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HIERARCHY-OF-MODELS APPROACH FOR AGGREGATED-FORCE ATTRITION

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ABSTRACT

This paper presents some innovations for overcoming shortcomings in the current state-of-the-art for the hierarchy-of-models approach to modeling aggregated-force attrition in ground-combat models. The basic concept of such an approach for modeling large-scale system behavior is presented, together with the theoretical underpinnings for modeling attrition in large-scale ground combat. The output of an entity-level discrete-event combat simulation is fit to a Lanchester-type aggregated-replay model. Use of a reliable statistical-estimation technique for determining model parameters is emphasized. The main innovation is to show how use of more detailed output data (e.g. line-of-sight (LOS) data) from the high-resolution simulation allows one to develop maximum-likelihood estimates. The methodology is applied to a current high-resolution DoD combat model, and a Lanchester-type aggregated-force replay model is developed.

1 INTRODUCTION

Aggregated-force combat models are widely used in DoD for modeling military campaigns (particularly those for ground forces) to support defense decision making. The theoretical basis of such models are systems of deterministic differential equations (usually called Lanchester-type equations), which represent an approximation to the mean numbers of the various combat systems. Such differential-equation models, however, are implemented in practice as finite-difference equations for computational reasons. The simple fact is that analytical solutions are quite elusive for essentially all cases of practical interest (see Taylor (1983) for further details).

Moreover, a major problem for applications is the development of numerical values for each and every so-called Lanchester attrition-rate coefficient, the rate at which an individual weapon-system type kills enemy targets of a particular type. Two approaches for determining such numerical values are (Taylor 1983, Section 5.1)

(1) the freestanding-analytical-model approach (which generates these values from an analytical submodel, independent of any high-resolution model),
(2) the hierarchy-of-models approach (which estimates parameter values for such an attrition-rate coefficient from the output of a high-resolution Monte-Carlo combat simulation).

This paper presents methodological improvement of the state of the art for the latter approach. The hierarchy-of-models approach has also been called the fitted-parameter-analytical-model approach (e.g. the attrition-calibration (ATCAL) methodology implemented in the U.S. Army's CEM model (CAA 1983)).

Since the pioneering work of G.M. Clark (1969), no substantial theoretical improvement in the hierarchy-of-models approach has appeared in the open literature. Collecting times between casualties from a high-resolution Monte-Carlo simulation (DYNTACS), Clark had to assume that every target type on a side had the same target availability in order to implement maximum-likelihood estimation of model parameters (target availability and conditional kill rates). If one could not accept such a strong assumption, in the past there was no alternative except to abandon maximum-likelihood estimation (Stockton 1973, CAA 1983). This paper shows how to implement such MLE without having to assume that all target types on a side have the same target availability.

2 HIERARCHY-OF-MODELS APPROACH

The hierarchy-of-models approach for Lanchester-type models consists of the following

(1) output from high-resolution Monte-Carlo simulation,
(2) set of aggregated replay equations,
(3) reliable methodology for estimating parameters in replay equations from data (1),
(4) situation matching/extrapolation methodology,
(5) solution of aggregated replay equations.

This concept is depicted graphically in Figure 1.

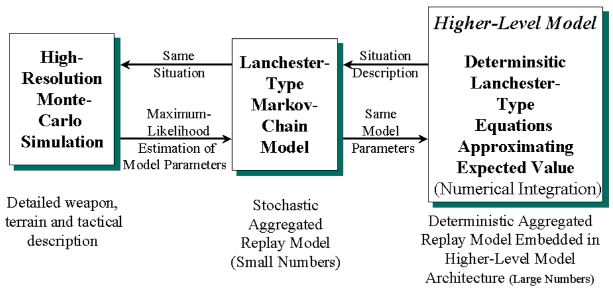

Figure 1: Hierarchy-of-Models Concept

3 AGGREGATED-REPLAY EQUATIONS

For the case of two homogeneous forces, the following Lanchester-type equations were used to replay the mean course of combat in the high-resolution simulation (Clark 1969, Yildirim 1999)

$$\begin{cases} \dfrac{dx}{dt} = -\alpha\left\{1-(1-A)^x\right\}y & \text{with } x(0)=x_0, \\ \dfrac{dy}{dt} = -\beta\left\{1-(1-B)^y\right\}x & \text{with } y(0)=y_0. \end{cases} \tag{1}$$

