A PERSPECTIVE OF BATCHING METHODS IN A SIMULATION ENVIRONMENT OF MULTIPLE REPLICATIONS IN PARALLEL

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ABSTRACT

Discrete event simulation is frequently time-consuming either because modern dynamic systems, such as telecommunication networks, are becoming increasingly complex and/or a great number of observations is required to yield reasonably accurate results. An interesting approach to reduce the time duration of simulation is that of concurrently running multiple replications in parallel (MRIP) on a number of processors connected via networking and averaging the results adequately. We present the results of our research on the suitability of batch-means-based procedures in such distributed stochastic simulation.

1 INTRODUCTION

Analysis of output data from steady-state discrete-event simulation has attracted a considerable attention. A sound methodology can be found in the literature (refer to Pawlikowski (1990) for a thorough review of problems and solutions survey), but there are still open questions that deserve attention, especially concerning the quality of the results produced by a plethora of methods proposed for simulation output data analysis.

Estimation of results during steady-state simulation is a nontrivial problem because the output data are neither independent nor identically distributed, but usually highly correlated. In order to apply the classical statistical analysis, one can (for example) group observations in the output sequence in such a way that means over groups of observations, are nearly uncorrelated. Such transformation of original output sequences is applied in a class of methods of simulation output analysis known as Batch Means.

Batch Means techniques are widely used due their conceptual simplicity and intuitive implementation. It is based on the concept that if batches are large enough then their mean values can be practically uncorrelated. The main challenge of this method is the determination of batch size; see e.g. Schmeiser (1982).

The classical nonoverlapping batch means estimator (NOBM) is constructed by dividing a sequence $\{X_i\}$ of n steady-state observations, into b contiguous batches of size m. Provided that correlations among batch means can be considered negligible at a certain significance level β, the interval estimator for a performance parameter μ is given as

$$P(\bar{X}(n) - H \le \mu \le \bar{X}(n) + H) = 1 - \alpha \qquad (1)$$

where $\bar{X}$ is the arithmetic mean of the sample of size n, and H is the half-length of the confidence interval defined as

$$H = t_{df,1-\frac{\alpha}{2}} \hat{\sigma}[\bar{X}]. \qquad (2)$$

The standard deviation of the batch means estimator of μ is $\hat{\sigma}[\bar{X}(n)]$, and $t_{df,1-\frac{\alpha}{2}}$ is the upper $(1-\frac{\alpha}{2})$ critical point of the t distribution with df degrees of freedom.

In NOBM, df equals b-1. In this paper, we discuss three additional sequential methods based on Batch Methods that can be run under MRIP, and use estimators with more degrees of freedom than in the case of NOBM. This suggest that they could require shorter simulations to get final results with an acceptable statistical error.

In the following sections, our discussion is based on the results from sequential stochastic simulation of an $M/M/1/\infty$, in which the mean waiting time was esti-

mated, with the relative statistical error not greater than 5%, at 95% of confidence level.

The main performance criterion for a confidence interval procedure is its coverage, defined as the frequency of the confidence intervals containing the true parameter μ. Usually, the coverage is assessed on a fixed-sample basis, but we are of the opinion that coverage analysis should be analysed sequentially. Therefore, we have applied the sequential coverage analysis proposed in (Pawlikowski et al. 1998). Following this approach, the coverage analysis begins when a minimum number (we assumed 200) of bad confidence intervals, i.e. confidence intervals that do not contain theoretical value, are collected, and results from too short runs are discarded.; It stops when the relative precision of the confidence intervals for the coverage is less than 5%.

2 SEQUENTIAL ANALYSIS

Sequential procedures of simulation output data analysis are widely recognized as the only effective techniques for controlling the final precision of simulation results. In the sequential procedures analysed here we apply sequential versions of batch-means-based techniques together with sequential test for detecting the length of the initialtransient period, proposed by Schruben (1982).

In NOBM, in its version presented in Pawlikowski (1990), for reducing dependence between batch means, correlation coefficients for lag k (k=1, ..., n) are tested sequentially. If all these correlations cannot be considered negligible at a significance level $\beta = 0.01$, more observations are collected and the test is repeated.

To improve the quality of this test, we applied a nonparametric method based on jackknife estimators, for calculating the correlation coefficients.

When independence test succeeds, observations are grouped into 25 batches according to Schmeiser's results, and estimation phase initiates. From now on, when NOBM collects a batch of size m^*, the optimal batch size found in the previous phase, a checkpoint is reached and, if desired relative precision is detected, simulation stops. This sequential method behaves acceptably when system load $\rho < 90\%$, but as load increases its quality worsens (see Table 1).

