. When a sufficient majority of students decided to count their samples and not just guess, this method was agreed upon as an effective method. All students adopted this method. The classroom achieved a high-accuracy guess. In Figure 8 (see below), the classroom mean input for the population greenness is 91.2% and the true population value is 92%. For the remaining rounds, students carefully counted the sampled squares. Guessing was no longer considered a sufficiently accurate mathematical practice. Students were now mathematizing their perceptual judgments.

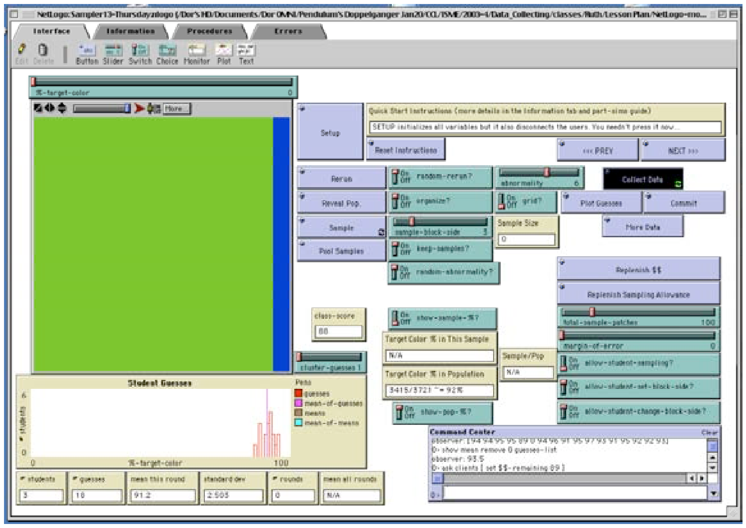


Figure 8. Accurate classroom guess achieved through mathematizing samples (screenshot)[4]

Note in Figure 8 (see above), the low variance of the classroom input distribution. The low variance is primarily due to some student groups coordinating a common value. Yet, other students did not coordinate and, instead, input their own guess. Thus, a final issue students had to resolve was whether they should each input their personal value or first compute the classroom average and then each input that mean value. Several participants claimed that students should each input their own guess, because the program will compute the average more accurately. Other students thought that the classroom average would be "stronger" if many students guessed the calculated average rather than inputting guesses that are above or below the average (as though a mean that is also the mode is in some sense a more powerful estimate). Yet the majority of the students were content to input their own guess, and so this practice was announced as the improved strategy that all students were expected to follow. Indeed, Figure 9, below, demonstrates that on the subsequent round, classroom guesses were normally distributed and that the mean guess was precisely the population value.

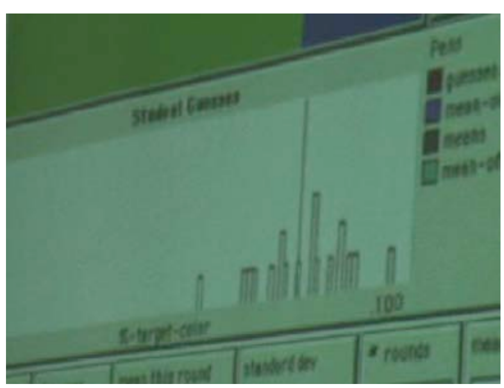


Figure 9. Normally distributed and highly accurate classroom guess.

[4] In the current version of S.A.M.P.L.E.R., this interface has been simplified.

The classroom persisted in this strategy on the last round and, again, their mean guess was precisely on target. Thus, through five iterations under the four competition rules, the optimal mathematical practice emerged as a classroom collaborative optimization solution.

Summary

We have described the implementation of S.A.M.P.L.E.R., a participatory simulation activity in the mathematical domain of statistics. We intend our description to demonstrate the range,

Table 1.
Learning Affordances of Non-Networked and Networked Designs for Collaborative Learning