The constants α, β > 0 denote conditional kill rates (e.g. α denotes the rate at which an individual Y firer kills acquired X targets), while the constants A, B > 0 denote probabilities of target availability (e.g. A denotes the probability that a typical Y firer has a particular X target available to engage). However, for simplicity a target-availability probability will be referred to as "target availability." Clark's (1969) nonlinear model is depicted in Figure 2.

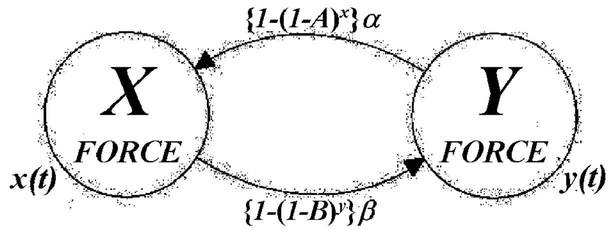

Figure 2: Clark's Nonlinear Attrition Model

Lanchester-type rate-based attrition can be played as either a deterministic process (modeled by deterministic Lanchester-type equations) or a stochastic process (modeled as a continuous-time Markov chain). However, it is basically an unresolved problem as to whether the aggregated replay model should be deterministic or stochastic for other than small numbers of combatants. For a discussion of such issues, see Clark (1969), (1982), Taylor (1983, Chapter 4). The work at hand (being more oriented towards large-scale combat than Clark's original work (Taylor 1983, Chapter 4)) considers a deterministic replay model. However, estimation of parameters in this model depends on a corresponding Markov-chain model (see Figure 1 above).

For future purposes it is convenient to record here a Markov-chain model that corresponds to the above deterministic model. This model will be used to develop maximum likelihood estimates that can be used in the deterministic replay model. Thus, the Markov chain corresponding to the above nonlinear Clark Equations (1) is given for $0<m<m_0$ and $0<n<n_0$ by

$$\frac{d}{dt}P(t,m,n) = F(m+1,n)P(t,m+1,n) + G(m,n+1)P(t,m,n+1) - \left\{F(m,n)+G(m,n)\right\}P(t,m,n), \tag{2}$$

where P(t,m,n) denotes the probability that there are m of the X combatants and n of the Y combatants alive at time t, F(m,n) denotes the total-X-force attrition rate (and is given by $\alpha\{1 - (1\text{-}A)^m\}n$), and similarly for G(m,n). The deterministic equations (1) may be thought of as representing an approximation (which is fairly poor for small numbers (Clark 1969, Taylor 1983, Chapter 4)) to the mean force levels of this Markov chain.

For the case of heterogeneous forces, the following Lanchester-type equations were used (Clark 1969, Yildirim 1999)

$$\begin{cases} \dfrac{dx_i}{dt} = -\displaystyle\sum_{j=1}^{n_Y}\alpha_{ij}\left\{\left(1-p_{ij}^{x_i}\right)\prod_{k\in I_{ij}}p_{kj}^{x_k}\right\}y_j & \text{for } i=1,\ldots,n_X, \\ \dfrac{dy_j}{dt} = -\displaystyle\sum_{i=1}^{n_X}\beta_{ji}\left\{\left(1-q_{ji}^{y_j}\right)\prod_{k\in J_{ji}}q_{ki}^{y_k}\right\}x_i & \text{for } j=1,\ldots,n_Y, \end{cases} \tag{3}$$

where α_{ij} denotes a conditional kill rate (the rate at an individual Y_j firer type kills acquired X_i target types), p_{ij} denotes the probability that a typical Y_j firer type does not have a particular X_i target type available to engage (=1-A_{ij}), I_{ij} denotes the set of indices for X target types with higher priority than X_i for Y_j, and n_X denotes the number of X target types. For doubly subscripted variables, the first subscript denotes the target type, while the second denotes the firer type. Here, A_{ij} (=1-p_{ij}) denotes the availability of X_i targets to a Y_j firer.