This asymptotic failure was expected by Glynn and Whitt (1991), when they showed that "there is no variance estimator based on a fixed number of batch that is consistent." In light of that, we introduced a variant of NOBM to get rid of this *anomaly*, and called it NOBM/GW : at each checkpoint, when the relative precision is not achieved, the number of batches increases by 2. This increment is enough to keep in touch with Schmeiser's findings and, at the same time, to guarantee a better asymptotic behavior. Table 1 shows that NOBM/GW indeed improves somewhat

Table 1: Performance of Batching Methods under MRIP: cov $\pm H$ Is a Confidence Interval for the Coverage; E[O] Is the Average Sample Size, and CoV{H} Is the Coefficient of Variation of the Confidence Interval Half-length for the Coverage

NOBM			
ρ (%)	cov $\pm H$	E[O]	CoV{H}
91	91.10 ± 1.4	545.27	0.0390
92	88.90 ± 1.7	661.25	0.0393
93	87.60 ± 1.9	829.99	0.0398
94	86.60 ± 2.0	1079.23	0.0407
95	86.00 ± 2.2	1474.18	0.0381

NOBM/GW			
ρ (%)	cov $\pm H$	E[O]	CoV{H}
91	90.06 ± 1.5	557.78	0.0297
92	88.80 ± 1.7	674.15	0.0307
93	88.60 ± 1.7	851.09	0.0313
94	88.40 ± 1.8	1095.49	0.0320
95	86.50 ± 2.0	1501.23	0.0339

SBM			
ρ (%)	cov $\pm H$	E[O]	CoV{H}
91	91.20 ± 1.4	559.37	0.0294
92	89.90 ± 1.5	677.01	0.0308
93	89.20 ± 1.6	858.13	0.0313
94	88.10 ± 1.8	1101.58	0.0318
95	87.60 ± 1.9	1505.77	0.0335

OBM			
ρ (%)	cov $\pm H$	E[O]	CoV{H}
91	94.90 ± 0.8	747.76	0.0252
92	94.50 ± 0.9	925.23	0.0257
93	95.20 ± 0.8	1191.40	0.0254
94	94.70 ± 0.9	1573.26	0.0259
95	94.20 ± 1.0	2210.58	0.0258

the coverage in very high-loaded systems, when compared to classical NOBM.

As an attempt to weaken the strong correlations, we also designed and implemented a sequential version of a fixed-sample size technique proposed by (Fox et al. 1991), in which one discards some observations between consecutive batches. This is the so-called *Spaced Batch Means* (SBM) technique. As in NOBM, the number of batches increases in the estimation phase, if needed.

When the number **s** of discarded observations is **s**=0, we have the classical NOBM. Theoretically, the greater **s**, the better should be the coverage of final results, but that imposes an obvious problem of throwing out many observations. We have adopted a spacing equal 20% of the initial batch size but the best solution would probably be to determine **s** according to the properties of the underlying

stochastic process. Table 1 shows that, in the reported cases, SBM is a bit better in the sense of coverage than NOBM and NOBM/GW.

Finally, we implemented a sequential version of *Overlapping Batch Means* (OBM), proposed by Meketon and Schmeiser (1984), which reuses some observations of one batch for constructing the next (overlapping) batch of observations. The original work suggested that after finding m^*, each observation should initiates a new (overlapped) batch. Of course, batch means are becoming strongly correlated but the number of batch means is much larger and that compensates the negative effect of correlation. Welch (1987), studied the possibility of achieving the same results with partial overlapping.

One can clearly see in Table 1 that, in terms of coverage, OBM behaves much better than the other methods. Its final confidence intervals are more stable and achieve almost perfect coverage for high values of ρ.

3 MULTIPLE REPLICATIONS

Research on speeding up execution of simulation models is a challenging problem which has attracted a considerable scientific interest and effort. A simple yet effective way to exploit computer networks for computationally intensive discrete-event simulation is to run multiple independent replications in parallel (Pawlikowski 1994), on multiple processors and to average the results appropriately. Of course, each replication should be run using different, independent sequence of (pseudo) random numbers. This way, by using P processors simulation output data can be generated P times faster. The only communication overhead of MRIP is associated with loading of the model into different processors (at the beginning of simulation) and with transmissions of data to a central analyser whenever a checkpoint is reached, to calculate global estimates. This approach is statistically efficient provided that the initialization bias is not severe (Heidelberger 1986).

MRIP approach offers a quite simple solution concerned with the credibility of the final simulation results. Refer to (Pawlikowski et al. 1994), and (Mota et al. 1999, 2000) for other investigation issues in these area.

4 ACHIEVABLE SPEEDUP

According to Amdahl's law (1967), if a fraction f of a computation is inherently sequential, then the speedup S(P) is bounded above by

$$S(P) = \frac{1}{f + \frac{1-f}{P}} \qquad (3)$$

where P is the number of processors and f is defined to be the ratio of the service demand of sequential parts

of the computation to the service demand of the entire computation.