	Combinations Tower	S.A.M.P.L.E.R.
Participation	Some students not engaged yet this is not immediately evident to facilitator. Loose group leadership and/or unsuccessful within-group distribution of labor allows for 'hangers on.'	All students consistently engaged as evident in their sampling and input. The technology plays a major role in mitigating classroom management issues, thus allowing the facilitator to devote time to individual students.
Support	Students do not have access to the reasoning of most of their fellow students, and, in any case, they do not have access to *all* students' mathematical reasoning. The teacher must be physically near each student to elicit their reasoning.	Student output is anonymously displayed for all to see and respond; student interdependency embedded in activity—it is within the classroom's interest to mentor strugglers. The teacher can elicit student strategies by discussing the histogram.
Student-centered	Once assigned to task forces, most students no longer initiated solution methods	All students continuously had opportunities to invent, explore, and refine solution methods
Inclusion	Weaker student opportunity to contribute is sensitive to the math-ability make up of their groups	Mathematical "outliers" received special attention, because every voice was critical for overall success
Leveraging social dynamics	Social dynamics worked against weaker students, who, through self selection and tacit peer pressure, adopted more menial and less mathematical roles	Peer pressure was turned towards each student, yet the learning zone did not become stratified, since all students worked with the same tools
Equity	Students self-organize by role and many develop delimited expertise	All students engaged in the same task; all input is weighed equally

richness, and depth of classroom discussion enabled by HubNet networked-classroom technology. We focused on several brief classroom episodes both to exemplify classroom progress along a collaborative problem-solving track and to show how the design supports an ongoing assessment of many students' understanding that is expressed through their electronic gesture as well as their verbal commentary. Abrahamson and Wilensky (2005c), looking at data from the same classroom, investigated learning opportunities in a non-networked collaborative construction project in the domain of probability (the combinations tower). Arguably, these two designs are comparable in terms of the level of mathematical reasoning demanded for successful completion of their respective core tasks. Table 1, below, presents a summary comparison between findings from that study and the present study along dimensions of participation and collaboration.

Post-Intervention Questionnaire

Following the completion of the mini-unit, students filled out questionnaires designed to elicit their personal strategies and the statistical reasoning behind these strategies. Table 2 (see, below) summarizes the number of students who responded affirmatively, negatively, or otherwise on two items. The table is structured so as to group students according to their "compound" response on both of these items. That is, the rows group students by their response on Item #1, and the columns group students by their response on Item #2. Thus, each student is assigned to a single cell in the table.

Student reasoning, as expressed in their written responses, was grounded in their personal classroom experiences. For instance, students who recommend betting points on one's own guesses explained that they had lost points when betting on the classroom guess, because some students did not perform well on this collaborative task (e.g., "They guessed way off"). Students who preferred betting on the classroom guess explained that doing so saved them the loss of points. Students who responded that one should bet either on one's personal guess or on the group guess explained that one's choice should depend on: (a) one's confidence level per population (e.g., one might feel insecure when the population appears to be non-randomly distributed in terms of the location of green squares); (b) one's sense of personal skill in guessing and calculating (e.g., if you do not feel that you are an able mathematician, you should go with the group guess, because you may well be considerably off mark); (c) one's interpretation of other students' work, such as if you are informed of their samples or guesses (e.g., if there are outliers, you may decide to go alone); and (d) whether the activity has been declared as a competition within the classroom or between classrooms (e.g., if it is a between-classroom competition, one need not mind if one's guesses deviates from the classroom mean as long as the classroom is doing better as a whole).

Table 2.
Student Response on Two Post-Intervention Questionnaire Items

	2. "Should students pool their data before guessing?"		
	Yes	No	Other
1. "What should you bet your points on?"			
On your own guess	10(5)*	0(2)	1(0)
On the group guess	7(3)	0(3)	--
It depends	3(6)	--	--

Note. The items were open ended (not multiple choice). For the sake of clarity in this table and discussion, we have modified the original text of the items. Item #1 was, "Some students chose to go with their own guess. Other students went with the group guess. What is better—to go alone or to go with the group guess? Why?" Item #2 was, " Some students wanted to see other students' sample before inputting in their guess. Is that a good idea or not? Why?"
*Afternoon classroom values appear in parentheses

Note, in Table 2 (see above) that only in the afternoon classroom did some students respond that one should not attend to data other than one's own. This difference between the classrooms maps well onto students' differential experiences in these classrooms: On the last implementation day, students in the afternoon classroom conducted their last two runs by *not* sharing data and achieved higher-accuracy group guesses (they also beat the morning classroom, with 88 vs. 86 points after five rounds—this despite one "give away" round in the morning classroom, in which the greenness was a stark 0%). Given the fit between student explanations and classroom experiences, the two items appear to be reliable in eliciting learning experiences in the two classrooms.