4 ESTIMATION OF MODEL PARAMETERS

For the case of two homogeneous forces, the four parameters α, β, A, and B for the model given by Equation (1) are to be estimated from the output of the high-resolution combat simulation. One can develop maximum-likelihood estimates (MLEs) for these parameters by considering the corresponding Markov chain given by Equation (2). If one has recorded the times at which each casualty has occurred (and also the casualty type) in the Monte-Carlo combat simulation, then one can develop MLEs for these four model parameters (Taylor 1998).

Thus, computing the natural logarithm of the so-called likelihood function and setting its first derivative with respect to α equal to zero (Taylor 1983, Section 5.15, Taylor 1998), one finds that the MLE for α is given by

$$\hat{\alpha} = \frac{c_T^{XY}}{\sum_{k=1}^{K}\left\{1 - \left(1-\hat{A}\right)^{m_{k-1}}\right\} n_{k-1}\left(t_k - t_{k-1}\right)}, \qquad (4)$$

where

$$c_T^{XY} = \sum_{k=1}^{K} c_k^{XY}.$$

Here, we have assumed that the stochastic simulation has been run until a total of K casualties has occurred, t_k denotes the time (a realization of a random variable) at which the $k^{\underline{th}}$ casualty has occurred, m_k (=$m(t_k)$) denotes the number of X combatants alive after the occurrence of the $k^{\underline{th}}$ casualty, and

$$c_k^{XY} = 1$$

if the $k^{\underline{th}}$ casualty produced by a Y firer is an X casualty and 0 otherwise (a realization of a random variable). Other quantities are similarly defined. For doubly superscripted variables (here as well for combat between heterogeneous forces below), the first superscript denotes the target type, while the second denotes the firer type. Further details about notation and also the determination of MLEs appears in Taylor (1983, Section 5.15) (also Yildirim (1999)).

Setting the first derivative with respect to A equal to zero at the MLE value of α given by Equation (4), one finds that the MLE for A satisfies the following nonlinear equation

$$f(\hat{A}) = \sum_{k=1}^{K} c_k^{XY}\left\{\frac{m_k\left(1-\hat{A}\right)^{m_{k-1}-1}}{1-\left(1-\hat{A}\right)^{m_{k-1}}}\right\} - c_T^{XY}\frac{\left[\sum_{k=1}^{K} m_{k-1}\left(1-\hat{A}\right)^{m_{k-1}-1} n_{k-1}\left(t_k - t_{k-1}\right)\right]}{\left[\sum_{k=1}^{K}\left\{1-\left(1-\hat{A}\right)^{m_{k-1}}\right\} n_{k-1}\left(t_k - t_{k-1}\right)\right]} = 0. \qquad (5)$$

Computational experience with high-resolution-simulation data has always yielded that Equation (5) has a unique real root. It is easily solved by numerical methods (Yildirim, 1999).

For the case of heterogeneous forces, one finds that the MLE for α_{ij} is given by

$$\hat{\alpha}_{ij} = \frac{c_T^{X_iY_j}}{\sum_{k=1}^{K}\left\{1-\left(1-\hat{A}_{ij}\right)^{m_{i,k-1}}\right\}\prod_{l\in I_{ij}}\left(1-\hat{A}_{lj}\right)^{m_{l,k-1}} n_{j,k-1}\left(t_k - t_{k-1}\right)}, \qquad (6)$$

where all notation is an extrapolation from that for Equation (3), except that $n_{j,k-1}$ denotes the number of Y_j firer types alive after the occurrence of the $(k-1)^{\underline{st}}$ casualty. Unfortunately, like Clark (1969) before us, we were not able to compute MLEs for target availabilities A_{ij} and B_{ji}, using only the times between casualties. However, G.M. Clark (1998) kindly suggested that this difficulty could be overcome by considering a continuous-time Markov-chain model of the line-of-sight (LOS) process and target acquisition for a particular firer-type and target-type pair from which target availability can be calculated in terms of its transition rates (for which MLEs are well known) (see also Clark (1982)). Since any function of MLEs is also an MLE (Zehna 1966), this Markov-chain model readily leads to MLEs for target availabilities.