In steady-state simulation, considering that results are analyzed sequentially, MRIP imposes a limit to the average speedup that should be incorporated into above expression.

Let N_{min} be the number of collected observations, sufficient for achieving the required precision of the final results. One could think of a situation in which there are so many processors employed that each one achieves just the first checkpoint only, and the stopping rule is reached. Let this number of processors be P_{min}.

Let N_o be the length of the transient phase, N_1 be the amount of observations collected until the first checkpoint is achieved, and D be the distance between consecutive checkpoints (Figure 1). A truncated version of Amdahl's law for the MRIP scenario, formulated in (Pawlikowski and McNickle 2000) states that:

$$S(P) = \begin{cases} \dfrac{1}{f + \frac{1-f}{P}} & if \quad P \le P_{min} \\[3mm] \dfrac{1}{f + \frac{1-f}{P_{min}}} & \text{otherwise} \end{cases} \qquad (4)$$
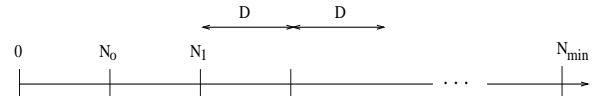

Figure 1: Steady-State Structure

In Batch Means techniques, observations collected during initial transient phase are not used by steady state estimates, thus:

$$f = \frac{N_0}{N_{min}}$$

Note that if D means the number of observations collected between consecutive checkpoints, then P_{min} times D observations is needed to stop the simulation. That is

$$P_{min} = \frac{(1-f)N_{min}}{D} = \frac{N_{min} - N_0}{D} \qquad (5)$$

Considering the classical NOBM, NOBM/GW and SBM, D is at least equal to m^*,

OBM offers greater flexibility as D can be smaller than m^*. Namely, in the case of complete overlapping, the distance between checkpoints can be, theoretically, as short as 1. Thus, for f remaining the same as in the previous methods, P_{min} can be considerably greater in OBM.

5 EMPIRICAL INVESTIGATION

The following discussion will be based on the results obtained by AKAROA-2, an MRIP implementation developed

at the University of Canterbury, New Zealand, and used at the Technical University of Berlin, Germany.

5.1 Speedup

To assess the average speedup obtained when the sequential batching techniques considered are applied in *Multiple Replications in Parallel* scenario, we simulated an $M/M/1/\infty$ queuing system, with traffic intensity 95%, and constructed a confidence interval at 95% of confidence level.

A specific feature of sequential techniques is that randomness of data collected at the output of the model being analyzed can fortuitously yield the stopping condition much earlier than it could be expected, and that can lead to wrong results. In light of that, we adopted a rule of thumb proposed by (Ruth Lee et al. 1999). Namely, while using P processors we:

1. run the simulation experiment 3 times;
2. accepted the results produced by the longest simulation run only;
3. recorded the average length of the transient phase, measured by the number of transient observations N_o discarded by each of P processors;
4. recorded the average number of observations N_1 required to achieve the first checkpoint;
5. recorded the average total number of observations N_{min} when simulation was stopped.

Each time when these steps were followed, we obtained an n-uple (N_o, N_1, N_{min}). To improve the accuracy of the results we repeated the above sequence 100 times and averaged the results at the end. We repeated the whole experiment for P=2, 4, 6, 8, 10, 15 and 20 processors, as shown in Table 2.

P_{min}, the number of processors that could still give a speedup, was calculated from the truncated Amdahl's law, using the results obtained from simulations on P=1 processors.

5.2 Granularity

In AKAROA-2, each simulation engine collects a number of observations before calculating an estimate that is sent to a global analyzer. In the case of methods based on *batching*, one should wait until a number of batches are collected since only then an estimate can be obtained.

The complete overlapping version of **OBM** offers an attractive alternative in terms of batching methods under **MRIP**, as each new observation can be used to form a new overlapped batch, and an estimate can be obtained.

To investigate the complete overlapping version of **OBM**, by varying the distance D between checkpoints, we have simulated an $M/M/1/\infty$ queuing system, with

Table 2: Achieved Speedup S(P) by Varying the Number of Replications in Parallel

			D = m*		
P	P_{min}	N_o	N_1	N_{min}	S(P)
1	832	626	85300	710110	1.000
2	832	707	78100	698875	1.998
4	832	602	76658	715507	3.989
8	832	651	64322	665697	7.946
10	832	648	63628	666770	9.913
15	832	641	62459	656010	14.798
20	732	638	63496	659464	19.639

			D = m*/10		
P	P_{min}	N_o	N_1	N_{min}	S(P)
1	8655	629	81800	708609	1.000
2	8655	666	72550	709144	1.998
4	8655	617	78913	732444	3.990
8	8655	656	71787	725743	7.950
10	8655	647	67763	698836	9.917
15	8655	673	65203	682858	14.796
20	8655	639	66727	686178	19.652

			D = m*/50		
P	P_{min}	N_o	N_1	N_{min}	S(P)
1	43551	617	81600	711375	1.000
2	43551	661	81650	722998	1.998
4	43551	647	77000	743142	3.990
8	43551	671	76001	749429	7.950
10	43551	660	69405	721601	9.918
15	43551	664	69427	708131	14.806
20	43551	635	60686	629966	19.624

traffic intensity 95%, stopping the simulation when the relative precision reached 5% or less. Figures 2, 3 and 4 summarize this experiment.