Of the 9 students who replied that "It depends" on the item, "What should you bet your points on?," all 3 morning-classroom students and 3 of the afternoon-classroom students are among the highest-achieving students of their respective classrooms. Seven of these 9 students were female students. Nine of the 13 students who replied that one should go with the group guess were female students. The 3 students who replied that one should not attend to other people's guesses were all female and were either middle- or low-achieving students. There is *no* difference between the "On your own" and the "On the group guess" rows in terms of mathematical achievement, gender, ethnicity, or SES. We interpret this result as indicating that the design offered equal experiences for students irrespective of their background or coming-in mathematical level. This latter finding stands in stark contrast to findings from a comparative analysis of the combinations-tower implementation (Abrahamson & Wilensky, 2005c), in which the classroom was stratified in terms of their post-intervention achievement by mathematical level, ethnicity, and SES.

Conclusions

In comparing two designs for collaborative learning, one that was enabled by the HubNet networked-classroom technological infrastructure (S.A.M.P.L.E.R.) and another that was implemented in traditional media (the combinations-tower project), we found that the HubNet classrooms were: (a) more <u>demanding of student participation</u>—each and every student was expected to share the products of their mathematical reasoning (b) more <u>supportive</u>—the product of student reasoning was exposed to many eyes, and products that deviated from the classroom norm were easily assessed as such and received facilitator and student attention; (c) more <u>student-centered</u>—students each had opportunities to pursue their individual sampling strategies; (d) more <u>inclusive</u>—the activity supported classroom-level discussion as well as group level discussion, so all students were exposed to more explanations from their classmates, including the reasoning of the higher-achieving students and not only to the input of like-achieving cohort members; (e) more <u>suited to capitalize on classroom social dynamics</u>—the significance and quality of individual student work was built and contextualized by classmate electronic action, with proximal classmates streaming information orally/gesturally and distal classmates interacting through the computer medium and classroom discussions; and (f) more <u>equitable</u>—students all engaged in the same mathematical challenges, were not marginalized to fringe tasks, and were not dominated by higher-achieving students (so even the most reticent students had a voice and vote in the collaborative activity).

CLZ \ SLZ	Ann	Bert	Cathy	David
Ann	Skill 1	Skill 2	Skill 3	Skill 4
Bert	Skill 1	Skill 2	Skill 3	Skill 4
Cathy	Skill 1	Skill 2	Skill 3	Skill 4
David	Skill 1	Skill 2	Skill 3	Skill 4

Figure 10. Learning trajectories in continuous (CLZ) and stratified (SLZ) learning zones.

Figure 10 (see above) represents two different types of student learning trajectories that may result from the implementation of designs for collaborative projects. In a *stratified learning zone* (SLZ; Abrahamson & Wilensky, 2005c), students self-organize into a production assembly line, with each student engaging in the practice of a delimited skill. For example, Cathy (see above, the second-from-the-right column) repeatedly engages in a project-based action by which she practices “Skill 3,” which might be “counting” or “coloring in little squares.” In a *continuous learning zone* (CLZ), Cathy works from “Skill 1” through to “Skill 4” (see above, second-from-bottom row). We contend that participatory simulation activities in networked classrooms allow for continuous and not stratified learning zones. These networked activities are more equitable in that they are both more demanding and more supportive of all students. *The key to equitable learning in collaborative classrooms is to furnish all students with the same learning tools and build facilitation infrastructure that provide enough personal space and time for students each to problem solve individually as well as share their ideas and receive feedback from the facilitator and many classmates*. Student participation was channeled, expressed, and conditional on particular modes of using specific tools—taking samples from a population and inputting a value representing the sample mean. That is, students each had to mathematize their own perceptual judgments. The design created both a personal and group-mediated incentive to achieve accuracy, so all students were encouraged to develop the core skill targeted by the design. Students operated like an orchestra, in which they all played the same instruments yet each played it in their own style and as best they could. What was it exactly about the networked activity that allowed for more learning opportunities? The following sub-section addresses this question.