Initially the following Markov-chain model for target availability was considered (see Figure 3). For simplicity this situation is depicted for the case of homogeneous forces, with the case of heterogeneous forces being handled in a straightforward manner by adding the appropriate subscripts. For the Markov chain depicted in Figure 3, the steady-state probability that the particular target is visible and acquired (i.e. the target is available) is given by

$$A_{ij} = p_{VA_{X_iY_j}}(\infty) = \frac{\eta_{X_iY_j}\lambda_{X_iY_j}}{\left(\eta_{X_iY_j} + \mu_{X_iY_j}\right)\left(\lambda_{X_iY_j} + \mu_{X_iY_j}\right)}.$$

Hence, an MLE for target availability, for example, is given by

$$\hat{A}_{ij} = \frac{\hat{\eta}_{X_iY_j}\hat{\lambda}_{X_iY_j}}{\left(\hat{\eta}_{X_iY_j} + \hat{\mu}_{X_iY_j}\right)\left(\hat{\lambda}_{X_iY_j} + \hat{\mu}_{X_iY_j}\right)},$$

where the MLE for η is given by the mean time that the target is in the invisible state, and similarly for μ and λ.

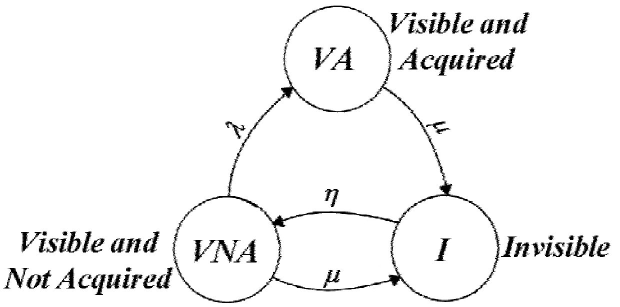

Figure 3: Markov-Chain Model for Interaction of LOS and Target-Acquisition Processes (Single Observer versus Particular Target)

Unfortunately, peculiarities in the high-resolution discrete-event simulation (Janus) that was used in this work necessitated that one had to consider transitions occurring directly from the invisible state to a target being acquired. (This point is elaborated upon at the end of this section.) Hence, the above Markov-chain model had to be modified to accommodate such transitions (Yildirim 1999). Denoting the transition rate from the state of the target being invisible to it being visible and acquired as τ, one finds that an MLE for target availability, for example, for the modified model is given by

$$\hat{A}_{ij} = \frac{\hat{\eta}_{X_iY_j}\hat{\lambda}_{X_iY_j} + \hat{\tau}_{X_iY_j}\left(\hat{\lambda}_{X_iY_j} + \hat{\mu}_{X_iY_j}\right)}{\left(\hat{\tau}_{X_iY_j} + \hat{\eta}_{X_iY_j} + \hat{\mu}_{X_iY_j}\right)\left(\hat{\lambda}_{X_iY_j} + \hat{\mu}_{X_iY_j}\right)}. \quad (7)$$

Once target availabilities have been estimated by equations like Equation (7), one can readily estimate conditional kill rates with equations like Equation (6).

Janus (as well as many, if not most, other simulations of ground combat) uses both the time-step method (employing a hierarchy of so-called sensor scans and sweeps) for the target-acquisition process and the event-step method for essentially all other processes. The use of such time steps for simulating target acquisition is caused by the fact that the range between an observer and a particular target is usually continuously changing and acquisition rates depend upon this range (see Yildirim (1999) for further details). Because of this time-step scheme for target acquisition (with time steps so small that acquisition rates, through their dependence on observer-target range, could be safely assumed to be constant), target acquisition could occur in one sensor scan after the target had become unmasked. Such target acquisition had to be played as occurring directly from the invisible state because a sensor scan was negligible with respect to the length of time that a target was invisible.