Figure 2 shows the rate within which a sequential confidence interval procedure based on OBM converges to the desired relative precision. Clearly, this convergence becomes slower as the distance D between consecutive
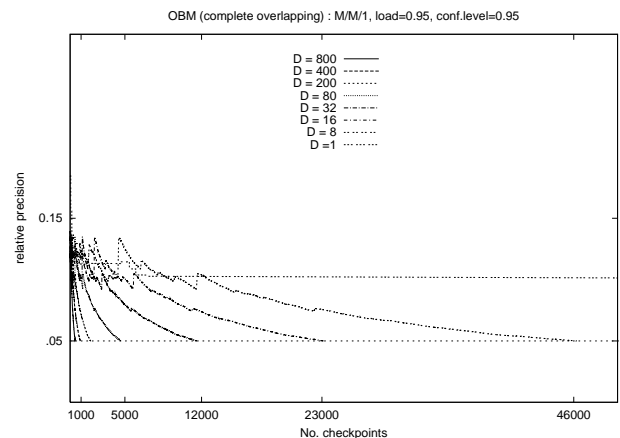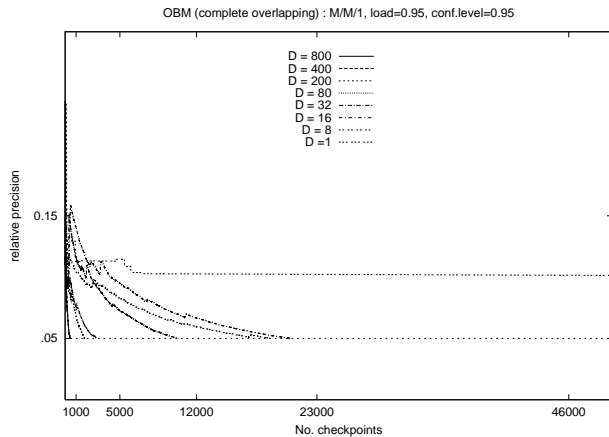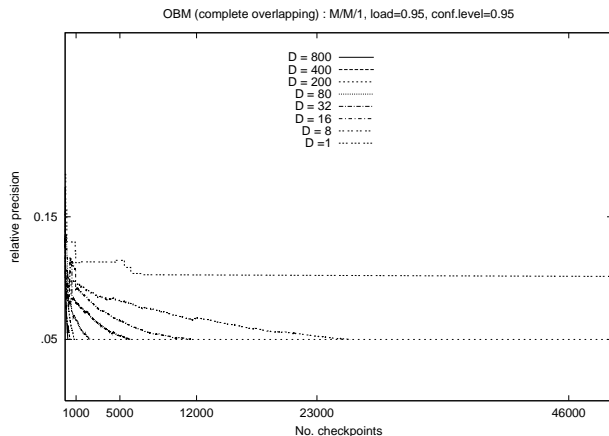

Figure 2: Convergence of Relative Precision : P = 1

OBM (complete overlapping) : M/M/1, load=0.95, conf.level=0.95

Figure 3: Convergence of Relative Precision : P = 2

OBM (complete overlapping) : M/M/1, load=0.95, conf.level=0.95

Figure 4: Convergence of Relative Precision : P = 4

checkpoints decreases (D is measured in the number of observations). Either the variance increases as we try to reduce the spacing between consecutive checkpoints, or the variance reduces very slowly. Figures 3 and 4 show the same effect but the degree of parallelization was increased, and convergence was somewhat faster than for P=2 and P=4, respectively.

6 FINAL REMARKS

Our investigation of methods based on the concept of Batch Means show that OBM is very robust in the sense that it yields confidence intervals with probability close to the nominal confidence level, especially when traffic intensity is very high. An analysis of the performance of this method applied to queuing networks can be found in (Fitzek et al. 2000).

MRIP is a promising, though somewhat unpopular, approach for speeding up stochastic simulation experiments of complex dynamic systems. We have applied this methodology on simulation of modern telecommunication problems,

and we have gotten very impressive results in terms of both speedup and quality of final results. The additional effort required from the analyst was minimal, since MRIP implementation in AKAROA-2 appeared to be very user-friendly. Currently, we are investigating other methods of simulation output data analysis, including those based on the concepts of *standardized time series*.

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