Activity, design, and learning affordances. The premise of any comparison-based study is that the study-related essential factors are common to the two objects of comparison. The combinations-tower and S.A.M.P.L.E.R. activities are both in the domain of probability and statistics and both require of students careful analysis, mathematizaton, sophistication, and articulation of intuitive strategies. However, central to the combinations-tower activity and not to S.A.M.P.L.E.R. was the objective of producing a material object as the “deliverable” of the classroom collaboration. It is this production process, by and large, that gave rise to the stratification of the classroom learning zone, in which the more menial tasks were completed by the lower mathematical achievers of the classroom. In that sense, the combinations-tower design for collaborative learning is a straw man in this comparison—ostensibly, if one were to strip the combinations-tower activity of its materiality, it would not lead to stratification. Yet this straw dog bites back: It is the very nature of the medium in which the combinations tower was implemented that allowed for the stratification to emerge. The combinations-tower traditional activity-medium has a:

1) physical limitation on the number of people who can co-operate simultaneously on the same minute objects. Yet in S.A.M.P.L.E.R., all students could dip their hands at once into a 10cm-by-10cm object without seeing what they each pulled out.
2) procedural limitation on the engineering, distribution, and coordination of the production and assembly of these physical objects. In S.A.M.P.L.E.R., computer procedures controlled the coordination of student input.
3) feedback limitation on the goodness of a method—in producing many 9-blocks, you can eliminate duplicates through visual comparison, but unless you patently witness a possible flaw in your combinatorial strategy, you may be oblivious that this strategy either creates duplicate items or does not exhaust the sample space. In S.A.M.P.L.E.R., the effectiveness of sampling-and-calculation techniques emerges through classroom analysis of the histogram created by the computer program in comparison to the true population value that is disclosed (note that in common statistical practice, the trueness of the inferred population value is intrinsically tenuous).
4) motivation limitation—students who were committed to a personal method that was proving ineffective were loath to cut their losses and abandon their method, because they had created many 9-blocks that they would not know how, subsequently, to reorganize according to a new method. In S.A.M.P.L.E.R., errors were relatively cheap—the multiple runs enabled students to begin each run from scratch and, thus, explore multiple strategies.
5) exploration limitation—once students were committed to contributing by following a particular group strategy, any deviation was "expensive." In S.A.M.P.L.E.R., students were free to work exploratively within their private domain, as long as they contributed to the public domain according to the consensual methodology.

Thus, networked participatory simulation activities appear to address and all but obviate spatial, physical, and logistical constraints on collaborative participation imposed by the materiality of concrete learning tools.

An ideal facilitation of the combinations-tower activity would put a mathematical spin on the challenges inherent in traditional media. Yet, classroom pragmatics, such as consideration of time and sustaining student engagement, challenge the implementations of facilitation solutions that do not tradeoff between the activity product (the combinations tower) and the process (learning) that it shapes.[5] The results of this study suggest that network technologies such as HubNet mitigate the teacher's dilemma between product and process. These technologies are still relatively young as classroom supports, and much R&D is still needed to improve these technologies and understand their advantages as well their constraints. However, this study indicates the learning affordances of this medium for facilitating mathematics content with student inclusion and engagement.

---

[5]Recently, we have designed a HubNet networked version of the combinations-tower activity.

A final point of comparison between the designs regards student access to the learning tools. Students could take the combinations-tower tools home or even re-create and investigate 3-by-3 grids using pencil and paper. S.A.M.P.L.E.R. materials, at least in the implementation reported in this study, did not leave the classroom space. We consciously chose not to assign computer-based homework, even though these materials are available as a resource on the internet, both as interactive simulations and as part of the NetLogo free download package.[6] This decision was made in corroboration with the teacher, who informed us that a significant portion of her classroom does not have reasonable access to computers outside of school.[7] Student access to technology is in itself an equity issue that must be addressed by policy makers. It is our hope and vision that student access to our materials will grow with the increasing penetration, into the public domain, of technology as well as the promise of technology-enabled educational materials. Currently, development efforts are under way to massively scale this technology, and future studies will examine these efforts.

Pedagogy, design, and equity. We are not claiming that in participatory simulation activities all students perform at the same level—in any body of students there will be some range and diversity in mathematical reasoning. The design problem is what happens when this mathematical heterogeneity is plotted onto free-range collaborative activity: Can we embrace this heterogeneity or do we inadvertently increase it? Our research is aimed at addressing student heterogeneity by designing facilitation infrastructure that pull up the mean of this achievement distribution whilst decreasing its variance. In S.A.M.P.L.E.R., as in the combinations-tower project, classroom-level success increased as some students adopted other students' solutions. But in S.A.M.P.L.E.R., more students worked over more time on the core problems and not on peripheral tasks, so more students could share in the discovery process of their classmates' more advanced solutions as well as understand and appreciate the efficacy of these solutions. So, we do claim that in the networked activities, students have equal *opportunities* to think hard, progress at their own pace, and bring to bear and develop their personal skills, and that consequentially more students are engaged at a higher mathematical level as compared to their engagement in collaborative projects where stratification is liable to emerge. The thrust of this paper is not about the higher-achieving students—these students flourished in both designs. Rather, this paper is about the middle- and, especially, the lower-achieving students—it is more with them in mind that designers should strive to build learning environments that afford equitable opportunity.