5 DATA REQUIRED FROM HIGH RESOLUTION SIMULATION

For the case of two homogeneous forces, in order to compute the MLEs given by Equations (4) and (5) one must first collect the following data from the output of the high-resolution simulation: the time and casualty type (i.e. whether X or Y casualty) for all casualties. All high-resolution combat simulations routinely produce such output.

For the case of heterogeneous forces, it is convenient to first compute the MLEs for the Markov-chain model for the LOS and target-acquisition process for every (firer-type)-(target-type) pair from which the MLEs for target availability can be computed. In the simplest case, one must first collect the following data from the output of the high-resolution simulation in order to estimate the transition rates of this Markov chain (see Figure 3):

(1) the time at which a target in the invisible state becomes visible,
(2) the time at which a visible target becomes invisible,
(3) the time at which a visible target becomes acquired.

The above data must be collected for every (firer-type)-(target-type) pair. Additionally, peculiarities of the high-resolution combat simulation (Janus) required the collection of

(4) the length of time that a target was invisible for a target that was acquired in one sensor scan.

The mean value of this quantity then estimated the rate τ at which targets formerly invisible were acquired. One must also (of course) collect the following data:

(5) the time, casualty type, and shooter type for all casualties.

Only this latter data is routinely produced by high-resolution combat simulations.

6 THE JANUS SIMULATION

The high-resolution combat simulation that was used in the work reported here is called Janus, which was originally developed for the U.S. Army by the Lawrence Livermore National Laboratory (LLNL). Janus is an interactive, six-sided, closed, Monte-Carlo simulation for ground combat between essentially battalion-sized units and smaller. It is called interactive because a military analyst must control the position and movement of forces and also input decisions as to what to do during critical situations of the scripted combat. Direct-fire engagements can be entirely scripted so that Janus can be run in an automatic mode for them. However, the user must run Janus in the interactive mode to plan artillery missions or move his forces in response to enemy actions. Entities in Janus are individual weapon systems (e.g. tank, machine gun, dismounted infantry soldier, etc.). Janus has situation displays, one for each side in combat. On a situation display for a particular side is shown the location of each friendly unit and those enemy units detected by these friendly units. Janus is written in FORTRAN with some C routines. The version of Janus used for this research was Version 6.88. Further details about Janus can be found in Yildirim (1999).

7 TARGET ACQUISITION IN JANUS

Target acquisition in Janus requires further discussion, since it is a complex, compound process and major difficulties were encountered in obtaining the data (1) through (4) discussed in Section 5 above from Janus for the case of heterogeneous forces. First of all, Janus uses time steps for the simulation of the target acquisition process in order to represent the effect of moving targets on detection rates and hence target-detection process. (Ironically, except for Taylor's (1985) methodology, the authors know of no methodology for estimating such detection rates from experimental data for moving targets. This fact would cast doubt that the rates played in Janus for moving targets are based on any type of empirical evidence. Also see discussion at end of Section 4 above.) Here the term "detection rate" refers to the rates for transitioning to the various levels of target recognition discussed below. In Section 4 and above in this paper the term acquisition rate has been used for simplicity.

Janus considers that an observer can obtain different distinct levels of knowledge about a target (referred to as levels of target discrimination). Thus, Janus plays four levels of target discrimination

(1) detection (target detected at such a level of resolution/discrimination that observer can distinguish an object of military interest that is foreign to the
(2) background in its field of view, e.g. distinguish a vehicle from a bush),
(3) aimpoint (target detected at such a level of resolution/discrimination that observer can distinguish an object by its class, e.g. a tracked vehicle versus a helicopter or a wheeled vehicle; observer can thus establish an aimpoint on the object),
(4) recognition (observer can categorize targets discriminated at aimpoint within a given class, e.g. recognize a tank versus an armored personnel carrier in the tracked vehicle class),
(5) identification (observer can distinguish between specific recognized target models, e.g. a T-72 tank versus a T-80).

