# MONTE-CARLO STATISTICAL MODELLING METHOD USING FOR INVESTIGATION OF ECONOMIC AND SOCIAL SYSTEMS

**Vladimirs Jansons, Vitalijs Jurenoks, Konstantins Didenko (Latvia)**

## 1. THE COMMO№ SCHEME OF USI№G OF TRADITIO№AL METHOD OF STATISTICAL MODELLI№G

Using traditional methods of statistical modelling for investigation of economic and social systems it is possible to set the task of creating an efficient procedure for generating incidental parameter values constituting factors of a simulation model, to effectively use up-to-date information technologies, to ensure continuous control of the behaviour of the specific economic system that is being researched. The traditional scheme of simulation modelling is the generation of a mass of incidental parameter values featuring the changes of model factors (Figure 1).

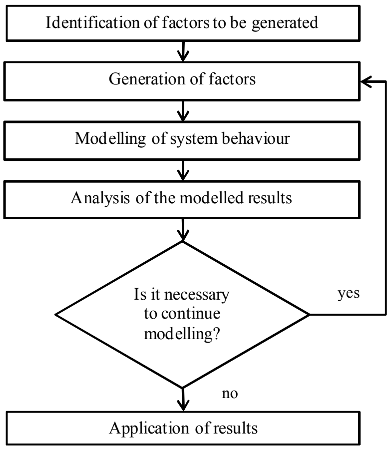


**Figure 1. Algorithm of generation of incidental parameters**

The algorithm of generation of incidental continuous value X (Figure 1), having continuous distribution function F, can be described in the following steps:

1. Let us generate, within an interval (0.1), an evenly distributed incidental parameter u ~U(0.1).
2. Let us calculate $X = F^{-1}(u)$.

The value of $F^{-1}(u)$ will always be definite, since $0 < u < 1$, but the area of defining the function F is the interval [0,1]. The figure below presents the essence of the algorithm graphically; here incidental value may be assumed to be either positive or negative. This depends on the specific value of parameter u. In the figure, the value of parameter $u_1$ produces a negative incidental value $X_1$, but parameter $u_2$ yields a positive incidental value $X_2$ (Figure 2).

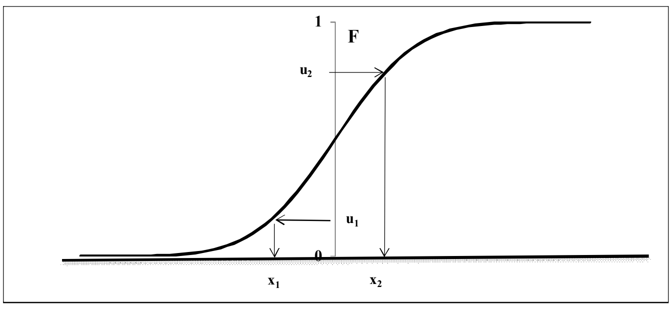


**Figure 2. Scheme of reverse transformation**

The method of reverse transformation may be also used if value X is discrete. In this case the distribution is as follows:

$$F(x) = P\{X \leq x\} = \sum_{x_i \leq x} p(x_i), \qquad \textbf{(1)}$$

where $p(x_i)$ is probability $p(x_i) = P\{X = x_i\}$.

It is admitted that incidental parameter X may have only such values as $x_1$, $x_2$, ..., for which $x_1 < x_2 < ....$

Thus the algorithm of developing the values of incidental parameter X will have the following consequences:

- Let us generate, within the interval (0.1), uniform distributed incidental parameter u ~ U(0. 1);
- Let us establish the least positive round value I, for which $u < F(x_i)$, and assume that $X=x_i$.

**Both options of the method of the reverse transformation for continuous and discrete values (at least formally) can be combined in one formula:**

$$X = \min P\{x : F(x) \geq U\}, \qquad \textbf{(2)}$$

**which is true also for mixed distributions (i.e., containing both continuous and discrete components). In contrast to commonly used direct methods of generating incidental values (the method of the reverse transformation composition and implosion), for imitating the factors of the simulation model it is recommended to use the so-called indirect methods, namely, the acceptance-refusal method. This method may turn out to be suitable if due to certain reasons it is impossible to apply direct methods or if these methods are inefficient.**

## 2. STOCHASTIC MODELLI№G OF I№SURA№CE I№ AGRICULTURE

Let us to describe the using of stochastic modelling method in insurance. In order to ensure steady growth of agricultural production, especially in the private sector, it is necessary to introduce insurance products with regard to operation of agricultural enterprises (Figure 3).

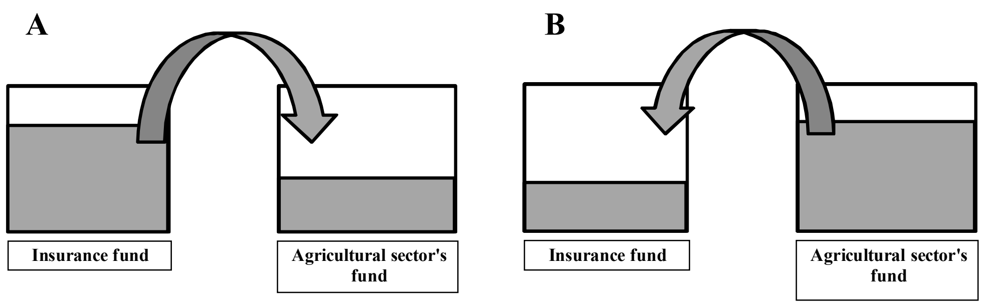


**Figure 3. In unfavourable years (case A) the insurance fund allows to stabilize the agricultural sector, in favourable years (case B) the insurance fund can be increased using funds from the agricultural sector**