S.A.M.P.L.E.R. was designed with the assumption that mathematics content may be introduced to students as enhancing their intuition and everyday good reasoning. We wish for students to adopt mathematical tools only once they recognize the ultimately limited capacity of their prior intuition and reasoning. In this study, students began counting their samples and computing proportions only once they recognized that doing so is in their interest; when students who were

---

[6] S.A.M.P.L.E.R., along with all NetLogo software and models, is available for free download at http://ccl.northwestern.edu/netlogo/ . For a standalone "solo" version of S.A.M.P.L.E.R., go through the models page at http://ccl.northwestern.edu/curriculum/ProbLab/index.html .
[7] For a balanced view of the "digital divide" in the U.S.A, see Compaine (2001), but see also Kuttan and Peters (2003).

using these methods were consistently achieving higher precision. A prerequisite for adopting new tools may be, thus, that students value and wish to engage in the social dynamics of progress. In S.A.M.P.L.E.R., we attempted to stimulate and leverage such social dynamics by embedding student playful interaction into the activities. Student response suggested that the design resonated with students' models of "group play." It is still necessary to understand how students' participation in S.A.M.P.L.E.R. activities interfaces with learning goals, such as those delineated by state and national standards. More generally, we identify a need to articulate design principles for learning environments that intrinsically motivate students as well as vouchsafe that students leave schools equipped to enter the better echelons of the work market.

Future work. In this paper, we has focused on opportunities and participation and not yet analyzed the learning results. These results will be reported in a future paper. Also, in future data analysis, we will describe several case studies of students who participated both in the non-networked and networked activities to have richer pictures of the differential affordances of the designs and their relation to student mathematical achievement. Also, we will continue to improve the design of our networked activities so as to model, emulate, and enable features of authentic student-to-student and student-to-teacher collaboration practices that occur spontaneously outside of the virtual space. For instance, we will create virtual spaces where students can "step aside" to consult with specific classmates, log and share their individual strategies, query the facilitators, and access classroom-data-base and online information. This development work carried out at the Center for Connected Learning and Computer-Based Modeling is intended to augment traditional classroom learning, not replace it. At the same time, virtual learning spaces will help us reach broader populations of learners, including learners in remote locations.

Acknowledgement

This study was partly sponsored by the NSF ROLE grant 0126227.

References

Abrahamson, D. (2004). *Keeping meaning in proportion: The multiplication table as a case of pedagogical bridging tools*. Unpublished doctoral dissertation, Northwestern University, Evanston, IL.

Abrahamson, D., Blikstein, P., Lamberty, K. K., & Wilensky, U. (2005, June). Mixed-media learning environments. *Proceedings of the annual meeting of Interaction Design and Children 2005,* Boulder, Colorado. (Manuscript in press) http://ccl.northwestern.edu/papers/Ab+al_IDC2005.pdf

Abrahamson, D. & Wilensky, U. (2002). *ProbLab*. The Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL. http://ccl.northwestern.edu/curriculum/ProbLab/index.html

Abrahamson, D. & Wilensky, U. (2002). *S.A.M.P.L.E.R.* The Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.

Abrahamson, D. & Wilensky, U. (2004). S.A.M.P.L.E.R.: Collaborative interactive computer-based statistics learning environment. *Proceedings of the 10th International Congress on Mathematical Education*, Copenhagen, July 4 – 11, 2004. http://ccl.northwestern.edu/papers/Abrahamson_Wilensky_ICME10.pdf

Abrahamson, D. & Wilensky, U. (2005a). *Is a disease like a lottery?: Classroom networked technology that enables student reasoning about complexity*. Manuscript in preparation.

Abrahamson, D. & Wilensky, U. (2005b). The combinations tower: A design for probability. Manuscript submitted for publication.