Every time the level of knowledge about a target changes, Janus decides whether or not to engage the target, based on rules of engagement (referred to as firing criteria). There are three different rules of engagement that can be played in Janus. These are

(1) engage any target that you have detected and can aim at (aimpoint firing criterion),
(2) engage any target that you have recognized (recognition firing criterion),
(3) engage any target that you have identified (identification firing criterion).

Once such a level of target discrimination has been reached that the rule of engagement allows the target to be engaged, the target is considered to be acquired. Hence, playing a different rule of engagement results in different output from Janus. The combination of level of discrimination and rule of engagement determines when a target is considered to be acquired in Janus (Yildirim 1999, pp. 63-64). Further details are to be found in Yildirim (1999, Chapter III).

8 SCENARIO CONSIDERED

A hypothetical combat situation was developed and run on Janus. The scenario was for an attack of two US-equipped mechanized task-force battalions (Blue), with 8 combat engineer vehicles and supported by 12 155mm self-propelled howitzers against a static defense of two Soviet-equipped armored companies (Orange) supported by 6 self-propelled artillery units. Blue had a total of 132 units, while Orange had 38 units, resulting in more than a 3:1 ratio in favor of the attacker. The artillery fire of each side was pre-planned for simplicity and for independence of the simulation runs with a no-man-in-the-loop design. Hypothetical values for weapon-performance data and other inputs were developed based on military judgement.

The initial disposition of the two forces is shown in Figure 4. Further details are to be found in Yildirim (1999).

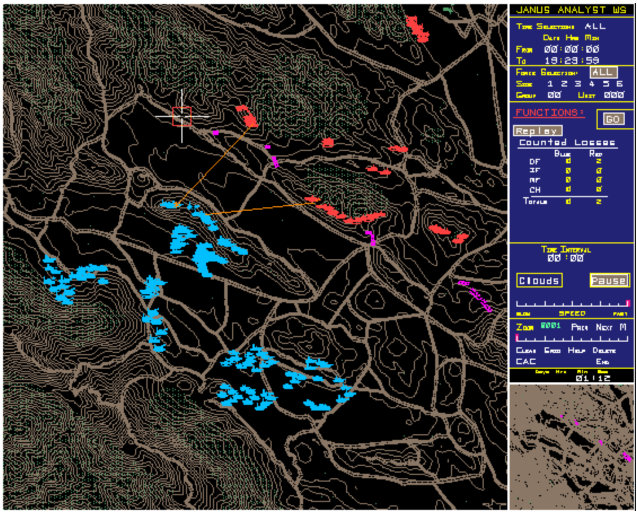

Figure 4: Initial Disposition of Forces at Start of Battle Considered in the Work at Hand

9 DIFFICULTIES ENCOUNTERED WITH JANUS

Janus outputs a file for each detection event and also each firing event. For the work at hand only firing events that resulted in a kill needed to be considered. Moreover, detection means that a change from one level of target discrimination to another had occurred (see Section 7 above). Such detection events (including when a target became unmasked so that LOS existed) had to be considered at each time step of the target-detection scans. Although voluminous, the files for these detection events could generate all the times required to estimate the transition rates of the LOS-target acquisition Markov chain except the times at which LOS was lost for an observer-target pair (see Section 5 above). Obtaining appropriate data to be able to generate the times at which LOS was lost was a major difficulty encountered in the work reported here, as well as the complexity of the program required to calculate the MLEs for estimating the Markov-chain transition rates.

Although it existed internally to Janus computer program, the Janus code had to be modified to obtain appropriate data for generating the times at which LOS was lost. Thus, the Janus code was modified to generate special text files (referred to as modified detection files) to replace the standard post-processing files for detection. These text files were then processed by a JAVA program that generated the appropriate transition times (see Section 5 above), MLEs for the Markov-chain transition rates, and finally target availabilities required for the estimation of conditional kill rates by formulas similar to Equation (6). Only because the TRADOC Analysis Center (TRAC)-Monterey had been involved in similar work that involved modification of the Janus code, was it possible to generate these modified detection files. (In particular, TRAC-Monterey was involved in a project that included reprogramming Janus from FORTRAN to C++.) See Yildirim (1999, Chapter IV, Section 1) for further details.