Let us consider the modelling scheme of the agricultural insurance fund, which later will allow us to model the process of developing the model and to establish the minimum amount of the insurance fund U (without a state subsidy). The minimum fund amount U guarantees that with a certainty γ (probability γ) agricultural losses will be compensated. For modelling the insurance fund, we will use the simplest individual risk modelling scheme. Let us assume that the insurance fund is satisfactory, given the following conditions:
The number of registered farms in the fund is constant;
Risks of individual farms are independent;
Payment of premiums is effected at the beginning of the period;
The loss distribution function is equal for all farms.
Let us designate that:
n – number of agreements in the fund;
j – ordinal number of the farm;
p – probability of setting in of the insurance event;
$Y_j$ – possible losses of the farm j. Value $Y_j$ has probability distribution function F(x);
$X_j$ – satisfied loss of the farm j. $X_j = Ind_j*Y_j$.
$Ind_j$ – binary index of the insurance event of the farm j:
By using variable Ind, we can calculate total number № of farms incurring losses:

$$N = \sum_{j=1}^{n} Ind_j \qquad \textbf{(3)}$$

Total amount of losses is:

$$Z = X_1 + X_2 + ... + X_n \qquad (4)$$

Or by using indices of setting in of the events:

$$Z = Ind_1 \cdot Y_1 + Ind_2 \cdot Y_2 + ... + Ind_n Y_n = \sum_{j=1}^{n} Ind_j \cdot Y_j \qquad \textbf{(5)}$$

Figure 4 shows that total losses are formed in n farms during one time period.

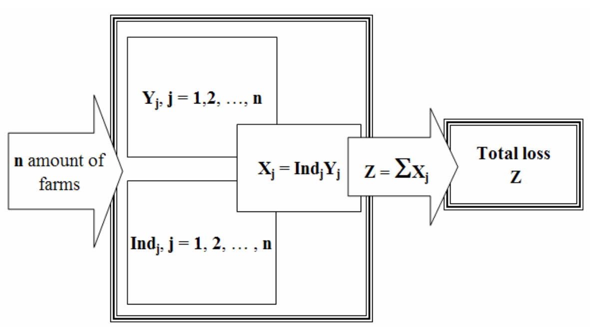


**Figure 4. Illustration of process of loss formation**

We are to compensate losses to farms with a certainty γ and are to ensure the required operation of the fund with cash funds L. It means that the amount of the fund after compensations must be positive with a certainty γ ( $P(U - Z \geq 0) = \gamma$ ). The degree of risk (stability) of the insurance fund can be established by the variation coefficient:

$$K_{\text{var}}(Z) = \frac{\sigma(Z)}{E(Z)} = \frac{\sqrt{D(Z)}}{E(Z)} \tag{6}$$

where $\sigma(Z)$ – standard deviation from the amount Z (standard error);

E(Z) – mathematical expectation of value Z, which in practice is measured with average value of Z;

D(Z) – variation of value Z.

If the number of farms in the fund is big, it is possible to use the central marginal theorem and to establish value U. Let us consider the inequality:

$$U - Z \geq 0, \tag{7}$$

which has to be valid under probability γ:

$$P(U - Z \geq 0) = \gamma. \tag{8}$$

From the inequality $U - Z \geq 0$ derives inequality $Z \leq U$, and after that inequality $Z - E(Z) \leq U - E(Z)$. Dividing it by a positive value σ(Z), we obtain:

$$\frac{Z - E(Z)}{\sigma(Z)} \leq \frac{U - E(Z)}{\sigma(Z)}. \tag{9}$$

Value $S = \frac{Z - E(Z)}{\sigma(Z)}$ normally distributed with E(S) = 0 and σ(S) = 1. Then the following formula can be applied:

$$P(S \leq \alpha) = \int_{-\infty}^{\alpha} \frac{1}{2\pi} e^{-\frac{x^2}{2}} dx \tag{10}$$

From the equation $\frac{U - E(Z)}{\sigma(Z)} = \alpha(\gamma)$, the required insurance fund amount U is

$$U = \alpha(\gamma)\sigma(Z) + E(Z) \tag{11}$$

The amount of the insurance coverage in cereal sowings insurance depends on the average amount of crop received by years, in which no relevant losses took place. The calculations show that very often variation coefficient $K_{var}$ fluctuates within the range from 20% to 50%, which testifies to the fact that insurance fund, is often not so stable and additional financing is required from the state. In the age of up-to-date information technologies the application of the Monte-Carlo statistical method is simple and frequently allows avoiding from complicated theoretical calculations as well as allows obtaining sufficiently accurate practical results to take appropriate decisions on insurance parameters.

## CO№CLUSIO№

The application of statistical modelling is connected with the fact that frequently it is not possible to provide a definite description of the behaviour of the economic and social system being investigated. When investigating the dynamic behaviour of the economic and social system, i.e. by making definite changes of parameters of the system under investigation, we frequently observe the existence of incidental factors affecting the character of the behaviour of the system. In addition, it should not be forgotten that the very character of the research also brings its incidental elements into the research process. The process of investigation of economic and social systems using statistical modelling, dynamic programming and Monte Carlo method allows set of alternative strategies supports stable functioning of economic and social systems in the conditions of uncertainty. The theoretical and practical results obtained as a result of this research can be applied in practical activities of companies for making effective decisions.

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