Abrahamson, D. & Wilensky, U. (2005c, April). The stratified learning zone: Examining collaborative-learning design in demographically-diverse mathematics classrooms. In D. Y. White (Chair) & E. H. Gutstein (Discussant), *Equity and diversity studies in mathematics learning and instruction*. Paper presented at the annual meeting of the American Educational Research Association, Montreal, Canada.

Abrahamson, D. & Wilensky, U. (2005d). ProbLab goes to school: Design, teaching, and learning of probability with multi-agent interactive computer models. In D. Pratt, R. Biehler, M. B. Ottaviani, & M. Meletiou (Eds.), the *Proceedings of the Fourth Conference of the European Society for Research in Mathematics Education*. Manuscript in press.

Abrahamson, D. & Wilensky, U. (2005e). *Learning axes and bridging tools in a technology-based design for statistics.* Manuscript in preparation.

Ares, N., Brady, C., Haertel, G., Hamilton, E., Kaput, J., Nasir, N., Pea, R., Sabelli, N., Tatar, D., & Wilensky, U. (2004). *Paper presented at the SRI Cataalyst Workshop*, Palo Alto, CA: Stanford Research Institute, April 1 – 3, 2004.

Ares, N., Stroup, W., & Schademan, A. (2004, April). *Group-level development of powerful discourses in mathematics: Networked classroom technologies as mediating artifacts*. Paper presented at the American Educational Research Association, San Diego, CA.

Berland, M. & Wilensky, U. (2005). Complex play systems: Results from a classroom implementation of VBOT. In W. Stroup and U. Wilensky (Chairs) & C. D. Lee (Discussant), *Patterns in group learning with next-generation network technology*. The annual meeting of the American Educational Research Association, Montreal, Canada, April 11 – 15, 2005.

Compaine, B. M. (Ed.) (2001). *The digital divide: facing a crisis or creating a myth?* Cambridge, MA: MIT Press.

De La Cruz, Y. (1999). Social and cultural contexts in mathematics pedagogy. In F. Hitt & M. Santos (Eds.) *Proceedings of the Twentieth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environment Education.

Fuson, K. C. & Lo Cicero, A. M. (2000). El Mercado in Latino primary math classrooms. In M. L. Fernandez (Ed.), *Proceedings of the Twenty-Second Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Vol. 2* (p. 453). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Hooper, P. K. (1996). "They have their own thoughts.": A Story of constructivist learning in an alternative African-centered community school. In Y. Kafai & M. Resnick (Eds.), *Constructivism in practice: Designing, thinking, and learning in a digital world* (pp. 241–255). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Kuttan, A. & Peters, L. (2003). *From digital divide to digital opportunity*. Lanham, MD: Scarecrow Press.

Lee, C. D. (2003). Toward a framework for culturally responsive design in multimedia computer environments: Cultural modeling as a case. *Mind, Culture, and Activity, 10(1),* 42 – 61.

Roschelle, J., Penuel, W. R., & Abrahamson, L. A. (2004). The networked classroom. *Educational Leadership, 61(5)*, 50 – 54.

Stroup, W., Ares, N., & Hurford, A. (2004) *A Dialectic analysis of generativity: Issues of network supported design in mathematics and science. Mathematical Thinking and Learning, An International Journal.* Mahwah, NJ: Lawrence Erlbaum Associates.

Surowiecki, J. (2004). *The wisdom of crowds.* New York: Random House, Doubleday.

Wilensky, U. (1993). *Connected mathematics—Building concrete relationships with mathematical knowledge.* Doctoral thesis, M.I.T.

Wilensky, U. (1997). What is normal anyway?: Therapy for epistemological anxiety. *Educational Studies in Mathematics 33(2)* 171 – 202.

Wilensky, U. (1999). *NetLogo.* Northwestern University, Evanston, IL. http://ccl.northwestern.edu/netlogo/

Wilensky, U. & Stroup, W. (1999a). *Calculator–HubNet.* Northwestern University, Evanston, IL. http://ccl.northwestern.edu/netlogo/hubnet.html

Wilensky, U. & Stroup, W. (1999b). Learning through Participatory Simulations: Network-based design for systems learning in classrooms. *Proceeding of the conference of Computer-Supported Collaborative Learning*. Stanford University.

Wilensky, U. & Stroup, W. (2002). *Computer–HubNet.* Northwestern University, Evanston, IL. http://ccl.northwestern.edu/netlogo/hubnet.html