10 DATA REDUCTION

For the case of heterogeneous force, two JAVA programs were developed to process the Janus output files. The first JAVA program takes the modified detection files from Janus as input. It first computes MLEs for the transition rates in the Markov-chain model for LOS and target acquisition and then uses them to compute MLEs for target availabilities, which are then manually imported into the Excel spreadsheet that computes MLEs for the conditional kill rates. The second JAVA program takes the standard kill files from Janus as input and computes the times between kill events and keeps track of the numbers of remaining entities in the high-resolution simulation (cf. Equation (6)). The program exports this information via a text file to an Excel spread sheet (combining it with MLEs for target availabilities) that computes MLEs for the conditional kill rates. Further details can be found in Yidirim (1999, Chapter V).

11 NUMERICAL RESULTS

The scenario described above was input into Janus. It was replicated 20 times by using different starting random-number seeds for each of two so-called firing criteria (i.e. rules of engagement). Numerical results for the scenario described above for one of the rules of engagement (identification firing criterion) are shown in Figure 5 below. Force levels computed according to the aggregated replay model given by Equation (3), denoted as Lanchester attrition results, are shown in these figures, as is a realization of such force levels for a particular Janus run. In Figure 5, in the legend on the right-hand side are given the notations for each of the various weapon systems on each side involved in the scenario. For each such unit, the first letter (either a “J” for the Janus run or an “L” for the replay model) denotes whether the force level is for the aggregated-replay model (Lanchester-type attrition) or for the realization of the Janus run. Further details are to be found in Yildirim (1999).

12 FINAL COMMENTS

This paper has presented the salient features of the hierarchy-of-models approach to modeling aggregated-force attrition. It has presented an important methodological advance in this approach by showing how

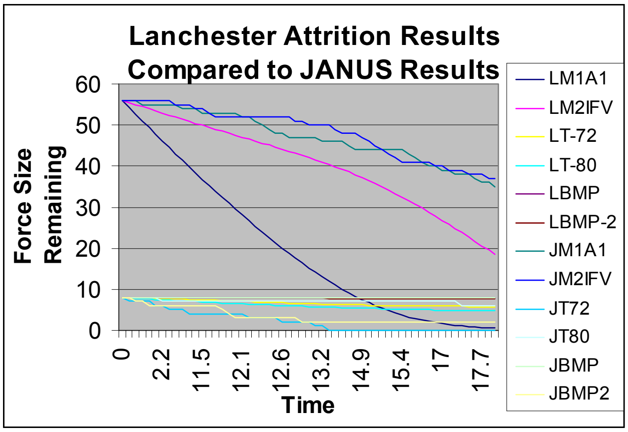

Figure 5: Force-Level Decays for Aggregated-Replay Equations (3) Compared with Realization of Original Janus Battle for Identification Firing Criterion

maximum-likelihood estimates (MLEs) can be obtained for parameters in heterogeneous-force replay equations (3) if more-detailed LOS/acquisition data is extracted from the high-resolution combat simulation. Unfortunately, current high-resolution simulations (such as Janus that employs a time-step approach for simulating target acquisition) do not explicitly keep track of the times at which LOS is established or lost for specific target-observer (unit) pairs. This situation is quite understandable, since there was never any requirement for recording such data.

Obtaining such data for the work at hand was only possible due to the fact that TRAC-Monterey had recently done some projects that had required detailed knowledge of the Janus code. Even then, data reduction to compute MLEs turned out to be a Herculean task. Until it is recognized that there is a need for a theoretically-sound hierarchy-of-models approach (i.e. a need for obtaining statistically-sound estimates for replay-model parameters) and that there is a need for this approach for models to be used by DoD, it is unlikely that such LOS data will even be output from entity-level combat simulations. In turn, one will not in practice be able to use the methodology presented here. Unfortunately, the DoD modeling community has not been aware of this situation.

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